BIG COMPOSITION FACTORS IN RESTRICTIONS OF MODULAR REPRESENTATIONS OF CLASSICAL ALGEBRAIC GROUPS TO SUBSYSTEM SUBGROUPS

I.D. Suprunenko

Institute of Mathematics, National Academy of Belarus
11 Surganov str., 220072, Minsk, Belarus suprunenko@im.bas-net.by

The goal of the talk is to discuss constructing of composition factors with certain special properties in restrictions of modular irreducible representations of classical algebraic groups to subsystem subgroups with two simple components. We shall deal with factors that are in a certain sense big enough (or not too small) for both components of a subgroup under consideration. The existence of such factors yield effective tools for solving a number of questions, in particular, for finding or estimating various parameters of the images of individual elements in representations of such groups, and not only for elements of relevant subsystem subgroups. Often the analysis of restrictions to subsystem subgroups with several simple components yields a useful information that, probably, cannot be obtained if we deal with simple subsystem subgroups only. It was A.E. Zalesskii who has drawn the author's attention to investigating restrictions of representations of simple algebraic groups to non-simple subsystem subgroups. In a joint paper [1] we have proved that the restriction of a nontrivial representation of a simple algebraic group to a subsystem subgroup with two simple components almost always has a composition factor that is nontrivial for both components.

In what follows K is an algebraically closed field, $G = A_r(K)$ or $C_r(K)$, ω_i $(1 \le i \le r)$ are the fundamental weights of G, $\omega(\varphi)$ is the highest weight of an irreducible representation φ, and φ^* is the representation dual to φ. If $\omega(\varphi) = \sum_{i=1}^r a_i\omega_i$, set $s(\varphi) = \sum_{i=1}^r a_i$, put

$$\Sigma(\varphi) = a_1 + 2(a_2 + \ldots + a_{r-1}) + a_r$$

for $G = A_r(K)$ and

$$\Sigma(\varphi) = a_1 + 2(a_2 + \ldots + a_r)$$

for $G = C_r(K)$, in both cases set $t(\varphi) = \Sigma(\varphi) - s(\varphi)$. A subgroup in G is called a subsystem subgroup if it is generated by the root subgroups associated with all roots of a subsystem in the root system. We write an irreducible representation ρ of a semisimple group H with two simple components H_1 and H_2 in the form $\rho_1 \otimes \rho_2$ where ρ_i is an irreducible representation of H_i, $i = 1, 2$. Some of the results that will be discussed are stated below.

Theorem 1. *Let $2 \le l \le r-3$ for $G = A_r(K)$ and $2 \le l \le r-2$ for $G = C_r(K)$, and let φ be an irreducible representation of G. Assume that H_1 and $H_2 \subset G$ are commuting subsystem subgroups of types A_l and A_{r-l-1}, respectively, for $G = A_r(K)$ and of types C_l and C_{r-l} for $G = C_r(K)$. Set $H = H_1H_2$. If $\psi = \psi_1 \otimes \psi_2$ is a composition factor of the restriction $\varphi|H$, then $s(\psi_1) + s(\psi_2) \le \Sigma(\varphi)$. The representation $\varphi|H$ has a composition factor $\tau = \tau_1 \otimes \tau_2$ with $s(\tau_1) = s(\varphi)$ and $s(\tau_2) = t(\varphi)$.*

Theorem 2. *In the assumptions of Theorem 1 if φ is nontrivial, then $\varphi|H$ has a composition factor $\rho = \rho_1 \otimes \rho_2$ with $s(\rho_1) \ge s(\varphi) - 1$ and $s(\rho_2) > 0$.*

Now let K be a field of positive characteristic p. The parameter $s(\varphi)$ is important for describing the behavior of unipotent elements in p-restricted irreducible representations, but for arbitrary representations another parameter appears more useful. By the Steinberg tensor product theorem, an irreducible representation φ of G is equivalent to a tensor product $\bigotimes_{i=0}^j \varphi_i Fr^i$ where Fr is the Frobenius morphism determined by raising the elements of K to the pth power and φ_i are p-restricted irreducible representations of G (all coefficients of their highest weights are less than

p). Set $s_1(\varphi) = \sum_{i=0}^{j} s(\varphi_i)$. One easily observes that $s_1(\varphi)$ is correctly determined. We call φ p-large if $s_1(\varphi) \geq p$. For $p > 2$ it is proved in [2, Theorem 1.1] that for every unipotent element $x \in G$ the degree of the minimal polynomial of $\varphi(x)$ is equal to the order of x if φ is p-large. Hence for applications in the analysis of the behavior of unipotent elements in modular representations it is worth to get an analog of Theorem 1 that for p-large representation φ would yield a factor τ with $s(\tau_1)$ and $s(\tau_2)$ close to the values from Theorem 1 and big $s_1(\tau_1)$. We have such result for $G = A_r(K)$ and a certain class of representations.

Theorem 3. *Let $G = A_r(K)$. In the assumptions of Theorem 1 let φ be a p-restricted representation with highest weight $\sum_{i=1}^{r} a_i\omega_i$, $s(\varphi) \geq p$, $\sum_{i=1}^{m} a_i \neq 0$, and $\sum_{i=m+2}^{r} a_i \neq 0$. Then $\varphi|H$ has a composition factor $\varphi_1 \otimes \varphi_2$, where φ_i is an irreducible representation of H_i, $i = 1, 2$; φ_1 is p-large, $s(\varphi_1) > s(\varphi) - p$; for $t(\varphi) \geq p$ the representation φ_2 is p-large and $s(\varphi_2) > t(\varphi) - p$; if $t(\varphi) < p$, the parameter $s(\varphi_2) \geq t(\varphi)$.*

We shall also discuss how the analysis of restrictions of representations of classical algebraic groups to subsystem subgroups with two simple components can be used for investigating the behavior of unipotent elements in representations. In particular, we use this approach for finding estimates for the number of certain Jordan blocks in the images of such elements in irreducible representations (the blocks of the maximal possible size or the blocks whose order is equal to the order of an element under consideration).

This research has been supported by the Belarusian Republican Foundation for Fundamental Research, project F14-043.

References

1. Suprunenko I.D., Zalesskii A.E. On restricting representations of simple algebraic groups to semisimple subgroups with two simple components // Trudy Instituta matematiki. 2005. V. 13. No 2. P . 109-115.

2. Suprunenko I. D. The minimal polynomials of unipotent elements in irreducible representations of the classical groups in odd characteristic // Memoirs of the AMS. 2009. Vol. 200. No. 939.