Теорема. *Система дифференциальных уравнений* (1), (2) *удовлетворяет формальному тесту Пенлеве.*

Рассмотрен вопрос о сходимости полученных при использовании метода резонансов формальных рядов (удовлетворяющих системе (1), (2))

$$q = a_{-1}\tau^{-1} + a_1\tau + a_2\tau^2 + \ldots, \quad \tau = s - s_0, \quad u = c_{-1}\tau^{-1} + c_0 + c_1\tau + c_2\tau^2 + \ldots,$$

$$p = b_{-1}\tau^{-1} + b_1\tau + b_2\tau^2 + \ldots, \quad w = d_{-1}\tau^{-1} + d_0 + d_1\tau + d_2\tau^2,$$

содержащих четыре произвольных параметра s_0, $a_{-1} \neq 0$, a_1, b_2.

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A MATHEMATICAL DESCRIPTION OF A ONE-DIMENSIONAL DISCRETE VELOCITY MODEL OF NON-SYMMETRIC PARTICLE SYSTEMS

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Non-equilibrium states of infinite particle systems can be described by infinite sequences of distribution functions that are solutions to the Cauchy problem of the BBGKY hierarchy of equations.

The BBGKY hierarchy solutions can be constructed as the iteration or the functional series.

A solution of the Cauchy problem for the BBGKY hierarchy of equations can be represented in the form of an expansion over particle groups whose evolution is governed by the cumulants (semi-invariants) of the evolution operator of the corresponding particle group [1–3].

Consider a one-dimensional discrete velocity model of mixture of gases, i.e. a non-symmetric system of many particles interacting as hard rods of lengths $\sigma > 0$ and masses $m = 1$. The configurations of such a particle system must satisfy the inequality $q_{i+1} \geqslant q_i + \sigma$, $i \in \mathbb{Z}^1 \setminus \{0\}$.

Denote by $W_s = \{(q_{-s_2}, \ldots, q_{s_1}) \in \mathbb{R}^s \mid q_{i+1} < q_i + \sigma$ at least for a single pair $(i, i+1) \in \in ((-s_2, -s_2+1), \ldots, (-1, 1), \ldots, (s_1 - 1, s_1))\}$ the set of forbidden configurations. The set $M_s \equiv (\mathbb{R}^s \setminus W_s) \times V^s$ is the phase space of the particle system.

Consider the linear space $L^1(\mathbb{R}^s \times V^s)$ of summable functions $f_s(x_{-s_2}, \ldots, x_{s_1})$, each defined on the phase space M_s, non-symmetric under permutations of the arguments $(x_{-s_2}, \ldots, x_{s_1})$, equal to zero on the set W_s with the norm

$$\|f_s\| = \sum_{v_{-s_2}, \ldots, v_{s_1} \in V^s} \int_{\mathbb{R}^s} dq_{-s_2} \ldots dq_{s_1} \, |f_s(x_{-s_2}, \ldots, x_{s_1})|.$$

Define the set $L^1_0(\mathbb{R}^s \times V^s)$, everywhere dense in $L^1(\mathbb{R}^s \times V^s)$, of functions $f_s \in L^1(\mathbb{R}^s \times V^s)$ with compact support in the phase space M_s, which are continuously differentiable with respect to the configuration variables $(q_{-s_2}, \ldots, q_{s_1})$ and equal to zero in an ε-neighbourhood of the set W_s of forbidden configurations.

In the Banach space L^1 of infinite sequences $f = \{f_s(x_{-s_2}, \dots, x_{s_1})\}_{s=s_1+s_2\geqslant 0}$ we examine a one-dimensional discrete velocity model of the Cauchy problem for the BBGKY hierarchy of equations with initial data possessing the factorization property (the chaos property

$$F_1(t,x_1)|_{t=0} = F_1(0,x_1),$$

$$F_s(t,x_{-s_2},\dots,x_{s_1})|_{t=0} = \chi_s(q_{-s_2},\dots,q_{s_1}) \prod_{i=-s_2}^{s_1} F_1(0,x_i), \quad s \geqslant 2,$$

where $F_1(0,x_i) \in L^1_0(\mathbb{R}^1 \times V)$, $\chi_s = \chi_s(q_{-s_2},\dots,q_{s_1})$ is the characteristic function of the set $\mathbb{R}^s \setminus \{W_s \cup \partial W_s^\varepsilon\}$, the set ∂W_s^ε is an ε-neighbourhood of the forbidden configurations set W_s). The state of a considerable particle system is described by an infinite sequence of distribution functions that are solutions to a one-dimensional discrete velocity model of the Cauchy problem for the BBGKY hierarchy of equations with initial data possessing the factorization property (the chaos property).

For the considerable Cauchy problem for the BBGKY hierarchy of equations in the Banach space L^1 we proved the following theorem.

Theorem. *If $F_1(0) \in L^1_0(\mathbb{R}^1 \times V) \subset L^1(\mathbb{R}^1 \times V)$ then there exists a unique strong, global in time, solution $F(t) = \{F_s(t,Y)\}_{s=s_1+s_2=|Y|\geqslant 0}$, where $F_s(t) \in L^1(\mathbb{R}^s \times V^s)$, $s \geqslant 1$, of the Cauchy problem for the BBGKY hierarchy of equations. It is given by the expansion over particle groups whose evolution is governed by the cumulants of the evolution operator of the corresponding particle group:*

$$F_{|Y|}(t,Y) = S_s(-t,Y)\chi_s(Q^s) \prod_{i=-s_2}^{s_1} F_1(0,x_i) + \sum_{n=1}^{\infty} \sum_{n_1+n_2=n} \frac{1}{v^n} \times$$

$$\times \sum_{v_{-(s_2+n_2)},\dots,v_{-(s_2+1)},v_{s_1+1},\dots,v_{s_1+n_1} \in V^n} \int_{\mathbb{R}^n} d(Q^{s+n} \setminus Q^s) \sum_{\substack{Z\subset X\setminus Y\\ Z\neq\varnothing}} (-1)^{|X\setminus(Y\cup Z)|} \times$$

$$\times \mathfrak{A}_2(t,Y,Z)\chi_{s+n}(Q^{s+n}) \prod_{i=-(n_2+s_2)}^{n_1+s_1} F_1(0,x_i), \quad |X\setminus Y| \geqslant 1,$$

where $1/v$ is the density,

$$Q^s = (q_{-s_2},\dots,q_{s_1}), \quad Q^{s+n} = (q_{-(n_2+s_2)},\dots,q_{s_1+n_1}),$$

$$Y = (x_{-s_2},\dots,x_{s_1}), \quad X\setminus Y = (x_{-(n_2+s_2)},\dots,x_{-(s_2+1)},x_{s_1+1},\dots,x_{s_1+n_1}).$$

Here $\sum_{\substack{Z\subset X\setminus Y\\ Z\neq\varnothing}}$ is the sum over all nonempty ordered subsets Z of the partially ordered set $X\setminus Y$, $Z\subset X\setminus Y$, and the group of $|Z|$ particles evolves as one element, $\mathfrak{A}_2(t,Y,Z)$ is the cumulant of the 2-nd kind, $\chi_{s+n}(Q^{s+n})$ is the characteristic function of the set $\mathbb{R}^s \setminus \{W_s \cup \partial W_s^\varepsilon\}$.

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