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INFORMATION THEORY AND POSSIBLE MATHEMATICAL DESCRIPTIONS OF ECONOMICAL AND SOCIAL SYSTEMS BASED ON REAL PHYSICAL PHENOMENA

M. GAETA, G. IOVANE, A. MAKARENKO

Recent approaches in informatics to model large complex systems are considered following the ideas from real phenomena explained by physical tools. The econophysics and sociophysics are considered. In particular, Master Equation approach and Markov chains approaches are discussed. Also the partial differential equations as the tool for modeling economical and social systems are represented. New approaches for modeling systems with memory and with accounting internal properties of system elements are considered and some new research problems are proposed.

INTRODUCTION

Now the wide development of informational technologies requires the application of mathematical modeling for qualitative and quantitative understanding of system behavior for predicting and forecasting and for the technical design of system structures. This clearly appear for social, economical and political systems. In fact, the recent systems had become very complex in their structure. The society transformation factors are besides the informational technologies, which follows to the necessity of mentality accounting. The electronical web may serves as the bright example.

Taking into account the increasing of complexity inside the considered systems, new adequate approaches need to make model better and better. Fortunately in the past some physicists considered systems with many interacting particles, which we can use also to supply corresponding models and investigation methodologies in other knowledge fields. The process for introducing physical concepts into informatics is rather new and will spread. But just now there are a great number of results which require the understanding and interpretation. Also searching of further ways of developments and implementations of physical ideas to modeling is necessary. Let us remark that one of the sources of fruitful concept is synergetic. So at present work we make the system analysis and evaluation of existing approaches related to physical concepts. We consider master equation, partial differential equations, ordinary differential equations, Ising-type models, artificial neural network and others.

1. MASTER EQUATION APPROACH

One of the rather general approach to considering social and economical systems had followed from theoretical physics (mainly from the physics of many-particle systems) [1–3]. In accordance with physical approaches in such case it is necessary to consider different scales of process and interactions. The basic are the micro level of local interactions between elements and macro level of aggregated macro parameters. Main mechanism in such approach is stochastic transitions between element states. These transitions are rooled by the set of transition probabilities. Usual description is the string of element's states ($\vec{s}(t) = \{s_1(t), s_2(t),...,s_N(t)\}$). Then the basic equation is the so called master equation for the probability distribution of the general parameters of the system. Here we display an illustrative simple example [1] for reaching the goal of better understanding of model structure. The general master equation form is described in [1, Chapter. 4, equation (4.7)] and is involved and lengthy.

In illustrative simple case the state of social (economical and so on) systems at the moment t is described by cells representation $\left(\vec{n},\vec{\theta}\right) = \{n_1^1,...,n_C^1,...,n_1^P,...$ $...,n_C^P;\ \theta_1^1,...,\theta_C^1,...,\theta_1^P,...,\theta_C^P$, where n, θ are integers, which represents the number of elements in *i*-th cell (numbers of individuals with *i*-th opinion) and θ trend parameter (propensity to opinion) in *i*-th cell of individuals. For simplicity in the case of two sates of opinion $(+)$ or (–) the master equation has the form (equation (5.20) in [1])

$$\frac{dP(n,\theta,t)}{dt} = \Big[w_{+-}(n-1,\theta)P(n-1,\theta,t) + w_{-+}(n+1,\theta)P(n+1,\theta,t) -$$

$$- w_{+-}(n,\theta)P(n,\theta,t) - w_{-+}(n,\theta)P(n,\theta,t)\Big] +$$

$$+ \Big[q_{\uparrow}(n,\theta-1)P(n,\theta-1,t) - q_{\downarrow}(n,\theta+1)P(n,\theta+1,t) -$$

$$- q_{\uparrow}(n,\theta)P(n,\theta,t) - q_{\downarrow}(n,\theta)P(n,\theta,t)\Big]$$

where $(\vec{n},\vec{\theta})$ $P(n,\theta,t)$ is the probability for system to be at state $(\vec{n},\vec{\theta})$ at moment t.

This equation corresponds to the probabilistic transition processes between cells in number of individuals which take opinion $(+)$ or (–). The coefficients $w_{+-}, w_{-+}, q_{\uparrow}, q_{\downarrow}$ account the probability of transitions and balance of such transitions. The previous equation is the basis for many investigations and consequences. We remark that the possibility of deriving the equations for mean values from master equation is very important. Maybe the main advantage of this approach is that in principle it can bring strong background for models (and another approaches). The advantage is accompanied with some drawbacks. First of all it is very difficult to collect the values of a lot of parameters for real problems (for example the transition probabilities). So, some simple analytical formulas usually are exploited. But the main problems still is the proof of applicability of theoretical concepts in social and economical systems. Moreover, usually the applications of master equations use some kind of regularity of space elements, which is poor adjusted to real geography, social structure and so on. Besides it is very difficult to include in master equation the mental factors (mentality, beliefs, decision-making, consciousness and so on). So, other approaches may be also useful and they may take into account more adequate other 'human' aspects. However the master equation may be used for aggregated (average) models. One of the developed models of such kind for example has the form of ordinary differential equations [4, 5], where the model for opinion formation on nuclear power problem is

$$\frac{dx}{dt} = -\gamma x + k y z ,$$

$$\frac{dy}{dt} = \beta y_0 - \beta y + \varepsilon x z ,$$

$$\frac{dz}{dt} = -\alpha z + \delta x ,$$

where $x(t)$ is the difference between the numbers of supporter and opponents on some opinion at time t; $y(t)$ is the difference between the number of journalists writing about nuclear energy topics and those not writing about nuclear energy topics; $z(t)$ is the difference between the number of scientist doing their research on innovations in nuclear energy; $\gamma, k, \beta, \varepsilon, \alpha, \delta$ — some coefficients. It is interesting and very illustrative that these equations may be obtained from the master equation by some averaging procedure. Also the probabilities of opinion change may be accounting explicitly by introducing the transition probabilities [4, 5]. It is very useful that in this problem some levels of description had been exploited and strict background for o.d.e. in principle may be received.

2. ECONOPHYSICS

Recently some other physical concepts became the key issues in modeling of non-physical systems. Strong support for applications of physics was the needs in economical applications, especially in financial problems. Such field of applications has a special name ‘econophysics’. Initially such term had been coined mainly for the phenomenon of fluctuations and critical phenomena in economical data sets and processes. But such title is useful also for more wide field of physical concepts for the economics. So here we describe some of such concepts which are or may be essential to understanding and modeling in economics.

2.1. **Mean field approach**. One of the useful idea is that in many-particles systems the dynamics of each particle is determined by the influence of all another particles (or in some surrounding region) [6, 7]. It is well known the fact that in economical and social systems the behavior of elements (participants) of such system is in many cases determined by surrounding influences following to possible analogies with physical concepts. Such idea had been already used explicitly or implicitly. One of the first explicit applications of such idea was the investigations of market [8, 9]. In fact in the cellular automata approach some restricted surrounding vicinity is exploited.

2.2. **Ferromagnetics models**. Important source for many concepts, which had lead as to the self-organization theory as particularly to social and economical modeling were ferromagnetic systems, that the system which consist from a large number of small interacting magnets [6, 10]. Main properties in such systems theory are mean field, phase transition, critical indexes, renormalization groups, domens (clusters), fluctuations caused by temperature influence, potential energy surfaces, order and disorder and spin glasses.

The most known before had been Ising model of ferromagnetics. Now we posed sketch of such model because of its importance for analogies with condensed physics phenomena. In classical Ising model N elements had been considered with two possible states $+1$ and -1 (directions of spins). The stats of the system (set of element's state) tend to the minimum of the system's energy

$$E = -\sum_{i \neq j} J_{ij} s_i s_j ,$$

where J_{ij} denote the interactions between the elements. In physics J_{ij} are random and symmetrical $J_{ij} = J_{ji}$ and some transition probability for state change exist for non-zero temperature [10, 11].

The dynamics of the models for variables $\vec{s}(t) = \{s_1(t), s_2(t), ..., s_N(t)\}$ ('pattern' of situation) follows to special dynamical laws and the variables tend to one of the minima of the energy. The graphic of the 'energy' E in the space R^N has the title 'landscape' (or 'potential landscape') and has very involved character with many local minima in case of random bounds J_{ij}. Of course all of these properties should be reconsidered for the case of social and economical applications because of peculiarities of non-physical systems.

2.3. **Critical indexes**. A large number of papers had been devoted to applications of physical concepts to the description on the behavior of economical and closely related to economical variables. The examples are numerous: size of firms, probability of price change, correlations and fluctuations in time series of parameters [12–15]. Remark that one of the typical problems is searching the laws of correlation functions in dependence on distance (time intervals). Frequently the experimental data is reconstructed as power law. Many different approaches had been exploited for such goals. For example in [15] the statistical model by Sherrington–Kirkpatric had been used for imitationing the correlations laws which has stock market. Also the models remembering spin glass had been developed for investigating the market fluctuations [16, 17]. Some applications of Ising model are in [18]. The results had been interpreted in terms of transitions ('flip-flop') of spins. Numerical investigations of quasi-equilibrium process followed by abrupt change (so-called 'punctuated equilibrium', which had been found in biology [17], are especially important.

2.4. **Complex networks and related problems**. Recently the problems from subsections 2.2–2.3 follow to new theoretical and practical results which have been connected with complex networks investigating by theoretical physics methods. Many definitions of complex networks exist now but intuitively many systems had been recognized as complex. The examples are electrical power networks, trophycal chains, ecological systems, world-wide web, phone-call networks, economics, language, ontological models of knowledge, social networks, communicational networks and many others [13, 19–21]. The complex network is the object which consists from many nodes (elements) and connections between elements; moreover, different elements can have different numbers of connections. We remark that frequently many problems had been reformulated as the graph theory problems.

An example of typical problem is the evaluation of $P(k)$ — the probability that a randomly selected node will have k connections with the rest elements. This problem had been intensively investigated since Erdosh and Reniy in the graph theory and now one of the results is the power law for the probability $P(k)$. Another important problem is the percolation problem [13, 22]. In such case the connected subpart of network is investigated (more frequently for regular networks). One of the most important facts is the existence of a thresholds for k. If k is less then some critical k^* then the network consists of isolated clusters (connected subparts of whole network). If $k > k^*$ than network has one large cluster. The examples of percolation problems supply the electrical networks or water channel systems, capillary networks in porous media. Spread of epidemic or resistance to the attacks (which imitates by some part of nodes removal) also is very interesting. Physical approaches, physical analogies are very useful for investigations of phenomena above. Remark here only the master equation approach for probability that the node added at the moment t has k connections [13, 23]. Also the models for number of connections for *i*-th node of o.d.e. type may be applied.

Recently new sub branch of network investigations had been created namely small-world networks [13, 19, 24]. This term is related to the network with intermediate extent of regularity between regular network with small number of connections and random graph with a great number of connections between nodes. But now it is recognized that a lot of natural, social, technical systems have the formal characteristics of small worlds. One of the basic models is the small-world network on the ring [13, 19]. In this case each element is connected to $2k$ neighbors (usually $k < N/2$). Also some part of bounds randomly relates far elements. One of the questions is the next: How many bounds need to reach one element from another? If N is the number of nodes, k — number of neighbor connected nodes and p — probability to randomly rewire each node, then average path length evaluation is

$$P(N,p) \sim \frac{N}{k} f(pkN^{\alpha}),$$

$$f(u) = \begin{cases} \text{const} \ldots \text{if} \ldots u \ll 1, \\ \ln(u)/u \ldots \text{if} \ldots u \gg 1. \end{cases}$$

Maybe the most intriguing property is that if $p \geq 2/Nk$ the l begin increases and the transition level depends on the system size. Remark that such kind of results had been received by statistical physics methods. As another special type of growing network behavior, the power law distribution $P(k) \sim 2m^{1/\beta} k^{-\gamma}$ had been proved for asymptotical case ($t \to \infty$), where k is the degree of node (number of connections), $\gamma = \frac{1}{\beta} + 1$. Continuum theory and master equations are the tools for such derivations.

Another problem closely connected to networks is the attack tolerance (that is to the disconnecting of network after removing some edges (or nodes)). This phenomenon is also named as defragmentation of network. From the physical point of view it is related to the percolation of the network in dependence of network parameter. Another aspect is the robustness of networks (especially WWW). And finally we should remark the epidemical process on network for which propagation laws had been considered in dependence on parameters [13, 25]. The most important is the threshold for epidemic spreading.

2.5. **Synchronization**. Important for interpretation and understanding the behavior of considered systems are synchronization phenomena. The synchronization is collective behavior of systems elements when some elements behave in familiar type [26, 27]. Earlier the synchronization had been considered in pure physical systems. The classical example is the field of connected oscillators. Whole synchronizations, clusters of synchronization, chaotic synchronization, defragmentation are different types of such behavior. Now the point of interests if pushed to coupled nonlinear maps. Remark that such type of models has already some economical applications [26].

3. SOCIOPHYSICS

Applications of physical ideas to considering social systems is more rare (although the term 'sociophysics' had been known since early 80 th of XX century). But the way of considering is the same as for the econophysics. First class of models is the master equation and related equations. Remember also as the example the problem of public opinion formation model [1, 4, 5, 28]. More recently the analogies to Ising systems had been applied occasionally. One example is again opinion formation and one of the first papers is [29] with two state of opinion. Further such approach has been developed in the works by S. Galam, J.A. Holist, Stauffer D and others. Here we consider the simplest case of equation

$$s_i(t+1) - s_i(t) = \begin{cases} -2s_i(t), & f_i(t) \le 0, \\ 0, & f_i(t) > 0 \end{cases}$$

for states of element, f some nonlinear function which corresponds to accounting of the mean influence. The ferromagnetic analogies also had been exploited in the investigations of international relations [30–32].

Recently new approach to social problems follows from physics, namely from the theory of Brownian particles. Classical theory of Brownian motion considers the movement of mechanical ('test') particles in the gas of small particles [33]. Now the same problem is formulated for hard particles with internal structure [34]. In most interesting case active particles had been considered. The 'active' particles can search its neibourhood and make some decision on the movement direction in dependence of own goal. So the model consists from three blocks of equations: for probability of individual opinion (master equation), parabolic equations for communicational field and Langeven equation for individual movement [34].

4. DIFFERENTIAL EQUATIONS AS MODELS

In the previous sections we consider some models which are very sophisticated and sometimes non-usual for scientific community. Vice versa the models based on ordinary or partial differential equations are more familiar. Such type of equation had been introduced phenomenologically. But now the possibility for deriving them by physical analogies exists. The first possibilities is in exploiting conservation laws. Economical and social sciences take from physics as model equations as ideas for leading concepts. Also synergetics is the source of many ideas in interpretation. Especially fruitful is the idea of dynamical chaos. So here we consider the realization of such ideas at the level of differential equations.

4.1. **Lorents system and dynamical chaos**. Maybe the most known model with the complex behavior is the so-called Lorents system [35, 36]

$$\frac{dx}{dt} = \sigma(y - x)\,,$$

$$\frac{dy}{dt} = -xz + rx - y\,,$$

$$\frac{dz}{dt} = xy - bz\,.$$

The investigation of its solutions follows to the concept of dynamical chaos - the behavior which remembers realization of stochastic processes and has the sensitive dependence to disturbances [10, 36]. Next very important concept now is attractive sets (attractor) especially strange. Just understanding of attractor's possibilities in social and economical systems may be important in the case of lack of information and models.

Further as the models as the idea of dynamical chaos had been applied to econonomics [37–39], sociology [40, 41]; especially it should be stressed financial applications [38]. Lorents system and many others are representative of concentrated systems. Of course the space and time aspects are important.

4.2. **Models for distributed systems.** At the present some applications of partial differential equations to socio-economical systems already exist; frequently we find models for spreading, diffusion of parabolic type (diffusive equation). For illustrating that we pose here one of the examples in [37, Chapter 3]

$$\frac{\partial p}{\partial t} = \nabla\left(\frac{q}{p}\nabla p\right) + p\left(1 + q - \gamma p\right),$$

where $p(x,t)$ is the distribution of economical parameter. Remark that in [37] also applications of wave equations to finance had been described. Another class of models are considered for describing the concurrence in a time process. The source of all such field of investigations are the so-called Lottka–Volterra equations for the concurrency of species

$$\frac{dN_i}{dt} = N_i\left(\varepsilon_i - \sum_{j=1}^{N}\gamma_{ji}N_j\right), \quad i = 1, 2, \ldots, N\,,$$

where ε_i, γ_{ij} are coefficients N_i number of representatives of *i*-th species (commodities, firms and so on) [28, 42–44].

In the case of a continual second independent variable τ and continual analog equation the model has the form [43]

$$\frac{\partial x_i(t,\tau)}{\partial t} + \frac{\partial x_i(t,\tau)}{\partial \tau} = -d_i(\tau)\, x_i(t,\tau)\,,$$

$$x_i(t,o) = \int_0^\infty d\tau b_i(\tau) x_i(t,\tau)\,.$$

We also remark the work by [45] where such models had been applied to some industrial processes and just chaotic solutions had been considered. Closely related to models in this subsection are integral and integro-differential equations. Some other models for demography belong to considered classes of models [46, 47] o.d.e.:

$$\frac{dN}{dt} = \frac{C}{(T_1 - T)^2 + \tau^2}$$

or to partial differential equations [48].

It should be stressed in socio-economical interpretations the notion of blow-up solutions (or solutions with collapses) [49, 50]. In the blow-up regimes the solutions of the correspondent models can tends to the infinity by the finite time. In demography blow-up regimes had been interpreted as acceleration in the population growth of the Earth [46]. Also blow-up regimes sometimes relate to the financial crashes. At the considered level of models (o.d.e. and p.d.e.) frequently the account of external disturbances should be investigated. One of the analytical tools for such research is found in partial differential equations with impulsive right hand [51, 52] which are reformulated as operator equations.

4.3. **Dissipative structures and self-organization**. One of the most important concepts from physics is self-organization and types of self-organized solutions. The examples of self-organization objects are dissipative structures, auto waves, collapses, sinchronisation and so on [1, 2, 28, 42, 43, 47, 53, 54]. At the first such examples had been recognized and investigated primarily in physics and biology. Now the fields of applications naturally spread on socio-economical systems where the next qualitative and quantitative problems are important: origin of structures, complexity of the structures, bifurcations of the solutions, and stability and transformation linguistics of system and many others. The examples of applications are city development processes, demography, epidemiology [52, 53]. Usually such problems had been considered on the background of diffusive systems.

5. ASSOCIATIVE MEMORY APPROACH WITH MENTALITY ACCOUNTING

The approaches and models, described before in this paper including Ising model, had its origin in physics and biology. It leads to many advantages of exploiting more developed existing tools from physics. But in such case some implicit drawbacks exist. Namely, physical concepts pose some frame of consideration; socio-economical systems as it also stressed in references should have specific features in its description and behavior. So if the investigations were start from the original system following the physical tools then we were received new intrinsic models and knowledge. Here we describe some of such interesting new features in modeling approach. The examples of such peculiarities are: evolutionary nature of society, evident hierarchical nature and the most important — the presence of human individuals in the system. The last theme assume the mentality accounting of different forms, types in different circumstances. Namely such accounting follows to surprisingly new properties of models. These properties are prospective for understanding of such system behavior, decision-making, and management of social systems. Moreover it may be useful for traditional humanitarian sciences: sociology, politology, social psychology etc. Some of such considerations and concepts had been developed and formalized since the beginning of 90-th (see publications [55–59]). Very shortly, the approach is based on the associative memory analogies of social systems.

5.1. **System's description**. Let us consider a society consisting of $N \gg 1$ individuals and each individual characterising by vector of state $S_i\{s_1^i,...,s_{k_i}^i, s_{k_i+1}^i,...,s_{M_i}^i\}$ $s_l^i \in M_i^l$, $l = 1,...,M_i$, where M_i^l is a set of possible values s_i. There are many possibilities to compose the elements in blocks and levels in such models. In sufficiently developed society, individuals have many complex connections. Let us formalise this. We assume that there are connections between the individuals i and j. Let J_{ij}^{pq} be the connection between p components of element i and q component of element j. Thus the set $Q = \left(\{s_i\},\ \{J_{ij}^{pq}\},\ i, j = 1,...,N\right)$ characterises state of society. Analysis of recent models for media from sets of elements and bonds shows the resemblance of such society models to neural network models.

In reality the society is an evolutionary system with dynamical changes on time. Further for simplicity we will consider only discrete time models with moments of time: $0,1,2,...,n,...$. Following evolutionary nature of the considered systems, it is natural to consider as input of system at the moment n the values of parameters in n-th time moment and as output the values at next $(n+1)$ time moment (for $n = 0,1,2,...$). In the simplest case the model takes the form of well-known Hopfield model [60], and dynamical equations have the form:

$$S_i(t+1) = \operatorname{sign}(h_i),$$

where $h_i = \sum_{j \neq i}^{N} J_{ij} s_i s$ and $\operatorname{sign}(W) = \{+1 \ if \ \ W > 0,\ -1 \ if \ \ W = 0\}$.

Surprisingly the models are familiar with models of brain activity — the neuronets [55–59]. It is well known that Hopfield model is derived from the unctional called 'energy' of the form:

$$E = \sum_{i \neq j}^{N} J_{ij}\, s_i s_j \ .$$

In Hopfield's like neuronet the system tends to one of the few stable states (attractors) with minimum of 'energy' (energetical landscape). Many of possible initial condition lead to a little number of such minimal 'energy' states called attractors.

5.2. **Mentality accounting**. The mentality accounting requires considerations about the inner structures and incorporating them in global hierarchical models. The most natural way for implementing this task is to consider as model for internal structure also neuronet models. The simplest way consists in representing image of World in the individual's brain or in a model as collection of elements and bonds between elements.

5.3. **Anticipatory property**. Let us name the pattern of society $Q^{(1)}(t)$ in the section above as 'image of real world' in discrete moment of time t. We also introduce the $Q_{\text{wish}}(t)$ — 'desirable image of world in moment t by the first individual' as the set of element states and bonds which are desired by the first mathematical descriptions for socio-economical modeling. The analysis had lead to the choice of some new prospective class of interdisciplinary models which exploit the concepts from physics and biology to informatics and cybernetics. Such approach is familiar with artificial neural networks. Such models already had been applied to some systems: sustainable development, economics, geopolitics (see references in [54–59]). Adaptive possibilities of approach may be combined with approaches from sections 2–4, which will be the issues for further applications and theoretical investigations. Bonabeau

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