CURRENT CATHODE SPOTS IN THE GLOW DISCHARGE NORMAL REGIME AS A STATIONARY DISSIPATIVE STRUCTURE: MACROSCOPIC PARAMETERS

O.P. Ponomaryov¹, I.O. Anisimov²

Taras Shevchenko National University of Kiev, Radio Physics Faculty, Kiev, Ukraine
E-mail: alex.ponomaryov@gmail.com¹, ioa@univ.kiev.ua²

Effect of the normal current density in the dc glow discharge normal regime is studied using a phenomenological model of stationary dissipative structures caused by the distributed feedback. The cathode layer is described as a bistable medium, governed by the Fisher-type reaction-diffusion equation for ions' density. Stability of normal cathode current structure and existence of the current-voltage characteristic section with the current independent voltage drop is explained using the proposed analogy. The main parameters of the dissipative structure such as travelling wave velocity and critical current of abnormal regime transition are calculated. Obtained results are in good correspondence with the parameters obtained from the numerical simulation of dc glow discharge.

PACS: 52.65.Kj, 52.80.Hc

1. INTRODUCTION

Cathode layer of the glow discharge in normal regime can be described form the point of view of the active systems’ theory. The basic effects, observed in the cathode region such as (i) current independent voltage drop, (ii) normal current density, and (iii) existence of separate current spots can be treated as self-organization phenomena [1, 2]. In this work these effects are described using model of the dissipative structure caused by the distributed feedback [3, 4].

2. KINETIC EQUATION

It is well known that main processes in the cathode layer of glow discharge are impact ionization and the ions’ drift to the cathode governed by superposition of external and eigen electric fields. Since electrons’ relaxation time is smaller than ions’ one, we assume that electrons’ densities can be always found from the instantaneous ion distribution and external electric field. So the initial set of hydrodynamic equations can be reduced to the reaction-diffusion equation for mean ions’ density $<n_i>z$:

$$\frac{\partial<n_i>_z}{\partial t} = f(<n_i>_z, V) + \frac{1}{r} \frac{\partial}{\partial r} \left( r D_{eff} \frac{\partial<n_i>_z}{\partial r} \right),$$  \hspace{0.5cm} (1)

were $z$-axis is directed along the discharge gap ($z = 0$ on the cathode, $z = L_z$ on the anode, $h_z$ is the cathode layer thickness), $r$ is radius in the cylindrical coordinates. Below we omit the averaging symbol $< > z$ for ions’ density. Bistability of the cathode layer can be described by kinetic function $f(n_i, V)$ with the voltage drop $V$ as external parameter:

$$f(n_i, V) = -\frac{n_i \mu E_z(n_i)}{h_z} + \alpha(n_i) \gamma n_i \mu E_z(n_i) 0.5 \left( \frac{\mu}{\gamma} + 1 \right),$$  \hspace{0.5cm} (2)

were $\mu_i$ is the ions’ mobility, $\alpha$ is the Townsend coefficient, $\gamma$ is the secondary electron emission coefficient, and $\mu$ is the reproduction coefficient. Electric field magnitude along the cathode layer can be approximated by the expression:

$$E_z(n_i) = E_z^{cr}(n_i) + E_z^{ex} = 4 \pi q h_z n_i \left( \frac{1 - n_i}{n_i^{cr}} \right) + \frac{V}{L_z},$$  \hspace{0.5cm} (3)

were $q$ is the elementary charge and $0.5 n_i^{cr}$ is the ions’ density corresponding to the field magnitude $V/h_z$ in the cathode layer. $D_{eff}$ is the effective diffusion coefficient describing diffusion-drift motion of ions along the cathode surface:

$$D_{eff}(n_i) = D_i + 2 \pi e h_z^2 \mu_i n_i \left( \frac{1 - n_i}{n_i^{cr}} \right).$$  \hspace{0.5cm} (4)

3. CURRENT-VOLTAGE CHARACTERISTIC AND STABILITY ANALYSIS

Current-voltage characteristic can be obtained using the set of conditions [4]:

$$f(n_i^{critical}, V^{critical}) = 0; \int_{n_i}^{n_i^{critical}} f(n_i, V^{critical}) dn_i = 0.$$  \hspace{0.5cm} (5)

Values $(n_i^{critical}, V^{critical})$ correspond to the traveling wave with the zero velocity.

If the discharge voltage drop $V$ exceeds the critical value $V^{cr}$ then discharge self-maintenance condition

$$\mu = \rho \exp \left[ \int_{0}^{y} \exp \left( \frac{-B p}{E(x)} \right) dx \right] - 1 \geq 1$$  \hspace{0.5cm} (6)

is satisfied in larger region, and traveling wave described by equation (1) spreads along the cathode. As a result the cathode spot grows. The total discharge current increases, and consequently voltage drop on the external resistance $V_{Res}$ also increases. This effect results to the decrease of the discharge gap voltage drop $V$. When voltage $V$ decreases to the threshold $V^{cr}$, the traveling wave front stops, so voltage $V^{cr}$ and normal density $n_i^{cr}$ remain constant until the normal ion density doesn’t fill the whole cathode surface.

4. TRAVELING WAVE VELOCITY

Using the plane geometry ($r \rightarrow \infty$) and automodel substitution

$$n_i(y, t) = n_i(\xi), \quad \xi = y - c_v t,$$  \hspace{0.5cm} (7)

were $c_v$ is the travelling wave velocity.
where \( c \) is the velocity of auto-wave, one can obtain the following equation for \( n_i \) from (1):

\[
\frac{d^2 n_i}{d \xi^2} + b \frac{d n_i}{d \xi} \left( \frac{1}{n_{i,cr}} - \frac{1}{n_i} \right) + \left( c_c \frac{d n_i}{d \xi} + f \left( n_i, V \right) \right) = 0, \quad b = 2 \pi e h_0^2.
\]

Unknown constant \( c_0 \) can be obtained from the condition that phase trajectory on the plane \((n_i, \frac{d n_i}{d t})\) starting from the saddle point corresponding to the right maximum of the potential (Fig. 1) should come to another saddle point corresponding to the left potential maximum (Fig. 2) [4].

\[
I = n_{i,cr}^{(i)} \mu \xi q S' \left( 1 - S' \right) \left( n_{i,cr}^{(i)} + \frac{V}{L_c} \right)
\]

where \( S' \) is the full area of zero-current phase, and \( S \) is the full area of cathode, so coexistence of two phases is possible when condition

\[
\left( 1 - S' \right) \left( n_{i,cr}^{(i)} + \frac{V}{L_c} \right) \mu q S > I
\]

is fulfilled (in this case total number of separate current spots is not limited). Inequality (10) is transformed into equality with the discharge current increasing, and low current density regions disappear in this case.

\[
\text{Fig. 1. Kinetic function (1) and potential (2) for}
\]
\[
p = 5 \text{ Torr}, \quad V = 197 \text{ V}, \quad h_z = 0.065 \text{ cm}
\]

\[
\text{Fig. 2. Potential distribution and phase trajectories for}
\]
\[
different cases \( c > c_0 \) (1), \( c = c_0 \) (2) and \( c < c_0 \) (3)
\]

Calculated dependence of the velocity \( c_0 \) on the spot radius is presented on Fig. 3. It is similar to the usual dependence of the traveling front on its radius of curvature [4].

5. CRITICAL CURRENT VALUE

If the area of transitional region is much less then areas of the stationary phases, the current across the discharge gap is given by the relationship

\[
\text{Fig. 3. Obtained velocities of the traveling wave as a function of radius for different applied voltage:}
\]
\[
V = 200 \text{ V (1), } V = 190 \text{ V (2); } p = 5 \text{ Torr}
\]

6. COMPARISON WITH THE RESULTS OF HYDRODYNAMIC SIMULATION

Obtained parameters of the glow discharge cathode layers were compared with the numerical simulation results [5]. Main data are presented in the Table. It is clear from this table that analytical results are in good correspondence with results of numerical simulation in the hydrodynamic approach.

<table>
<thead>
<tr>
<th>( p, \text{Torr} )</th>
<th>( n_i, 10^9 \text{cm}^{-3} )</th>
<th>( E, \text{V/cm} )</th>
<th>( j, \text{cm}^{-2} \text{s}^{-1} )</th>
<th>( V, \text{B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
<td>4.7</td>
<td>0.79</td>
<td>0.80</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>26</td>
<td>1.46</td>
<td>1.56</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

The theory of dissipative structures caused by the distributed feedback was applied to analysis of the glow discharge normal regime. It permits to explain the effect of normal current density, conservation of the discharge voltage drop and increase of the current density at transition to abnormal regime.

1. New way to obtain the kinetic function for this case was proposed. Kinetic function was presented in the form of continuous analytic function.
2. Critical discharge current that moves to abnormal regime transition was obtained analytically.
3. Analytical results are in good correspondence with results of numerical simulation in the hydrodynamic approach.

REFERENCES

Article received 14.09.10