A CHARACTERIZATION OF NILPOTENT NONASSOCIATIVE ALGEBRAS BY INVERTIBLE LEIBNIZ-DERIVATIONS

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In 1955, Jacobson proved that a finite-dimensional Lie algebra over a field of characteristic zero admitting a non-singular (invertible) derivation is nilpotent. The problem of whether the inverse of this statement is correct remained open until work of Dixmier and Lister, where an example of nilpotent Lie algebra all of which derivations are nilpotent (and hence, singular), was constructed. For Lie algebras in prime characteristic the situation is more complicated. In that case there exist non-nilpotent Lie algebras, even simple ones, which admit nonsingular derivations (Benkart, Kostrikin and Kuznetsov).

In paper of Moens a generalization of derivations and pre-derivations of Lie algebras is defined as a Leibniz-derivation of order k. Moens proved that a finite-dimensional Lie algebra over a field of characteristic zero is nilpotent if and only if it admits an invertible Leibniz-derivation. After that, Fialowski, Khudoyberdiyev and Omirov proved that a finite-dimensional Leibniz algebra is nilpotent if and only if it admits an invertible Leibniz-derivation. It should be noted that there exist non-nilpotent Filippov (n-Lie) algebras with invertible derivations. In paper of Kaygorodov and Popov authors showed that the same result holds for alternative algebras (in particularly, for associative algebras) [1]. Also, in this article an example of nilpotent alternative (non-associative) algebra over a field of positive characteristic possessing only singular derivations was provided.

The main purpose of this talk is to talk about the analogues of Moens' theorem for Jordan, $(-1,1)$- and Malcev algebras.

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References

1. Kaygorodov I., Popov Yu. *Alternative algebras admitting derivations with invertible values and invertible derivations* // Izvestiya: Math. 2014. V. 78. No. 5. P. 922–935.

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