ON THE COMPLEXITY OF THE CLUSTERING MINIMUM BICLIQUE COMPLETION PROBLEM

O. Duginov

Institute of Mathematics, National Academy of Sciences of Belarus
11 Surganov str., 220072, Minsk, Belarus oduginov@gmail.com

We consider the complexity results for the Clustering Minimum Biclique Completion problem restricted to subclasses of bipartite graphs.

A finite undirected graph $G = (V, E)$ is bipartite if its vertex set V can be partitioned into two sets $X, Y \subseteq V$ (partite sets) such that every edge of G has its ends in different sets X, Y. For a vertex $v \in V$, the set of vertices of the graph G adjacent to v is denoted by $N_G(v)$. Let $G = (X \cup Y, E)$ be an arbitrary bipartite graph with non-empty partite sets X, Y and let p be a positive integer such that $p \leq |X|$. If we add all edges of the set $\overline{E} = \{\{x, y\} : x \in X, y \in Y, \{x, y\} \notin E\}$ to the graph G, we obtain a complete bipartite graph $G' = (X \cup Y, E \cup \overline{E})$ whose the partite sets X can be partitioned into p non-empty sets $X_1, X_2, \ldots, X_p$ with the following condition: $N_{G'}(x) = N_{G'}(x')$ for every pair of vertices $x, x' \in X_i$, $i \in \{1, 2, \ldots, p\}$. The Clustering Minimum Biclique Completion problem asks for a minimum cardinality set $E' \subseteq \overline{E}$ to be added to the graph G so that the partite set X of the resulting bipartite graph $G' = (X \cup Y, E \cup E')$ can be partitioned into p non-empty sets $X_1, X_2, \ldots, X_p$ with the same condition. Let $\xi_p(G) = |E'|$. The decision version of the problem can be stated as follows:

Clustering Minimum Biclique Completion

Instance: A bipartite graph $G = (X \cup Y, E)$ with non-empty parts X and Y, two positive integers $p \leq |X|$ and k.

Question: Can G be transformed by adding at most k additional edges connecting vertices from different sets X, Y into a bipartite graph G' whose the partite set X can be partitioned into p non-empty sets $X_1, \ldots, X_p$ such that $N_{G'}(x) = N_{G'}(x')$ for any two vertices $x, x' \in X_i$, $i \in \{1, 2, \ldots, p\}$? Equivalently, is $\xi_p(G) \leq k$?

This problem, also known as the Multicast Partition problem, has been introduced by N. Faure [1, 2] and arises in telecommunication network technologies [2]. The computational complexity of the Clustering Minimum Biclique Completion problem for various subclasses of bipartite graphs is little-studied. To the best of our knowledge, there is only two results. N. Faure et. al. in [2] showed that: (a) the problem is NP-complete for fixed $p = 2$ (by a reduction from the Maximum Edge Biclique problem) and (b) the problem restricted to bipartite graphs $G = (X \cup Y, E)$ with degrees of vertices of Y at most 1 can be solved in strongly polynomial time by a dynamic programming algorithm. On the other hand, the problem is well-studied from a mathematical programming point of view [3–6].

We provide several well-studied subclasses of bipartite graphs, for which the considered problem remains NP-complete. Recall that a graph G is H-free if G does not contain an isomorphic copy of the graph H as an induced subgraph.

Theorem 1. Clustering Minimum Biclique Completion *for P_4-free bipartite graphs is NP-complete.*

Corollary 1. Clustering Minimum Biclique Completion *is NP-complete for the following subclasses of bipartite graphs (for definitions we refer to [7, 8]): bipartite permutation graphs, convex graphs and chordal bipartite graphs.*

A bipartite graph $G = (X \cup Y, E)$ is $(3, 2)$-regular if the degree of every vertex of X is 3 and the degree of every vertex of Y is 2.

Theorem 2. Clustering Minimum Biclique Completion *for C_4-free $(3, 2)$-regular bipartite graphs and $p = 2$ is NP-complete.*

On the positive side, $\xi_p(G)$ for $2K_2$-free bipartite graphs $G = (X \cup Y, E)$ can be computed in $O(|X|^4|Y|p)$ time.

This work was supported by Belarusian Republican Foundation for Fundamental Research (project F14RA-004).

References

1. Faure N. *Contribution á la résolution de problémes de regroupement de sessions multicasts.* PhD thesis, Université Paris VI. 2006 (in French).

2. Faure N., Chrétienne P., Gourdin E., Sourd F. *Biclique completion problems for multicast network design* // Discrete Optimization. 2007. V. 4. P. 360–377.

3. Gualandi S. *k-Clustering Minimum Biclique Completion via a Hybrid CP and SDP Approach* // Proceedings of the 6th International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems, CPAIOR 2009. 2009. V. 5547. P. 87–101.

4. Gualandi S., Maffioli F., Magni C. *A branch-and-price approach to k-clustering minimum biclique completion problem* // International Transactions in Operational Research. 2013. V. 20. P. 101–117.

5. Gualandi S., Malucelli F. *Weighted Biclique Completion via CP-SDP Randomized Rounding* // Proceedings of the European Workshop on Mixed Integer Nonlinear Programming. 2010. P. 223–230.

6. Magni C. *Biclique completion problem: models and algorithms.* MSc thesis, Politecnico di Milano. 2009.

7. Spinrad J.P. *Efficient Graph Representations.* Fields Institute Monographs, 19. 2003.

8. Brandstädt A., Le V.B., Spinrad J.P. *Graph classes: a survey.* Society for Industrial and Applied Mathematic, 1999.