Nucleus Driven Electronic Pulsation

H. Ludwig,* R. Ruffini,† and S.-S. Xue‡

Dipartimento di Fisica and ICRA, Sapienza Università di Roma, Rome, ITALY

By the spectral method we derive and solve the equations for a neutral system of ultra-relativistic electrons that are compressed to the radius of the nucleus and subjected to a driving force. This driving force can be thought of as originating from a nuclear breathing mode, a possibility we discuss in detail.

1.   Introduction

The creation of electron-positron pairs through strong fields, which has been predicted theoretically by F. Sauter as early as 1931 [1] but not yet been observed, is nowadays of high interest for both laboratory scientists and astrophysicists [2, 3]. The obstacle to be solved is the creation of a sufficiently strong electric field. In this note we will present an approach to investigate the dynamic response of an ultra-relativistic compressed electron gas subjected to the breathing mode of the underlying nucleus, a system that promises to generate strong electric fields, depending on the mutual dynamics of negative and positive charges. This model can, with slight modification like an adjustment of proton density, be extrapolated to astrophysical scales. In [4] we presented the stationary solutions of pulsating electron gas in the framework of the Thomas-Fermi model, and we will now recapitulate the main results that are of importance for the present work. In this model the nucleus is modeled as a sphere of homogenous positive charge distribution. The

───────────
*E-mail: hendrik.ludwig@icranet.org; Also at ICRANet, P.zza della Republica 10, I-65122 Pescara, ITALY; Also ICRANet, Nice, FRANCE
†E-mail: ruffini@icra.it; Also at ICRANet, P.zza della Republica 10, I-65122 Pescara, ITALY; Also at ICRANet, Nice, FRANCE
‡E-mail: xue@icra.it; Also at ICRANet, P.zza della Republica 10, I-65122 Pescara, ITALY

electrons are considered as a charged perfect fluid with the equation of state of a Fermi gas at zero temperature.

2.   Spectral method formalism

Since we want to treat the regime around nuclear density we can assume the ultra-relativistic limit

$$\rho = \frac{3}{4}(3\pi^2 n^4)^{1/3}, \qquad p = \frac{1}{4}(3\pi^2 n^4)^{1/3} \qquad (1)$$

for the equation of state, where ρ, p, and n are the energy density, pressure, and number density of the electron gas respectively. Throughout the text, natural units $c = 1$, $\hbar = 1$ are used. When we assume the nucleus of proton number Z to have a radius $r_{\text{nuc}} = \Delta Z^{1/3}$ in units of the pion mass, and provide sufficient external pressure to restrain the electrons to the volume of the nucleus, we obtain the very simple equilibrium configuration

$$\left. \begin{array}{l} \rho \;=\; \dfrac{9}{16\Delta^4}\left(\dfrac{3}{2\pi}\right)^{2/3}, \; p = \dfrac{3}{16\Delta^4}\left(\dfrac{3}{2\pi}\right)^{2/3}, \\[4mm] n \;=\; n_{\text{nuc}} = \dfrac{3}{4\pi\Delta^3}. \end{array} \right\} \quad (2)$$

The parameter Δ measures the departure of proton density from nuclear densities. Positive and negative charges completely cancel, so there is no electric field in equilibrium. Introducing a time-dependent displacement field $\xi(t, r)$ and treating Euler's and Maxwell's equations to first

order, one obtains for stationary modes the second order ordinary differential equation

$$\xi''(r) + \frac{2}{r}\xi'(r) + \left[3\omega^2 - \left(\frac{12}{\pi}\right)^{1/3}\frac{3\alpha}{\Delta^2} - \frac{2}{r^2}\right]$$
$$\times\,\xi(r) = 0. \tag{3}$$

Here α is the fine structure constant, and ω is the frequency of the mode that has to be chosen according to appropriate boundary conditions. This leads to

$$\omega_n = \left[\left(\frac{12}{\pi}\right)^{1/3}\frac{\alpha}{\Delta^2} + \frac{\pi^2 n^2}{3\,r_{\mathrm{nuc}}^2}\right]^{1/2} \quad \text{and}$$

$$\xi_n(x) = \mathcal{C}\,j_1\left(\pi n\,r/r_{\mathrm{nuc}}\right), \tag{4}$$

where $\mathcal{C}$ is an arbitrary factor and $j_1(z)$ is the first-order spherical Bessel functions of the first kind. The last allows to construct an orthonormal basis (see Figure 1)

$$\xi_n(r) = n\,\pi\,\sqrt{2}\,r_{\mathrm{nuc}}^{-3/2}\,j_1(n\,\pi\,r/r_{\mathrm{nuc}}), \quad n = 1, 2, 3... \tag{5}$$

on the interval $[0, r_{\mathrm{nuc}}]$ with respect to the scalar product

$$\langle\xi_n(r), \xi_m(r)\rangle = \int_0^{r_{\mathrm{nuc}}} r^2\xi_n(r)\xi_m(r)\mathrm{d}r = \delta_{nm}. \tag{6}$$

To model a driving force acting on the system we can expand the Dirac delta function on the surface

$$\delta(r - r_{\mathrm{nuc}}) = -(2\,r_{\mathrm{nuc}})^{\frac{1}{2}}\sum_{n=1}^{\infty}(-1)^n\xi_n(r) \tag{7}$$

and the identity function

$$\mathrm{id}(r) = -\frac{3(2\,r_{\mathrm{nuc}}^5)^{\frac{1}{2}}}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\xi_n(r) \tag{8}$$

in terms of this basis. The spatial delta function at the surface can be used in combination with a temporal delta function to model an instantaneous impulse applied to the surface,
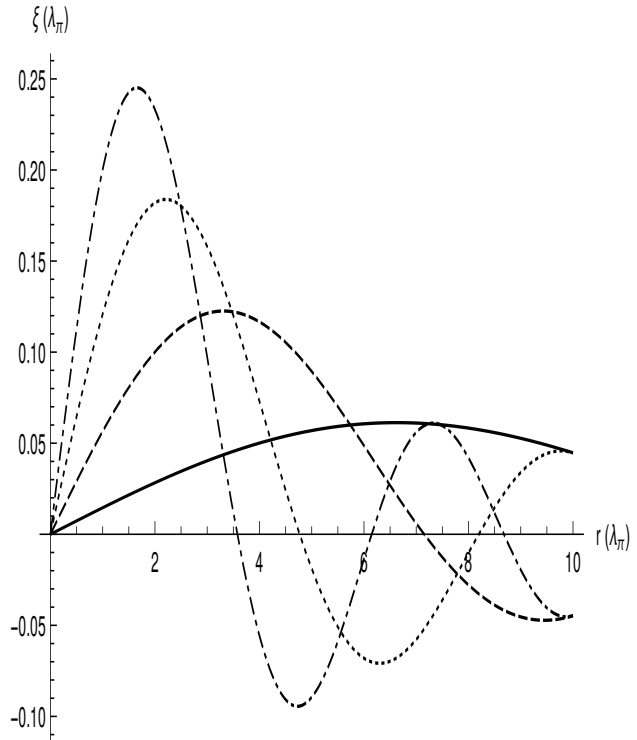

FIG. 1. The basis functions ξ_n for $n = 1, 2, 3, 4$ are shown for $r_{\mathrm{nuc}} = 10$, corresponding to $Z = 1000$ and $\Delta = 1$. For ascending n, the lines are solid, dashed, dotted and dash-dotted, respectively.

or in combination with a harmonic temporal function to drive the surface periodically. The identity function in combination with a harmonic temporal function models the breathing mode of a homogenously charged sphere. To first order, Gauss' law predicts an electric force proportional to the radius resulting from any contraction or expansion of the sphere.

3.   Solving the equation of motion

According to Eqs. (3) and (4), the basis functions fulfil the equation

$$\omega_n^2\xi_n(r) - \boxtimes\xi_n(r) = 0 \tag{9}$$

where $\boxtimes$ is a differential operator in r. Making the ansatz

$$\xi(t,r) = \sum_{n=1}^{\infty} T_n(t)\xi_n(r) \qquad (10)$$

and plugging it into the equation of motion with an external force term

$$\frac{\partial^2 \xi(t,r)}{\partial t^2} - \boxtimes \xi_n(t,r) = f(t,r) \qquad (11)$$

we can expand in the basis (ξ_n) to obtain the independent equations

$$\ddot{T}_n(t) - \omega_n^2 T_n(t) = f_n(t), \qquad (12)$$

where

$$f(t,r) = \sum_{n=1}^{\infty} f_n(t)\xi_n(r). \qquad (13)$$

If the force is applied just for an instant at $t = 0$, we can write $f_n(t) = f_n\delta(t)$, and assuming the system initially unperturbed ($T_n(t) = 0$ for $t < 0$) the solution to (12) is

$$T_n(t) = \frac{f_n}{\omega_n}\theta(t)\sin(\omega_n t). \qquad (14)$$

If the force is applied at a frequency Ω starting at $t = 0$ we can write $f_n(t) = f_n\theta(t)\sin(\Omega t)$, and assuming the system initially unperturbed the solution to (12) is

$$T_n(t) = f_n\theta(t)\frac{\omega_n \sin(\Omega t) - \Omega \sin \omega_n t}{\omega_n^3 - \omega_n \Omega^2}. \qquad (15)$$

Using for f_n now the coefficients of (7) or (8), we can construct various dynamical situations in the compressed atom.

4. Conclusions

Using the formalism developed above, it is now possible to investigate the response of the electron gas to the nuclear breathing mode for different system sizes from atoms and giant atoms to neutron stars. The proton density should be determined from beta-equilibrium, while the frequency of the breathing mode has to be inferred from studies of nuclear incompressibility such as [5, 6]. An investigation has to take into account the energy budget of the initial breathing mode, comparing it to the energy transferred to electron gas and electric field, to determine if dampening of the former occurs first.

Acknowledgement

References

[1] F. Sauter. Z. Phys. **82**, 742 (1931).

[2] G. Preparata, R. Ruffini, and S.-S. Xue. Astron. Astrophys. **338**, L87 (1998).

[3] A. Ipp, J. Evers, C. Keitel, and K. Hatsagortsyan. Phys. Lett. B **702**, 383 (2011).

[4] H. Ludwig, R. Ruffini, and S.-S. Xue. ArXiv:1402.3468 (2014).

[5] J. Blaizot, J. Berger, J. Dechargé, and M. Girod. Nuc. Phys. A **591**, 435 (1995).

[6] C. Wang, K. Chung, and A. Santiago. Phys. Rev. C **55**, 2844 (1997).