LOCALIZED FREE VIBRATIONS OF MULTI-WALLED CARBON NANOTUBE EMBEDDED IN NONHOMOGENEOUS ELASTIC MEDIUM

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The boundary-value problem on low-frequency free vibrations of a middle length multi-walled carbon nanotube (MWCNT) embedded in a nonhomogeneous elastic medium is considered. The MWCNT is modeled by a system of N concentrically nested elastic circular cylindrical shells. Free vibrations of this system is governed by the following equations

$$\begin{aligned}
&D\triangle_1^2 w_1 + \triangle_{R1}\Phi_1 - \rho h\omega^2 w_1 - p_{12} = 0, \quad \tfrac{1}{Eh}\triangle_1^2\Phi_1 - \triangle_{R1}W_1 = 0, \dots, \\
&D\triangle_n^2 w_n + \triangle_{Rn}\Phi_n - \rho h\omega^2 w_n - p_{n(n+1)} + \tfrac{R_{n-1}}{R_n}p_{(n-1)n} = 0, \\
&\tfrac{1}{Eh}\triangle_n^2\Phi_n - \triangle_{Rn}W_n = 0, \dots, \quad \tfrac{1}{Eh}\triangle_N^2\Phi_N - \triangle_{RN}W_N = 0, \\
&D\triangle_N^2 w_N + \triangle_{RN}\Phi_N + \rho h\omega^2 w_n + c(\varphi)w_N + \tfrac{R_{N-1}}{R_N}p_{(N-1)N} = 0.
\end{aligned} \tag{1}$$

with respect to the radial displacements w_n and the stress functions Φ_n for the n th wall $(n = 1, 2, \dots, N)$. In Eqs. (1), h, E, D, ρ are the effective thickness, Young's modulus, bending stiffness and density, respectively, $\triangle_n$ is the Laplace operator in the curvilinear coordinates (x, φ) (x and φ are dimensionless axial and circumferential coordinates), $\triangle_{Rn} = \frac{1}{A_nB_n}\left[\frac{\partial}{\partial x}\left(\frac{A_n}{R_nB_n}\frac{\partial}{\partial x}\right)\right]$ is the deferential operator depending on the wall number n, $A_n = R_N, B_n = R_N r_n$ are the Lame's coefficients, $r_n = R_n/R_N$, R_n is the radius of the n th wall, $c(\varphi)$ is the spring constant of the surrounding nonhomogeneous elastic medium represented by the Winkler-type foundation, and $p_{n(n+1)} = c^*(W_{n+1} - W_n)$, $\quad n = 1, 2, ..., (N-1)$ is the vdW forces acting between walls, where $c^* = 620 \times 10^{-10}\mathrm{N/nm}^3$ is the vdW interaction coefficient.

Four variants of boundary conditions are considered at the tube edges $x = 0, l$. Taking into account the nonhomogeneous response of the surrounding medium, the solution of Eqs. (1) with the appropriate boundary conditions is constructed in the form of functions decaying far from some weakest generatrix $\varphi = \varphi_0$ [1]. Depending on the magnitude of the outermost radius R_N, two different asymptotic estimates for the natural frequency ω are derived. The effect of the spring constant c, the outermost radius R_N, a number of walls N, and the type of boundary conditions on the eigenvalue ω is analyzed.

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Refrences

1. *Mikhasev G. I.* On localized modes of free vibrations of single-walled carbone nanotubes embedded in nonhomogeneous elastic medium. *Z Angew.Math.Mech.* **94** (1-2) (2014), 130-141.