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SINGER CYCLES IN COMPLEX REPRESENTATIONS OF THE GENERAL LINEAR GROUP OVER A FINITE FIELD

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Let $G = PGL(n, q)$ be the projective general linear group of degree n over a finite field of q elements. Let $t \in G$ be a Singer cycle in G, that is, an element of order $(q^n - 1)/(q - 1)$ whose preimage in $GL(n, q)$ is irreducible. Let ϕ be an irreducible representation of G over the complex numbers. We prove that 1 is an eigenvalue of $\phi(t)$, unless, possibly, the degree of ϕ is strictly less than $|t|$, the order of t. This answers a question raised by Pablo Spiga (University of Milan in Bicocca). Irreducible representations of G of degree less than $|t|$ are well known, and the inspection yields a more precise answer. Namely, the degree is either $|t| - 1$ or 1, or 3 for the case where $(n, q) = (3, 2)$.

Apart from an intrinsic interest, the result is assumed to be used as a base of induction for studying the occurrence of eigenvalue 1 for other semisimple elements of G. The method can probably be used to prove that the minimum polynomial degree of $\phi(t)$ equals $|t|$ with the same exceptions as above. In another direction, one can try to generalize the result to other classical groups. (The case $G = PSL(n, q)$ can be easily deduce to the above result.)

The proof is somehow by induction on the number of divisors of n. If n is a prime, the result follows by applying standard results of the Deligne-Lusztig theory of characters of groups of Lie type. The main difficulties arise in performing the induction step. In this situation, that is, when n is not a prime, an essential role in the proof is played by representation theory of groups with cyclic Sylow p-subgroup, not only over the complex numbers but also over the ring of p-adic integers. The starting point is the fact that the group $T = \langle t \rangle$ contains a cyclic Sylow p-subgroup, unless $n = 2$ and $q + 1$ is a 2-power, or $(n, q) = (6, 2)$. However, the reasonuing is not straightforward as the representation theory of groups with cyclic Sylow p-subgroup is efficient for analyzing eigenvalues of p-elements whereas t is not usually a p-element. Exactly this requires realization of the representation in question over the ring of p-adic integers, and some use of the theory of projective modules over such rings.