NEUTRON TEMPERATURE OSCILATIONS IN NEUTRON MULTIPLICATION SYSTEMS AND IN BLANKETS OF THERMONUCLEAR REACTORS

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Propagation of neutron and temperature oscillations in the neutron multiplication systems is studied. It is shown that the strongly coupled neutron temperature oscillations are resulting from the dependence of the reverse time of neutron capture by the fission nuclei on the "effective" temperature of thermal neutrons.

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The problem of controlling the processes that take place in the nuclear power plants acquired a particular urgency after an accident at Chernobyl. The wave and oscillating processes that occur in the nuclear reactor, give one of few opportunities for developing the methods of control and diagnostics of the in-core equipment.

In this connection, the neutron oscillation field excitation-and-transport processes in the neutron multiplication systems have been extensively investigated [1]. Measurements of neutron fluxes in the fission reactors showed their noise character.

Noise methods for analysis of neutron fields, influence on them of medium hydrodynamic parameter oscillations is the subject of calculations, experimental investigations and use in the diagnostics of reactors [1,2].

Today the research in the area of thermonuclear fusion is at the level of full-scale thermonuclear reactor (ITER) building. A thermonuclear reactor is an intensive source of neutrons, the energy dissipation of which should occur in the blanket-system surrounding the reactor plasma. There are projects of reactors the blanket of which plays the role of energy "multiplier" due to the processes of nuclear fuel fission. Therefore, to investigate the excitation of neutron field oscillations in the blanket of thermonuclear reactor also becomes an urgent problem.

The methods of such investigation developed in the theory of plasma oscillations are suitable for oscillations in the neutron systems. The oscillations in the multiplication medium with a coolant described by the related set of equations, including the equation of thermal diffusion of neutrons, taking into account their multiplication and nuclear capture, and the equations of compressible liquid dynamics [3]. In the blanket of hybrid fusion reactor with neutron multiplication/breeding the neutron field evolution is determined by the external source of neutrons, their inertia, diffusion, by the nuclear capture and multiplication, as a result of nuclear fission, by the convective transport of neutrons and heat. For description of the oscillating processes in such systems the next system of equations is applicable:

\[ \frac{\partial N}{\partial t} + \nabla \cdot (\bar{v} N) = \nabla (D_{ef} \nabla N) + \nu (K - 1) N + S; \]  

\[ \rho c_p \left( \bar{v} \frac{\partial T}{\partial t} + \bar{v} \cdot \nabla T \right) = \text{div} (\kappa \nabla T) + Q(\bar{r}, t, N); \]  

\[ \rho (\bar{v} \frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla) \bar{v}) = - \nabla P + \eta \Delta \bar{v}; \]  

\[ \frac{\partial \rho}{\partial t} + \nabla (\rho \bar{v}) = 0, \quad P = P(\rho, T). \]  

Here \( K = \nu \phi \theta \) is the coefficient of neutron production, including three factors: \( \nu \) is the number of emitted neutrons in one fission event, \( \phi \) is the probability to avoid a capture in the process of neutron slowing down. The thermal neutron utilization coefficient \( \theta = \nu_f \phi (\nu_f + \nu_a) \) is equal to the ratio between the frequency of their nuclear fission capture \( \nu_f = \sigma_f N_f v \) and the sum of all the processes of absorption: by fission nuclei and neutron absorption \( \nu_a = \sum_i \sigma_i \bar{n}_i \), where \( i \) is the index of sort of nuclei and \( \sigma_f \) is the nuclear fission reaction cross-section and \( N_f \) is the nuclear density, \( v \) is the average thermal neutron velocity.

The effective diffusion coefficient \( D_{ef} = D + K \tau v_c \) is equal to the sum of the classic coefficient \( D = (1/3) v l \) and the addition, related to the neutron age \( \tau = l^2 \ln (E_0 / E) \), where \( l^2 = 1/(3 \xi \Sigma_a \Sigma_{tr}) \), in which \( \xi = 1 \), \( \Sigma_a = \sigma_a N_a \) and \( \Sigma_{tr} \) is macroscopic transport cross-section \( \Sigma_{tr} \approx \Sigma_s \) approximately equal to the scattering cross-section \( \Sigma_s = N_s \sigma_s \), \( \sigma_s \) is a microscopic scattering cross-section, \( N_s \) is the density of scattering nuclei.

In the diffusion equation \( S \) is the thermal (external or delayed) neutron source density.

Here we ignore the radiation of delayed neutron reaction products, because their contribution to the wave propagation processes is insignificant unlike the contribution to the system stability. Equations of coolant hydrodynamics are given in standard denotations.

The symbol \( Q \) is the density of thermal output in unit time of nuclear fission reactions. We will enter the quantity \( EN = Q(\bar{r}, t)/(\rho c_p) \), where \( N \) is the neutron density. Note that in the case of slowing down of thermonuclear neutrons we denote by \( Q \) the density of their energy loss in unit time and in unit volume. The last expression in line (4) is the coolant condition equation.

Propagation of neutron temperature oscillations takes place against a background of the stationary or quasi-stationary multiplication medium. The simplest model of
the stationary state is a one-dimensional model with a constant density \( \rho_0 \) and velocity \( U \) of coolant, neutron density \( N_0 \) and temperature \( T_0 \), which satisfy the equations (we assume that variations of velocity and pressure are insignificant and ignore them):

\[
U \frac{dN_0}{dz} = \frac{d}{dz} \left( D_{\text{ef}} \frac{dN_0}{dz} \right) + \nu_c (K - 1) N_0 + S ,
\]

\[
U \frac{dT_0}{dz} = E_N^0 + \frac{d^2T_0}{dz^2} + q_T .
\]

Linearization of the system of equations (1) - (4) is performed by replacement

\[
N = N_0 + \eta(z,t), T = T_0 + T_1(z,t), \rho = \rho_0 + \rho_1(z,t) \quad \text{and} \quad \bar{\nu} = (0,0,U + U_1(z,t)) .
\]

Linearized equations of the coolant motion and continuity

\[
\rho_0 \bar{\partial}^2 T_1/\bar{\partial}t^2 = -S^2 \bar{\partial}^2 \eta/\bar{\partial}z^2 - \left( P_T \right)_\rho \bar{\partial} T_1/\bar{\partial}z ,
\]

\[
\bar{\partial} \eta / \bar{\partial} t + (\rho_0 U + \rho_0 U_1)/\bar{\partial}z = 0
\]

need explanations. As is shown in paper [3], perturbations of local coolant acceleration \( \partial^2 T_1/\partial t^2 \) and coolant density inertia \( \partial \eta / \partial t \) result in the description of acoustic wave excitation. However, a sonic branch of oscillations is separated from neutron-temperature oscillations and therefore it is neglected by the indicated derivatives in times.

Propagation of oscillations with a given frequency of external sources is determined in the quasi-classic approximation [3] by the values of the complex wave vector of the dispersion equation, which is the condition of solvability of the homogeneous system of linear equations for perturbation values. This equation is as follows

\[
\omega_D(k) \omega_\chi(k) = a_1 + a_2 + A .
\]

Separately equal to zero factors are

\[
\omega_D(k) = \omega - k U + i k^2 D_{\text{ef}} - i(K-1)\nu_c
\]

and

\[
\omega_\chi(k) = \omega - k U + i \xi ,
\]

where \( \xi = \Gamma - E_N^0 \delta / T_0 \) and \( \Gamma = \gamma \left( E_N^0 / \rho \right) s^2 \left( P_T \right)_\rho \)

describes the contribution of neutron eigen- and convective modes of hydrodynamic oscillations. Parameters \( \Gamma \) and \( \xi \) reflect the influence of hydrodynamic subsystem of a coolant. On the right in (6) a sum is presented by the coefficients of coupling between the neutron and hydrodynamic perturbations:

\[
a_1 = \Gamma (\lambda + i k U), a_2 = -D_{T,G} E(N_0 + i k N_0) ,
\]

\[
A = -E_{N_0} (\lambda + \nu_c) / \delta / T_0 , \delta < 0 , \lambda = \nu_c (K - 1) .
\]

The solution of equation (6) is found by the perturbation theory in the approximation of weak coupling between the neutron and temperature oscillations. In a zero approximation \( k = k_{1,2} \) are the solutions of quadratic equation.

\[
\omega - k U + i k^2 D_{\text{ef}} - i(K - 1)\nu_c = 0 .
\]

The complex roots of this equation contain the next real and imaginary parts

\[
\text{Re} k_{1,2} = \pm \frac{1}{2 \sqrt{D_{\text{ef}}}} \left[ \sqrt{\omega + \sqrt{\omega^2 + B^2}} + \sqrt{\omega^2 + B^2 - \omega} \right] ,
\]

\[
\text{Im} k_{1,2} = \frac{1}{2 \sqrt{D_{\text{ef}}}} \left[ -ib \pm \frac{1}{2} \left[ \sqrt{\omega + \sqrt{\omega^2 + B^2}} - \sqrt{\omega^2 + B^2 - \omega} \right] \right] .
\]

Here the denotations \( b = U / (2 \sqrt{D_{\text{ef}}}) \) and \( B = \nu_c (K - 1) - b^2 \) are entered.

The expressions for \( \text{Re} k_{1,2} \) describe the wave numbers of waves propagating in the mutually opposite directions. A wave moving in the direction opposite to the coolant motion undergoes the spatial damping at all values of parameters. The amplitude of a neutron wave moving along the coolant flux with oscillations of a low velocity and a high frequency decreases too. In the case of the coolant velocity increasing or the frequency \( \omega \to 0 \) decreasing this wave changes the character of propagation from the amplitude decrease to its increase.

From dispersion equation (6), taking into account the connection coefficients by the perturbation theory, we find the correction to the wave number of neutron waves

\[
\delta k_{\omega} = (A - \nu_c (K - 1) \Gamma) \left[ \left( \lambda \omega \right)_\omega ((\omega - i k D_{\text{ef}} )) \right] , \quad \text{where} \quad k_N \quad \text{is one of the complex roots of equation (7). Other parameters} \quad \Gamma \quad \text{and} \quad A \quad \text{are written above.}
\]

The temperature oscillation branch in zero approximation is described by the expression:

\[
k_T = (\omega + i \xi) / U .
\]

It is easy to see from this formula that the temperature oscillations propagate with a phase velocity equal to the coolant velocity \( \omega / k = U \) and have an insignificant spatial damping.

\[
\text{Re} k_{3}(\omega) \quad \text{and} \quad \text{Im} k_{3}(\omega)
\]

\[\text{Fig. 1. Real and imaginary parts of wave numbers of the temperature oscillation branch as a function of the frequency } \omega. \text{ Def}=10^7, \nu_c=10^6, v_c=10^6, K=1.00001, \delta=0.2 \]

In Fig. 1 the behavior of curves is approximated similarly to the case described for the dispersion and damping of an eigenvalue convective mode according to the solution of Eq. (6). The dispersion, \( \text{Re} k_3(\omega) \), being the dependence of the real part of wave number on the frequency, differs from the linear one. The coefficient of spatial damping, depending on the frequency, decreases with frequency increasing (not included in formula (8)).
Characteristic equation (6) is the cubic one in relation to the complex unknown $k$. Under conditions when it is not possible to ignore the right part of the equation, a close coupling takes place between oscillations of the temperature and neutron branches. In such case, the numerical solution of dispersion equation (6) is used. A similar solution is shown in Fig. 2.

The parameters given in captions of Fig. 2 approximately correspond to the operation modes of WWER-1000 reactors.

In the case of relatively low frequencies of oscillations ($\omega < 20$ 1/s) moving in the direction of coolant motion a neutron wave undergoes a spatial growth.

In the case of high frequencies, a wave undergoes the spatial decrease of its amplitude, and the value of the real part of wave number increases proportionally to the square root of the frequency. The wave propagating in the opposite direction is damping for all the values of parameters.

CONCLUSIONS

The medium oscillations arising form some spatial temporal structures (for example, the areas of the coolant-and-fuel superheating) which excite neutron oscillations propagating in the medium to the place of their recording. The fastened elements of equipment also sense vibrations, which modulate the oscillations of the neutron and hydrodynamic fields. The problem consists in the study of oscillation propagation conditions in order to record oscillations and to control the behavior of equipment and component parts of the multiplication medium.

Analysis of calculation results shows that in the case when the frequency of external sources increases the values of the real part of wave number, as well as, the coefficient of the spatial damping of neutron oscillations are increasing. In the case of low frequency values the neutron waves, propagating along the direction of coolant flux motion, undergo a spatial increase. This new result is due to the taking into account the convective transfer of the neutron field and temperature of the coolant. The range of increasing oscillation frequencies narrows as the supercriticality characteristic for industrial power-reactor decreases. However, the range of frequencies near to one hertz is important for the aims of noise diagnostics.

As the coefficient of neutron diffusion decreases, the wave numbers of the neutron branch of oscillation and their damping coefficients are increasing and can make the hundredth parts of inverse centimeter.

It is clear, that under such conditions the neutron field oscillations can move by a convective temperature mode being weakly damping at not low frequencies. Using the results of the analysis of characteristics describing the neutron wave propagation it is possible to measure the coefficients of neutron diffusion in nuclear-physical systems (for example, in the subcritical assemblies with external sources of thermal neutrons or moderated neutrons).

Strongly coupled neutron-temperature oscillations are resulting from the dependence of the frequency of neutron capture by fission nuclei on the "effective" temperature of thermal neutrons.

REFERENCES


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