THE ZIEGLER SPECTRUM OF A-INFINITY PLANE SINGULARITY

Gena Puninski

Belarusian State University
av. Nezalezhnosti 4, 220030, Minsk, Belarus punins@mail.ru

Let F be an algebraically closed field of characteristic zero and let $R = F[[x, y]]/(y^2)$ be the so-called A_∞ plane singularity. We consider the theory T of x-torsion free R-modules.

The classification of indecomposable finitely generated modules in T (i.e. finitely generated indecomposable maximal Cohen–Macaulay R-modules) is well known [1]. Namely each such a module is either isomorphic to the right ideal $I_n = (x, y^n)$ of R or to the right ideal $I_\infty = xR$.

We will calculate the Ziegler spectrum, Zg, of torsion-free R-modules, i.e. a topological space (see [2]) whose points are indecomposable pure injective models of T. Because the modules I_n and I_∞ are linearly compact they are (the only) finitely generated points of Zg.

Theorem. *Each infinitely generated point of* Zg *is the following:*

1) the ring of quotients Q of R;

1) the integral closure $\widetilde{R}$ of R in Q;

3) $G = F((y))$ the ring of Laurent power series.

Furthermore one can execute the Cantor–Bendixson analysis on the Ziegler spectrum.

Proposition.

1) The modules I_n, $n < \infty$ are the only isolated points in Zg*;*

2) the modules I_∞ and $\widetilde{R}$ have Cantor–Bendixson rank 1*;*

3) Q and G have CB*-rank* 2.

Thus the Cantor–Bendixson rank of Zg *equals 2.*

This note is an announcement of some results from a project, joint with Ivo Herzog, where infinitely generated points of the Ziegler spectrum are used to get a better (than the Auslander–Reiten quiver) understanding of the category of finitely generated R-modules.

References

1. Yoshino Y. *Cohen–Macaulay Modules over Cohen–Macaulay Rings*. Cambridge University Press, 1990.

2. Ziegler M. *Model theory of modules* // Ann. Pure. Appl. Logic. 1984. V. 26. P. 149–213.