ABOUT H-MODELS OF NONINERTIAL SYSTEM

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In this paper we consider the problem of modeling noninertial processes with stochastic dependence between the input variables. Such processes are called H-processes ("tubular" structure processes). A new class of parametric identification algorithms with the indicator function of multidimensional static objects is suggested to use. The results of some computational experiments are presented.

Keywords: discrete-continuous processes, nonparametric identification, parametric identification, H-processes.

INTRODUCTION

Of course, the problem of identification a long time remains one of the central problems of cybernetics. Statement of the problem identification depends on the class of the investigated process (static, dynamic, linear, nonlinear, and others types of processes) and on the amount of a priori information about the investigated process.

In this paper we will focus on a new class of processes called "tubular" (or H-processes). The first mention of the process with "tubular" structure appears from the author Medvedev A.V. at [1]. These processes were observed in the simulation of metallurgy processes. It was found that there was statistical dependence between the components of the entrance, so that the process proceeds not in all domain identified by the technological regulations but in some of its subdomains. Simulation of such processes is associated with many difficulties, in particular, the traditional parametric methods do not produce a satisfactory identification result. H-processes can be regarded as new and, to date, poorly understood. But the processes of this class are increasingly found in practice, and therefore, require further study. Some studies H-processes are presented in [2].

The main objective of this paper is to describe the processes of "tubular" type, as well as to present some results of their simulations using developed algorithm.

THE STATEMENT OF THE PROBLEM

On Fig. 1 the table of symbols are accepted: A is an unknown object operator, $x(t) \in \Omega(x) \subset R^1$ is an output variable of the process, $u(t) = \left(u_i(t), i = \overline{1,m}\right) \in \Omega(u) \subset R^m$ is a control action vector, $\xi(t)$ is a vector random action, (t) is continuous time, $G^{u_i}, i = \overline{1,m}$, G^x are channels of connection corresponding to different variables and including control tools, $g^{u_i}(t), i = \overline{1,m}$, $g^x(t)$ are random interference of measurements corresponding to variables of the process with zero means and limited dispersion, u_{it}, $i=\overline{1,m}$, x_t are measurement of process variables at discrete times.

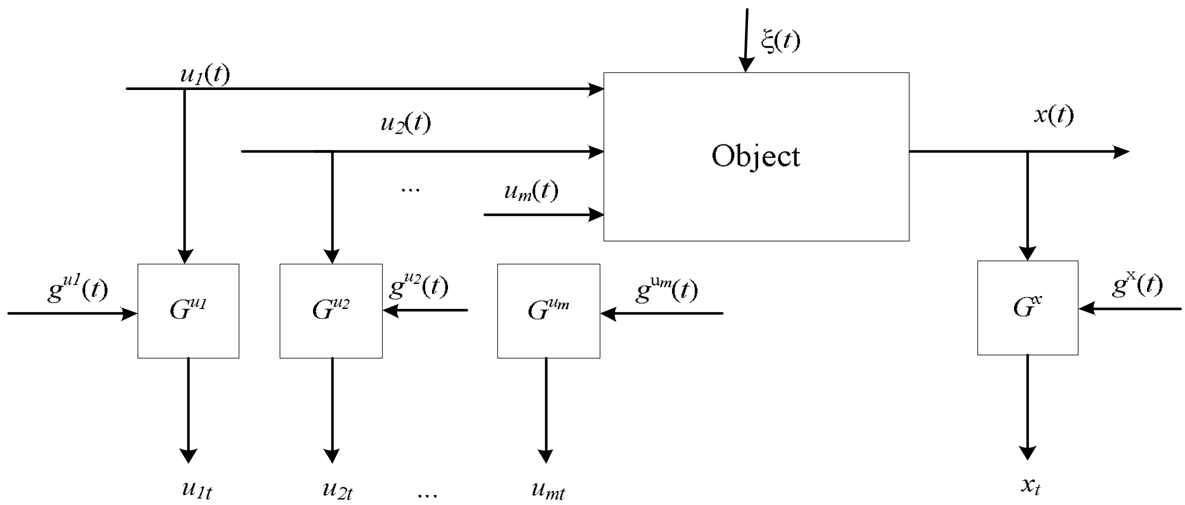

Fig. 1. The general scheme of multidimensional object

So, we have a sample of observations "input-output" process variables $\{u_i, x_i, i=\overline{1,s}\}$, where s – the sample size. Let the components of the input variables are linked by some unknown stochastic dependence. The case where the input vector components are independent can be illustrated by the following Figure 2:

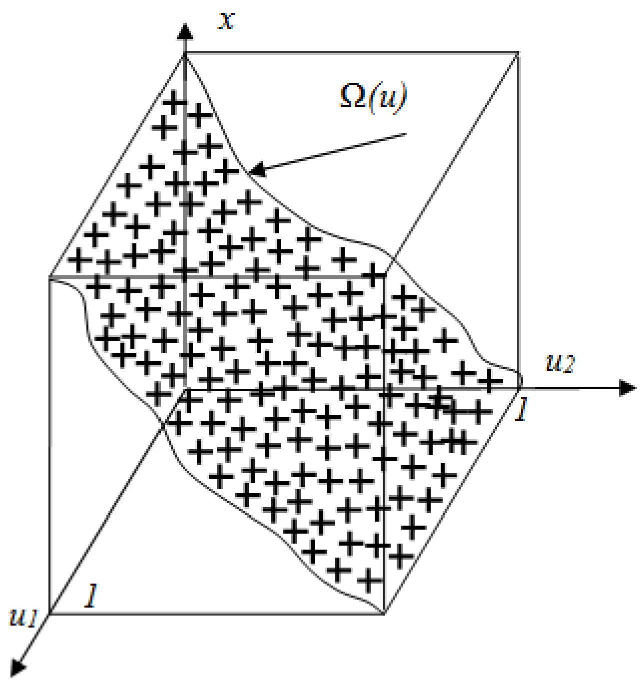

Fig. 2. Sample of observations in case of independent components of output vector $u(t)$

Here, for simplicity, we consider the three-dimensional case, when $x = f(u_1, u_2)$, $u_1, u_2 \in [0;1]$. The units of the sample are shown as sign "+" in the Fig. 2. As can be seen from the figure, the value of one component $u_1(t)$ may correspond to a set of values components, and vice versa. If there is dependence between the components of the input, the process has "tubular" structure. Such process is presented in Figure 3.

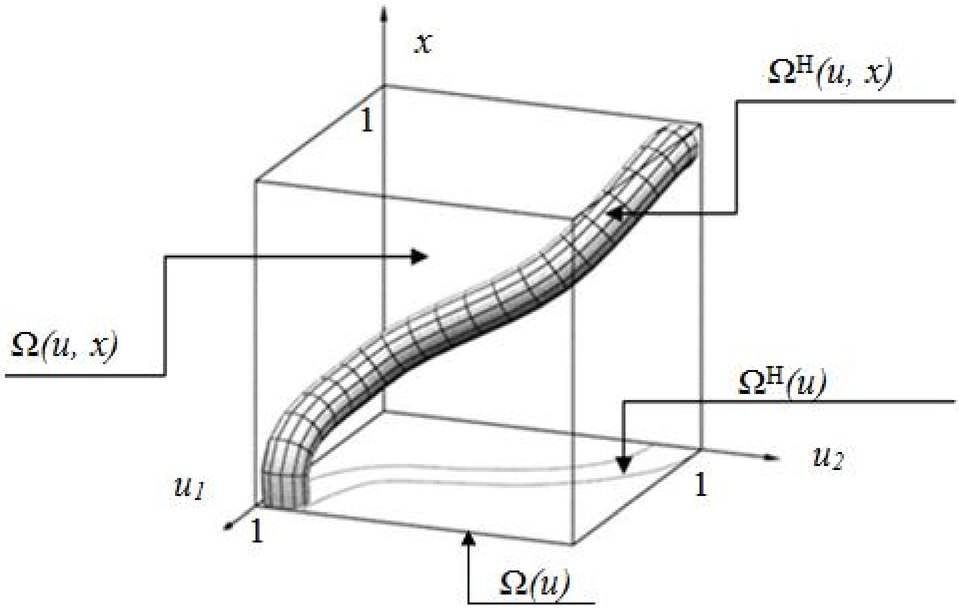

Fig. 3. Object with “tubular” structure

As can be seen from the Figure 3, the area of the process $\Omega(u, x)$ is, without loss of generality, the unit hypercube where $u = (u_1, u_2) \in R^2$, $x \in R^1$. The hypercube domain $\Omega(u, x)$ is always known in practice. For example, for the technological process values of "input-output" variable is limited by the technological regulations. However, if the investigated has a "tubular" structure, i.e. its input variables are linked stochastic dependence, the process proceeds not in all space of a hypercube $\Omega(u, x)$, but in some of its subdomain $\Omega^H(u, x) \in \Omega(u, x)$, which we have never known. Since the subdomain $\Omega^H(u, x)$ is not known, we do not know if the process is “tubular” or not. This is the main modeling difficulty of this kind of processes.

The complexity of the H-processes control is to select the input action $u(t)$. The input action $u(t)$ should belong a subdomain $\Omega^H(u)$, only in this case the output variable $x(t)$ would take an acceptable value. Otherwise, the value of the output $x(t)$ can be either outside of technological regulations and not physically be implemented (for example, the value of element content will be a negative number) or be out of the "tube". The first case is less dangerous because if the value of the output does not belong to technological regulations, this unit sample can be eliminated.

H-MODELS

Let consider modeling of process with "tubular" structure. Usually, the problem of identification of noninertial objects assumes some parameterized model which can be represented as a surface in the space of "input-output" variables:

$$\hat{x}_s(u) = \hat{f}(u, \alpha_s), \quad (1)$$

where α_s – the vector of parameters. In the case when the vector components of the input variable are statistically dependent, i.e. we deal with a "tubular" structure of the object, it is necessary to use the indicator function. Model foregoing type must also be adjusted as follows:

$$\hat{x}_s(u) = \hat{f}(u, \alpha_s) I_s(u), \quad (2)$$

The following approximation can be taken as indicator estimation:

$$I_s(u) = \mathrm{sgn}(sc_s)^{-1} \sum_{i=1}^{s} \prod_{j=1}^{m} \Phi\left(c_s^{-1}\left(u^j - u_i^j\right)\right), \quad (3)$$

where the smoothing parameter of kernel function c_s and bell-shaped function $\Phi\left(c_s^{-1}\left(u^j - u_i^j\right)\right)$ satisfy the conditions of convergence [3]. Parametric model (2) of "tubular" process containing the indicator function (3) hereafter would be called the H-model.

The logic of the indicator (3) is that for an arbitrarily given value of the current variable $u = u'$ indicator takes the value of unity if u' belongs to the "tubular" structure which is defined by the available sample $\{x_i, u_i, i = \overline{1, s}\}$. If the value does not belong to the "tubular" structure, the indicator is zero. Note that if the process is described by the surface in space, the model (2) and (1) coincide. If the process has a tubular structure in this space, it is necessary to use a model (2).

COMPUTATIONAL EXPERIMENT

Let the researched object be described by the system of the equations:

$$\begin{cases} x = 2\sin u_1 + u_2^2 + \psi; \\ u_2 = u_1 + \varphi, \end{cases} \quad (4)$$

where u_1, u_2 – input variables of the process, normally distributed in the interval [0; 3]; ξ and ψ – random numbers distributed by uniform law in the interval [–0.05; 0.05]; x – the output variable of the process. There is linear dependence between the components of input vector.

The parametric model (4) of the process (4) takes the following form:

$$x_\alpha = \alpha_1 \sin u_1 + \alpha_2 u_2^2, \quad (5)$$

where α_1, α_2 – the model parameters. Parametric model of the process is chosen correctly, this opportunity we have only within the computational experiments as we determines the nature of the process. Setting the coefficients α_1, α_2 of the model (5) was carried out by OLS. Consider the results presented in Figure 4.

$$\leq \frac{C.K'}{\varepsilon^p b_n^p}\sum_{j=1}^{n}|a_j|^p \; E\|U_{nj}\|^p$$

$$= \frac{C.K'}{\varepsilon^p b_n^p}\sum_{j=1}^{n}|a_j|^p \; E\|V_{nj}\|^p \; I(\|V_{nj}\|\leq c_n)$$

$$= \frac{C.K'}{\varepsilon^p b_n^p}\sum_{j=1}^{n}|a_j|^p \int_0^{c_n} pt^{p-1} I(\|V_{nj}\|> t)dt$$

$$\leq \frac{C.K'.D}{\varepsilon^p b_n^p}\sum_{j=1}^{n}|a_j|^p \int_0^{c_n} pt^{p-1} P(\|V\|> t)dt$$

$$= \frac{2C.K'.D}{\varepsilon^p}\frac{1}{b_n^p}\sum_{j=1}^{n}|a_j|^p \int_0^{c_n^{p/2}} tP(\|V\|^{p/2}> t)dt$$

$$= \frac{2C.K'.D}{\varepsilon^p}\left(\frac{1}{b_n^{p/2}}\frac{1}{|a_j|^{p/2}}\sum_{j=1}^{n}|a_j|^p\right) c_n^{-p/2}\int_0^{c_n^{p/2}} tP(\|V\|^{p/2}> t)dt = o(1).$$

(The last inequality follows from (3.1) and Lemma 3.1 with $f_n(x) = P(\|V\|^{p/2} > x),\ n = 1, 2, \ldots$).

Combining (3.4) we get (3.5) which completes the proof.

Remark: from the condition $E\|V\|^{p/2} < \infty,$ use the result in [2, p. 99] we get $n^{p/2}P(\|V\| > n) = o(1).$ So the condition (3.2) on the random element V can be replaced by the condition on sequences $\{a_n\}$ and $\{b_n\}$ as $n|a_n|^{p/2} = O(b_n^{p/2})$ or $n|a_n|^{p/2} = o(b_n^{p/2})$.

Corollary 3.2. Let $\{V_{ni}, F_{n,j};\ n = 1,2,\ldots; 1 \leq j \leq n\}$ be an array rowwise adapted of random elements in X, $\{V_{ni},\ n = 1,2,\ldots; 1 \leq j \leq n\}$ is stochastically dominated by random element V, $E\|V\|^{p/2} < \infty$. $\{b_n\}$ be a sequence of constants satisfy conditions $0 < b_n \uparrow \infty$, and $n = O(b_n^{p/2})$ (or $n = o(b_n^{p/2})$). Then

$$\frac{1}{b_n}\sum_{j=1}^{n}[V_{nj} - E(V_{nj}I(\|V_{nj} \leq b_n)/F_{n,j-1})] \xrightarrow{P} 0, \text{ as } n \to \infty.$$

Proof. Apply theorem 3.2 with $a_n = 1,\ n \geq 1.$ Easy to see that the sequences $\{a_n\}$ and $\{b_n\}$ satisfy the condition (3.1) in theorem 3.2.

From $E\|V\|^{p/2} < \infty$ and $n = O(b_n^{p/2})$ we have

$$nP(\|V\| > b_n) \sim b_n^{p/2}P(\|V\| > b_n) = o(1), \text{ as } n \to \infty,$$

which completes the proof.

In the case, when $X = R$ then $p = 2$, by the setting as $a_n = 1,\ n = 1, 2, \ldots,$ and $\{b_n\}$ be the sequence satisfying: $0 < b_n \uparrow \infty,\ \ n = O(b_n)$ (or $n = o(b_n)$), we get the weak law of large numbers type of theorem 2.13 in [3] for array of random variables:

Corollary 3.3. Let $\{V_{ni}, F_{n,j};\ n = 1,2,\ldots; 1 \leq j \leq n\}$ be an array rowwise adapted of random variables, $\{V_{ni},\ n = 1,2,\ldots; 1 \leq j \leq n\}$ is stochastically dominated by random variable V, with $E|V| < \infty$. $\{b_n\}$ be a sequence of positive numbers with $0 < b_n \uparrow \infty,\ \ n = O(b_n)$ (or $n = o(b_n)$). Then writing $U_{nj} = V_{nj}I(\|V_{nj}\| \leq b_n),\ 1 \leq j \leq n.$ If

$$\frac{1}{b_n}\sum_{j=1}^{n} E(U_{nj} / F_{n,j-1}) \xrightarrow{P} 0, \text{ as } n \to \infty, \tag{3.6}$$

then

$$\frac{1}{b_n}\sum_{j=1}^{n} V_{nj} \xrightarrow{P} 0, \text{ as } n \to \infty. \tag{3.7}$$

Proof. With $a_n = 1,\ n \geq 1,$ easy to see that the sequences $\{a_n\}$ and $\{b_n\}$ satisfy the conditions (3.1) and (3.2) in theorem 3.2.

By the same way of the first part in proof of theorem 3.2, for arbitrary $\varepsilon > 0$ we have

$$P(|\sum_{j=1}^{n}(V_{nj} - U_{nj})| > b_n\varepsilon) = o(1), \tag{3.8}$$

and

$$\frac{1}{b_n}\sum_{j=1}^{n}[U_{nj} - E(U_{nj} / F_{n,j-1})] \xrightarrow{P} 0, \text{ as } n \longrightarrow \infty. \tag{3.9}$$

Combining (3.6), (3.8) and (3.9) we get (3.7).

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