INTRODUCTION

A theoretical model of a semi-infinite dielectric waveguide is adequate for a number of pilot schemes for the acceleration of electrons by wavefields excited by a sequence of relativistic electron bunches. The presence of the front boundary leads to a number of features of the wavefields excitation in semi-infinite dielectric waveguide [1, 2]. First of all, in such systems, there is a “quenching” wave that restricts the number of bunches which wavefields are coherently summed. In addition there is a transition radiation bunches chain on the boundary of semi-infinite dielectric waveguide.

An important problem in the excitation theory of wavefields of electron bunch sequence in the segments of the dielectric waveguide is the dependence of the wavefield on the length of the segment. This relationship brightly reflects influence of the wave quenching on the excitation of wakefield sequence bunches in the waveguide segment.

This paper presents the results of theoretical investigations of wavefields excitation process, in total case, non-resonant sequence of electron bunches in semi-infinite dielectric waveguides. There is view sequence of bunches, frequency of which is close to the frequency of the excited wakefield.

Two schemes of semi-infinite waveguide are considered. In the first scheme, considered before in the 2nd quarter, the sequence of bunches are injected from the perfectly conducting diaphragm located at the face of semi-infinite waveguide. In the second scheme, the sequence of bunches moves in the vacuum region of the waveguide and then transverses the sharp boundary of semi-infinite dielectric waveguide. Dependence of amplitude of a wakefield on length of the segment of dielectric waveguide is presented too. This dependence brightly reflects influence of the quenching wave on the process of wavefields excitation by a sequence of bunches. To increase the accelerating gradient in dielectric waveguide the effect of coherent addition of the transverse modes of the excited wakefield can be used. Most effectively process of addition takes place at the frequency spectrum of the transverse modes which is closed to the equidistant one. In this case the wavefield looks like a sequence of narrow peaks of an opposite polarity of large amplitude.

In the paper of the process of wavefield excitation in a semi-infinite plane dielectric waveguide by sequence of electron bunches is studied too. It is known that in such slowing down structure the frequency spectrum of excited transverse wakefield modes is equidistant. Respectively, as a result of the coherent addition of wakefield transverse modes the resultant wakefield field looks like sequence of narrow peaks. Influence of boundary of semi-infinite plane dielectric waveguide (in the bunch propagation direction) upon the process of excitation of the large amplitude wakefield is considered.

1. WAKEFIELD OF NONRESONANCE SEQUENCE OF ELECTRON BUNCHES IN SEMI-INFINITE CIRCULAR DIELECTRIC WAVEGUIDE

Let’s consider the following model of semi-infinite dielectric waveguide. The semi-infinite dielectric waveguide occupies region \( \omega > z \geq 0 \). The butt \( z = 0 \) is short-circuited by perfectly conducting diaphragm. In the axial region there is a vacuum channel. In the waveguide from conducting diaphragm the sequence of relativistic bunches with initial value of a relativistic factor \( \gamma_0 >> 1 \) is injected. Longitudinal and transverse profiles of bunches density have rectangular profiles with sharp boundaries. Let’s neglect influence of the narrow vacuum channel on electrodynamics of system and for simplification of calculations we will consider dielectric filling continuous. The solution of the electrodynamics problem by method of Fourier transform gives the following expression of the wavefield excited by nonresonance sequence of bunches in a semi-infinite dielectric waveguide:

\[
E_x = E_x^{\Delta} + E_x^r,
\]

\[
E_x^{\Delta} = \frac{8Q}{b}\sum_{n=1}^{\infty} \sum_{j=0}^{\infty} \frac{J_0(\lambda_j r_p / b) J_0(\lambda_n r_p / b)}{\omega r_p J_0^2(\lambda_n)} \left[ Z_{0n}^{(\Delta)} - Z_{0n}^{(\rho)} \right],
\]

where \( Q \) is the bunch charge, \( r_p \) is the radius of the bunches, \( t_b \) is the duration of a single bunch, \( \omega = \lambda_n / b\sqrt{\varepsilon - v_0^2} \) are frequencies of radial modes, \( \varepsilon \) is dielectric constant, \( \lambda_n \) are roots of Bessel function \( J_0(x) \), \( b \) is waveguide radius,

\[
\sin \omega_z (t - z - v_0sT) > \sin \left[ \omega_z (z - v_0sT) / v_{gs} \right],
\]

\[
Z_{0n}^{(\Delta)} = \left\{ \begin{array}{ll}
\omega_g (t - sT) > z > v_g(t - t_b - sT), \\
\omega_g (t - sT) > z > v_g(t - t_b - sT),
\end{array} \right.
\]

\[
Z_{0n}^{(\rho)} = \left\{ \begin{array}{ll}
\omega_g (t - sT) > z > v_g(t - t_b - sT), \\
\omega_g (t - sT) > z > v_g(t - t_b - sT),
\end{array} \right.
\]

\[
z < v_g(t - t_b - sT),
\]

\[
E_x^r = E_x^{\Delta} + E_x^r,
\]

\[
E_x^{\Delta} = \frac{8Q}{b}\sum_{n=1}^{\infty} \sum_{j=0}^{\infty} \frac{J_0(\lambda_j r_p / b) J_0(\lambda_n r_p / b)}{\omega r_p J_0^2(\lambda_n)} \left[ Z_{0n}^{(\Delta)} - Z_{0n}^{(\rho)} \right],
\]
is the velocity of bunches, \( v_p \) is the group velocity of wakefield waves,

\[
Z_{mn}^{(s)}(\tau_s) = \left\{ \begin{array}{ll}
\sin \alpha_0 (t - z / v_0 - sT), & z > v_0 (t - L - sT), \\
\sin \alpha_0 (t - z / v_0 - sT) - \sin \alpha_0 (t - z / v_0 - t_0 - sT), & z < v_0 (t - L - sT), 
\end{array} \right.
\]

\( v_0 \) is the velocity of bunches, \( v_p = c^2 / v_0 e \) is the group velocity of wakefield waves,

\[
E_p^x = \frac{8Q}{b^2 e t_b} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} J_m (\lambda_m b r / b) J_n (\lambda_n r / b) \lambda_m J_n^2 (\lambda_n) S_{mn}, \quad (2)
\]

\[
S_{mn} = \frac{1}{2} \int_{0}^{\infty} \left[ \theta(t - z / v_p - sT) - \theta(t - z / v_0 - sT) \right] \times
\]

\[
\times \sum_{n=1}^{\infty} (-1)^n (r_{in}^2 - r_{in}^2) J_{2n+1}(y_r) +
\]

\[
+ \theta(t - z / v_0 - sT) \left[ J_1(y_r) + \sum_{n=1}^{\infty} (-1)^n (r_{in}^2 + r_{2n}^2) J_{2n+1}(y_r) \right].
\]

\( v_p = c / \sqrt{\varepsilon}, \quad y_r = \alpha \sqrt{\varepsilon \varepsilon - \zeta^2}, \quad \alpha = k_0 c / \sqrt{\varepsilon}, \quad \varepsilon = 1 - sT, \quad \zeta = z \sqrt{\varepsilon / c}, \quad \beta = (\varepsilon - \zeta) / (\varepsilon + \zeta).
\]

The term (1) describes Cherenkov field of bunches sequence with the quenching waves, the term (2) describes the transition radiation of the bunches chain, caused by existence of perfectly conducting butt \( z = 0 \). In expressions for fields (1), (2) external summing on index \( s \) corresponds to summing on bunches, and summing on index \( n \) corresponds to summing on radial modes of the dielectric waveguide.

Let’s consider now excitation of a wakefield in semi-infinite dielectric waveguide by periodic sequence from \( N \) infinitely thin bunches. We will be restricted to the most interesting case of existence of detuning between the frequency of bunches repetition and frequency of the excited wave. Numerical calculations were performed for different numbers of bunches in chain and value of the relative detuning. The pictures of the wakefield excitation field in semi-infinite dielectric waveguide depend on number of bunches in chain. Fig. 1 shows the spatial structure of the excited field for 22 bunches. It can be seen in semi-infinite waveguide as the distance from the conductive diaphragm two pulses of field, the following one after another are formed.

The first pulse of a wakefield is caused by the Cherenkov radiation of bunches and propagates with bunches velocity. The second pulse is caused by existence of boundary and propagates with group velocity which in dielectric waveguide always is less than velocity of bunches. As for the first pulse, in case of the chosen value of detuning the phase of wakefield changes on \( \pi \) after the tenth bunch. The Cherenkov wakefield in vicinity of the tenth bunch reaches the maximum value. After the twentieth bunch the phase of a field changes on \( 2\pi \). Thus wakefield is zero. Last 21 and the 22nd bunches excite in region between field pulses the monochromatic wake wave having relatively low level. With increase of number of bunches in chain amplitude of monochromatic wake wave increases and for 30 bunches reaches the maximum value. This case is illustrated by Fig. 2.a. It can be seen that amplitude of monochromatic wave wake behind the first pulse is equal in accuracy to amplitude of the first field. As a whole in region \( v_0 t > z > z_p = v_0 t \) picture of wakefield excitation is the same, as in infinite dielectric waveguide.

As always \( v_p < v_0 \), the second pulse will lag behind from chain of bunches and pulse wakefield connected with chain. The distance between pulses will increase and, respectively, length of a monochromatic wave wake behind the first pulse will increase also. In Figs. 1.a, 2.a full fields which include both a Cherenkov field, and transition radiation are shown. Pictures of the transition radiation are shown in Figs. 1.b, 2.b. The pulse of the transition radiation basically is in region \( 0 < z < z_e \) and level of this pulse is rather small. In region \( z > z_e \) pulse of the transition radiation extends to the plane.

\[
\begin{array}{c}
\text{Fig. 1. Spatial structure of wakefields at the moment of time } t / T = 100, \text{ system length is } L = 102 \lambda, \text{ number of bunches is } N = 22, \text{ value of detuning is } \alpha = 0.05, \\
\text{full wakefield (a); transition radiation (b)}
\end{array}
\]

\[
\begin{array}{c}
\text{Fig. 2. Spatial structure of full wakefields at the moment of time } t / T = 100, \text{ system length is } L = 102 \lambda, \text{ number of bunches is } N = 30, \text{ value of detuning is } \alpha = 0.05, \\
\text{full wakefield (a); transition radiation (b)}
\end{array}
\]

An important characteristic of the process of wakefields excitation in dielectric waveguide segment by sequence of relativistic electron bunches is dependence of the wakefield amplitude on the length of the segment of the dielectric slow-wave structure. Input end completely is short-circuited perfectly conducting diaphragm. The output end is assumed fully match with external RF line (the reflected wave from the output end is absent). Figs. 3.a, b shows the dependence of the amplitude of the Cherenkov wakefield at the output end of the segment of the dielectric waveguide length \( L = \lambda / 4 \) and \( L = \lambda / 2 \). The number of bunches is \( N = 10 \). The field is a non-sinusoidal oscillations with a constant amplitude. Such form of the field is caused by the fact that in the dielectric waveguide of length \( L = \lambda / (v_0 / v_p - 1) \), as a result of quenching waves (effect of group velocity), a bunch does not have time to excite the whole period of wakefield oscillation. So in the process of the sequence of bunches propagation in such a short waveguide Cherenkov pulses of separate bunches do not overlap each other. The coherent addition of Cherenkov fields of single bunches is absent. Therefore at an output end we observe pulses of Cherenkov radiation of single bunches. In Figs. 3.a, 3.b it is
shown that on every period of oscillations there is a period during which the Cherenkov field is absent. This period corresponds to an interval of time (coordinate) between this bunch and hind front of Cherenkov radiation of a previous bunch. In Fig. 3,c dependence on time of full field at output end of waveguide is presented. It is well seen that the transition radiation significantly perturbs oscillations. Besides decreasing ring appears, which has been completely caused by the transition radiation. Similar dependences of fields on time are presented in Figs. 4,a,b for segment of a dielectric waveguide by length $L = \lambda / 4$.

In this case in the region adjoined to output end pulses of Cherenkov radiation of three bunches have time to be overlapped. The transition radiation (see Fig. 4,b), as well as the previous case, strongly perturbs Cherenkov oscillations.

Further growth of field amplitude doesn't come due to removal of the field of previous bunches from wave guide with group velocity. Thus amplitude of the wakefield reaches stationary level. Value of maximum level of the wakefield amplitude depends on number of pulses of separate bunches which are able to overlapped on the waveguide length. After exiting of the last bunch the field amplitude falls down to null value in time which equals to the time rise of amplitude.

In Fig. 5,a it is presented dependence of amplitude of the longitudinal component of electric field $|E_z(t)|$ of wakefield on length of dielectric waveguide is presented. Averaging was performed on realization length. It is seen that with increasing of waveguide length amplitude of wakefield grows practically under the linear law. Such regularity is explained by that with waveguide lengthening as a result of lag of quenching wave increases the number of the bunches which fields coherently are added.

In all previous cases the semi-infinite dielectric waveguide which input end is short-circuited perfectly conducting diaphragm was considered. Important conclusion of the theory of wakefields excitation by electron bunches is formation in such semi-infinite slow wave structure of quenching wave of Cherenkov radiation. This wave plays a basic role in processes of wakefields excitation both for single electron bunches, and for sequences of bunches. Below we will consider a semi-infinite dielectric waveguide of other geometry [3]. In perfectly conducting metal pipe the region $z > 0$ is filled with dielectric with the vacuum channel at vicinity of axial region. In region $z < 0$ it is vacuum. From vacuum region into dielectric the relativistic electron bunch flies. Let's consider excitation of wakefield for considered geometry of semi-infinite dielectric waveguide. Let's solve the electrodynamics problem by method of expansion of field on Bessel functions with respect to radial coordinate and Fourier transform with respect to time. As a result for a longitudinal component of an electric wakefield in the region filled with dielectric $z > 0$, we will obtain the following expression

$$
E_m = E_m^{ch} + E_m^{ord}, \quad E_z = \sum_{n} E_n J_n(\kappa_n R / b),
$$

$$
E_m^{ch} = i Q \frac{4}{b^2} \frac{J_1(\kappa_n r_0)}{J_1(\kappa_n)} I_n^{ch}, \quad E_m^{ord} = i Q \frac{4}{b^2} \frac{\kappa_n^2}{J_1(\kappa_n)} I_n^{ord},
$$

$$
I_n^{ch} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{k_n^2 \omega - k_\perp^2}{k_n^2 - k_\perp^2} e^{-i\omega(z - v_0 t)},
$$

$$
I_n^{ord} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{1}{k_n^2 + \omega^2 + \omega k_n} \left[ \frac{1}{k_n^2} - \frac{k_n \omega + k_\perp}{e^{k_n^2 - k_\perp^2}} \right].
$$
where \( k_l = \omega / v_g \), \( k_{z0} = \sqrt{k_0^2 - \lambda_v^2 / b^2} \) are longitudinal wave numbers in vacuum and dielectric respectively, \( k_0 = \omega / c \). Integral \( I^0\) describes wake Cherenkov in boundless dielectric and integral \( I^{\text{bord}} \) takes into account the presence of a boundary \( z = 0 \) in semi-infinite dielectric waveguide. The integral describes wake Cherenkov in boundless dielectric, and the integral considers boundary existence at a semi-infinite dielectric waveguide. Wakefield in boundless dielectric waveguide we find, using the theorem of residues

\[
E_{z0} = \frac{4Q}{b^2} \int_0^L J_0(\lambda_l/b) J_l^1(\lambda_v) \cos \omega_z (t - t_0 - z / v_g) \d t,
\]

\[
\omega_z = \frac{\lambda_v v_g}{b \sqrt{v_v^2 / c^2 - 1}} \text{ are frequencies of dielectric waveguide eigenwaves.}
\]

The analysis of integral by the approximate method proposed by E.L. Burstein and G.V. Voskresensky [2], shows that in case of consideration of condition \( v_g(t - t_0) \gg z \gg v_g(t - t_0) \) it is necessary to take into consideration a pole in the integrand expression \( I^{\text{bord}} \). Residues in poles give a field of quenching wave

\[
E_{z0} = \frac{4Q}{b^2} \int_0^L J_0(\lambda_l/b) J_l^1(\lambda_v) \cos \omega_z (t - t_0 - z / v_g) \d t.
\]

The field of quenching wave in accuracy coincides on value with Cherenkov field, but has an opposite sign. Summing of these fields shows that the Cherenkov wakefield exists in region \( v_g(t - t_0) \gg z \gg v_g(t - t_0) \).

2. MULTIMODE REGIME WAVEFIELD SEMI-INFINITE PLANE DIELECTRIC WAVEGUIDE

For increasing of the field transformation ratio in dielectric waveguide may be used the effect of the coherent addition of a radial (transverse) harmonic of an excited wakefield. Most effectively process of addition takes place under frequency spectrum of transverse harmonic as much as possible the close to the equidistant. Let's consider a semi-infinite waveguide, having form of two parallel ideally conducting planes, distance between which is \( L \). The waveguide is completely filled with uniform dielectric with dielectric permittivity. The input end face is short-circuited by perfectly conductive transverse wall. In volume of waveguide from transverse wall the sequence of bunches with period \( T \) is injected. Let's chose the following model transverse profile of bunches

\[
R(x_0) = \left\{ \begin{array}{ll}
\cos \left( \frac{\pi x_0}{2 x_s} \right), & x_0 \geq x_s \\
0, & 0 \leq x_0 < x_s
\end{array} \right.,
\]

\( x_0 \geq x_0 \geq -x_s, \)

\( 0, L / 2 \geq x_0 \geq x_s, \)

\( -x_s \geq x_0 \geq -L / 2. \)

In the longitudinal direction bunches are infinitely thin. Expression for the wakefield in semi-infinite plane dielectric waveguide has form

\[
E_z = E_{z0}^0 + E_{z0}^r,
\]

\[
E_{z0}^0 = \frac{2\pi Q}{b c} \sum_{p = -n}^{n} \sum_{m = 0}^{\infty} R_m(x) \times
\]

\[
\times \left[ \theta(t - \frac{z}{v_g} - sT) - \theta(t - \frac{z}{v_e} - sT) \right] \cos \omega_z (t - \frac{z}{v_g} - sT),
\]

\[
E_{z0}^r = \frac{8\pi Q}{b c} \sum_{p = -n}^{n} \sum_{m = 0}^{\infty} R_m(x) \left[ \theta(t - \frac{z}{v_e} - sT) - \theta(t - \frac{z}{v_g} - sT) \right] \times
\]

\[
\times \sum_{\alpha = 1}^{\infty} (-1)^\alpha \left[ (r_{\alpha 0}^\alpha - r_{\alpha 0}^\alpha) J_{\alpha 0}(y) + \theta(t - \frac{z}{v_e} - sT) J_{\alpha 0}(y) \times \sum_{\alpha = 1}^{\infty} (-1)^\alpha \left[ r_{\alpha 0}^\alpha + r_{\alpha 0}^\alpha \right] J_{\alpha 0}(y) \right],
\]

where \( E_{z0}^0 \) is field of Cherenkov radiation, \( E_{z0}^r \) is field of the transition radiation, \( Q \) is charge of bunch, \( R_m(x) = \cos(k_{\perp 0} x_0) \cos(k_{\perp 0} x_0) / (\pi^2 - k_{\perp 0}^2 x_s^2) \quad k_{\perp 0} = \pi n / L \) are transverse wave numbers, \( \omega_0 = \pi v_g / L \sqrt{\varepsilon_0 / c^2 - 1} \) are equidistant frequencies of transverse modes, \( v_e = c / \sqrt{\varepsilon} \) is velocity of harbringer propagation.

Fig. 6. Spatial structure of multimode wakefields at the moment of time \( t / T = 60 \) (a); fragment of Fig. 6,a (b)

The spatial structure of wakefield excited by sequence of electron bunches in a multimode regime in the plane dielectric waveguide was calculated by numerical methods. Calculations were executed in case of the following parameter values: system length is \( L = 60 \lambda \), number of bunches is \( N = 20 \), number of transverse modes is \( N_{\text{mod}} = 21 \) (increasing of modes number didn't lead to notable change of results), frequency of the transverse fundamental mode is \( f = 2.8025 \text{GHz} \), transverse size of waveguide is \( L_x = 5.12 \text{cm} \), transverse size of a bunches is \( x_0 / L_x = 0.04 \), dielectric permittivity \( \varepsilon = 2.1 \). In Fig. 6a the spatial structure of a wakefield in a multimode regime is showed.

The qualitative picture of the wakefield in multimode regime is close to the similar picture of the single mode regime. In the beginning there is the linear growth of amplitude caused by the coherent addition of fields of all twenty bunches.

Then there is region of the wakefield with constant amplitude. And, at last, on output butt amplitude decreases practically to zero. Similarity of behavior of wakefield in multimode and single-mode approximation is explained by that for considered system group velocities of all radial harmonic identical and are equal \( v_e = c^2 / v_g e \). Therefore group fronts of all harmonics coincide. In Fig. 6,b the increased fragment of wakefield oscillations from region of constant value of amplitude is shown. It is seen that oscillations look like of sequence of narrow pulses with opposite polarity. Thus
amplitude of narrow pulses amplified, approximately, by 5 times in comparison with the single-mode approximation (see Fig. 5,c).

CONCLUSIONS

Thus, in the paper process of excitation of wakefields in a semi-infinite dielectric waveguide by sequence of relativistic electron bunches in the presence of detuning between the repetition frequency of bunches and frequency of the excited wave is considered. It is shown that in a semi-infinite dielectric waveguide two pulses, the following one after another are formed. The first pulse of a wakefield propagates with the velocity of bunches. The second pulse propagates with group velocity and is caused by existence of boundary. Between pulses of the field there is monochromatic wake wave of the constant amplitude, which value depends on number of bunches in sequence. In the first pulse of wakefield may be realized regime of auto acceleration of the electron bunches located in the region of wakefield decreasing.

The alternative scheme of semi-infinite dielectric waveguide which is the metal pipe, which half-space is filled with dielectric is considered. Exact expressions for Cherenkov radiation with taking into account quenching wave for wakefield are found.

Dependence of a wakefield on time at output end of the dielectric waveguide segment for different lengths of waveguide is investigated. Also dependence of average value of the electric field module on waveguide length is obtained. It is shown that with increasing of waveguide length the average wakefield grows, approximately, under the linear law. Process of wakefields excitation in semi-infinite plane dielectric waveguide by sequence of electron bunches in the multimode regime is considered. It is shown that the coherent addition of transverse modes with the equidistant frequency spectrum leads to excitation of a wakefield in the form of sequence of narrow pulses of field with opposite polarity. Thus there is the strong increasing of pulses amplitude. It is studied influence of waveguide input boundary which lead to first of all to excitation of quenching waves, on process of excitation of wakefield in multimode the regime.

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