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<td>Li, P; Jiang, L</td>
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Uncertainty Quantification for Electromagnetic Systems Using ASGC and DGTD Method

Ping Li, Student Member, IEEE, and Li Jun Jiang, Senior Member, IEEE

Abstract—In this paper, an adaptive hierarchical sparse grid collocation (ASGC) method combined with the discontinuous Galerkin time-domain method is leveraged to quantify the impacts of random parameters on the electromagnetics systems. The ASGC method approximates the stochastic observables of interest using interpolation functions over a set of collocation points determined by the Smolyak's algorithm integrated with an adaptive strategy. Instead of resorting to a full-tensor product sense, the Smolyak's algorithm constructs the collocation points in a hierarchical scheme with the interpolation level. Enhanced by an adaptive strategy, the Smolyak's algorithm will sample more points along important dimensions with sharp variations or discontinuities, resulting in a nonuniform sampling scheme. To flexibly handle different stochastic systems, either piecewise linear or Lagrange polynomial basis functions are applied. With these strategies, the number of collocation points is significantly reduced. The statistical knowledge of stochastic observables including the expected value, variance, probability density function, and cumulative distribution function are presented. The accuracy and robustness of the algorithm are demonstrated by various examples.

Index Terms—Adaptive hierarchical sparse grid collocation (ASGC) method, discontinuous Galerkin time domain (DGTD) method, DGTD-boundary integral (DGTD-BI) method, modified nodal analysis (MNA), Smolyak's algorithm, statistical knowledge, uncertainty quantification.

I. INTRODUCTION

E xtensive efforts has been devoted to develop efficient and reliable numerical solvers to characterize the physical behaviors of electromagnetics (EM)/circuit systems, which is the primary target and the research is still growing. Unfortunately, what has been much less considered is the understanding of the impact of uncertainties such as the geometrical parameters [1], the material properties [2], the values of lumped circuit elements [3], the biasing voltage for active devices like power amplifiers, initial and boundary conditions, etc. Therefore, it is very necessary to develop stochastic algorithms to quantify the impacts of these uncertainty parameters on the performance of system.

In general, the majority of these stochastic methods can fall into two groups [4]: the sampling-based statistical methods and the probabilistic methods. The most famous one for the first category is the classic Monte Carlo (MC) method [5]. The implementation of MC method is straightforward and nonintrusive: one only needs to repetitively execute the deterministic solver over a set of sampling points generated according to the probability distributions of random. Pitifully, the convergence of MC method is on the order of $1/\sqrt{N}$ with $N$ denoting the number of sampling points. Although various advanced MC methods such as the quasi-MC method [6], and Markov Chain MC method [7] were later proposed to accelerate its convergence rate, it’s still intractable for complex systems with multidimensional random inputs.

For the probabilistic method, it can also be called generalized polynomial chaos (gPC) method [8]. With this approach, the stochastic solutions are approximated by orthogonal polynomials (surrogate model) of the input random inputs. The quantities to be determined are the expansion coefficients. One typical approach called stochastic Galerkin (SG) method is to implement the Galerkin testing to minimize the error of the finite order gPC expansions [8]–[10], resulting in a set of coupled deterministic equations. Compared with the MC approach, the SG method is more accurate and converges exponentially. However, it is intrusive and would become very difficult to implement if the governing stochastic equations take very complicated forms. Furthermore, a huge coupling matrix system has to be solved if multidimension random inputs are involved. An alternative of SG method is the stochastic collocation (SC) approach [8], [9], which combines the advantages of decoupled MC and fast convergent SG methods. In the SC method, repetitive executions of deterministic simulations over a set of collocation points determined by Stroud [3], Smolyak-based Clenshaw-Curtis, Gauss-Patterson, and Gauss-Legendre quadrature rules are required. The SC method is nonintrusive and simultaneously it can achieve comparable accuracy as SG approach. However, the computational cost of SC approach is still unacceptable for nonsmooth stochastic solutions due to its global property. Thus, other alternatives have to be proposed.

One remedy is the adaptive multielement gPC (ME-gPC) method [11]–[17]. The basic idea of ME-gPC is to adaptively decompose the random domain into small subdomains. The adaptive strategy is guided by the decay rates of the stochastic observables’ local variances. Then, the gPC expansion is employed within each subdomain to locally approximate the stochastic observables. Discontinuities or sharp variations along the random dimensions can be effectively treated. Otherwise, its
efficiency will be discounted if the discontinuities are not along the random variable dimensions [18], [20].

In this paper, the adaptive hierarchical sparse grid collocation (ASGC) algorithm proposed in [18] is combined with our developed discontinuous Galerkin time-domain (DGTD) method [19]-based EM-circuit solver [34] and DGTD-boundary integral (DGTD-BI) algorithm to characterize quick-varying stochastic outputs of hybrid EM-circuit [34] and the scattering from composite structures. This approach seeks to approximate the stochastic outputs by interpolation functions [21] on a set of collocation points. The collocation points can either be in a full-tensor product sense or constructed by the Smolyak’s algorithm [24] based sparse grid scheme. For the full-tensor product case, the number of collocation points increases exponentially with the random dimensions. Therefore, a sparse grid scheme with Smolyak’s algorithm is employed in this paper. Even with Smolyak’s algorithm-based sparse grid strategy, unfortunately, a large number of collocation points are still required when the dimension of random inputs becomes higher, e.g., 25 370 753 support points are needed if the dimension of the random inputs and the interpolation level are ten [21], [22]. The computational cost is not affordable for practical engineering applications. To attack this issue, an adaptive approach guided by the decay rate of local hierarchical surplus is further implemented. Unlike [18], both the piecewise linear and the Lagrange polynomial [23], [25] basis functions are used in this paper to make the algorithm more flexible to handle different functions or systems. Generally, the piecewise linear function is more suitable to nonsmooth observables, while the Lagrange polynomial basis function is better for the slow-varying solutions [26]. Based on these strategies, significantly fewer support points than the conventional methods are needed.

To demonstrate the applicability and efficiency of this algorithm for the hybrid EM-circuit system and EM scattering analysis, the input impedance of a resistor (R), inductor (L), and capacitor (C) loaded parallel-plate waveguide and the dc operating property of a power amplifier are characterized by our developed hybrid EM-circuit solver [27], [28] based on DGTD [31] method and modified nodal analysis (MNA) [32], and the radarcross-section (RCS) of a dielectric sphere is evaluated from our proposed DGTD-BI method [36]. The local operation of DGTD results in a locally coupled EM-circuit matrix. When nonlinear circuit networks are involved, the coupling matrix becomes time-dependent. With the DGTD-based EM-circuit solver, we only invert a small coupling matrix each time step while the finite-element method (FEM)-based EM-circuit solvers need to solve a global matrix [29], [30]. As for the DGTD-BI solver, we use time-domain BI to calculate the field values at the truncation boundary used for the incoming flux evaluation based on the equivalent electric and magnetic currents over a Huygens surface enclosing the scatterer [33]. This method is mathematically rigorous and the truncation boundary can be conformal to scatterers with arbitrary shape.

The rest of this paper is organized as follows. Section II details the mathematical descriptions of the ASGC method, including the Smolyak’s algorithm-based sparse grid scheme, the choice of interpolation basis functions, the adaptive strategy, and the calculation of statistic information such as the expected (mean) value, variance, probability density functions (pdfs) and cumulative distribution functions (cdfs). Section III briefly introduces the hybrid EM-circuit solver based on DGTD and MNA. Section IV describes the basic theory of the hybrid DGTD-BI algorithm used for open space scattering analysis. Numerical examples are benchmarked in Section V. Conclusions are made at the end of the paper.

II. FORMULATION

With the ASGC method, the stochastic observables are approximated by the multidimensional interpolation functions of the random inputs, and Smolyak’s construction-based sparse grid algorithm is employed. Same notations in [21] and [25] are used to maintain consistency.

A. Smolyak’s Algorithm

Suppose that there is a smooth function \( f : [0, 1]^d \rightarrow \mathbb{R} \) to be approximated over a finite number of collocation points. Starting from the 1-D case \((d = 1)\), we can obtain the following interpolation formula to approximate \( f \):

\[
U^i(f) = \sum_{x^i \in X^i} a_{x^i} \cdot f(x^i)
\]

(1)

where \( U^i(f) \) denotes the approximation of function \( f \), \( X^i = \{x_j^i | x_j^i \in [0, 1] \text{ with } j = 1, 2, \ldots, m_i\} \) is the set of support points, \( a_{x^i} \in [0, 1] \) is the nodal basis function, the superscript \( i \in \mathbb{N} \) represents the interpolation level. For multivariate case, the full-tensor product-based interpolation formula is written as

\[
(U^{i_1} \otimes \cdots \otimes U^{i_d})(f) = \sum_{x^{i_1} \in X^{i_1}} \cdots \sum_{x^{i_d} \in X^{i_d}} a_{x^{i_1}} \cdots a_{x^{i_d}} \cdot f(x^{i_1}, \ldots, x^{i_d}).
\]

(2)

The total number of support points \( (m = m_{i_1}, \ldots, m_{i_d}) \) by the full-tensor scheme grows exponentially with dimensions of random variables. To avoid this deficiency, Smolyak’s algorithm is used instead. With \( U^0(f) = 0, \Delta^i = U^i - U^{i-1}, |i| = i_1 + \cdots + i_d \text{ for } i \in \mathbb{N}^d, \text{ and } q \geq d, q \in \mathbb{N} \), the Smolyak’s algorithm is given by

\[
A_{q,d} = \sum_{|i| \leq q} (\Delta^{i_1} \otimes \cdots \otimes \Delta^{i_d})(f)
\]

\[
= A_{q-1,d} + \sum_{|i| = q} (\Delta^{i_1} \otimes \cdots \otimes \Delta^{i_d})(f)
\]

(3)

\( \Delta A_{q,d}(f) \)

with \( q \geq d \). From (3), to compute the interpolation value at the interpolation level \( q - d \), it is clear that one only needs to compute the function at the newly generated points from the interpolation level \( q - d - 1 \) to \( q - d \), while the calculated results at the previous interpolation levels are kept. Thus, it is advantageous to choose the interpolation points in a nested fashion, namely \( X^i \subset X^{i+1} \) with \( X^0 = \emptyset \).

With these nested support points, a multivariate hierarchical formulation of Smolyak’s algorithm can be obtained. We
first begin with the 1-D case, and then extend to the multivari-
ate case straightforwardly. Based on the fact that $U^{i-1}(f) = U^i(U^{i-1}(f))$ and $f(x^i) - U^{i-1}(f)(x^i) = 0, \forall x^i \in X^{i-1}$ [21], we can write the interpolation difference $\Delta^i(f) = \sum_{x^i \in X^i, x^i \neq x^j} a_{x^j} \cdot f(x^j) - \sum_{x^i \in X^i} -U^{i-1}(f)(x^i)$ as

$$\Delta^i(f) = \sum_{x^i \in X^i} a_{x^j} \cdot [f(x^j) - U^{i-1}(f)(x^j)]$$

(4)

where $X^i \Delta = X^i \setminus X^{i-1}$. Obviously, there are $m^\Delta_i = m_i - m_{i-1}$ elements in the set $X^i \Delta$ due to $X^{i-1} \subset X^i$. By rearranging and consecutively numbering the elements in $X^i \Delta$, and indicating the $j$th element of $X^i \Delta$ and $a_{x^j}$ as $x^j$ and $a^j$, respectively, we can rewrite (4) as

$$\Delta^i(f) = \sum_{j=1}^{m^\Delta_i} a^j \cdot \left[f(x^j) - U^{i-1}(f)(x^j)\right].$$

(5)

It is noted that only support points not occurred in the previous sets $X^{i-k}$ with $1 \leq k \leq i - 1$ are required to evaluate $\Delta^i(f)$. The difference between the function value at the present and the previous interpolation levels denoted as $w^j_i$ is defined as hierarchical surplus, and $a^j$ is the hierarchical basis function.

For multivariate case, we rewrite (3) as

$$A_{q,d} = A_{q-1,d} + \Delta A_{q,d}.$$  \tag{6}

The hierarchical formulation in 1-D case [see (5)] can be directly extended to the multivariate case with a new expression for $\Delta A_{q,d} [18], [21]$

$$\Delta A_{q,d} = \sum_{i=q}^{d} \sum_{j=1}^{w^j_i} a^j \cdot \left[f(x^j_1, \ldots, x^j_d) - A^0_{q-1,d}(f)(x^j_1, \ldots, x^j_d)\right],$$  \tag{7}

where $j$ is a multi-index $(j_1, \ldots, j_d)$, $j_l = 1, \ldots, m^\Delta_{i_l}$, and $l = 1, \ldots, d$. $a^j$ represents the multivariate hierarchical basis function, and $w^j_i$ denotes the multivariate hierarchical surplus. For smooth functions, the hierarchical surplus will approach to zero as the interpolation level $k = q - d$ tends to infinity.

B. Choice of Support Points and Interpolation Basis Functions

For the purpose of hierarchy, the interpolation points have to be chosen in a nested fashion. One feasible choice is the Clenshaw-Curtis formula with nonequidistant abscissas given as the zeros or the extreme points of Chebyshev polynomials [18], [21], [35]. For any $i \geq 1$, $i \in \mathbb{N}$, the total number of support points $m_i$, and the corresponding sets $X^i = \{x^i_1, \ldots, x^i_{m_i}\}$ are
given by

$$m_i = \begin{cases} 1, & \text{if } i = 1, \\ 2^{i-1} + 1, & \text{if } i > 1. \end{cases}$$  \tag{8}

$$x^i_j = \begin{cases} 0.5, & \text{for } j = 1 \text{ if } m_i = 1, \\ \frac{\cos(\frac{(j-1)}{(m_i - 1)}) + 1}{2}, & \text{for } j = 1, \ldots, m_i, m_i > 1. \end{cases}$$  \tag{9}

Apparently, the resultant grid sets are in a nested nature. The corresponding univariate node basis functions for the above Clenshaw-Curtis formula with nonequidistant abscissas are Lagrange polynomials. Namely,

$$a^j_i = \begin{cases} 1, & \text{for } i = 1, \\ \prod_{k=1}^{m_i} \frac{x^j - x^k}{x^i - x^k}, & \text{for } i > 1 \text{ and } j = 1, \ldots, m_i. \end{cases}$$  \tag{10}

It is noted that the Lagrange polynomials are global in each dimension since each univariate Lagrange polynomial $a^j_i$ for the $l$th dimension involves all support nodes $x^l_k$ with $k = 1, \ldots, j - 1, j + 1, \ldots, m_i$. Here, we call the Lagrange polynomials as “locally global basis functions.” Also, the nonequidistant support nodes are not perfect for local refinement. These properties will degrade the performance of the Lagrange polynomials when nonsmooth functions encountered since strong local refinements are required.

Another possible choice could be the piecewise multilinear basis functions, which are defined over sets with equidistant nodes. The piecewise linear function has a local property, thus it can be potentially employed to attack discontinuous issues in stochastic solutions. For 1-D case, the support nodes are defined as

$$m_i = \begin{cases} 1, & \text{if } i = 1, \\ 2^{i-1} + 1, & \text{if } i > 1. \end{cases}$$  \tag{11}

$$x^i_j = \begin{cases} 0.5, & \text{for } j = 1 \text{ if } m_i = 1, \\ \frac{j-1}{m_i-1}, & \text{for } j = 1, \ldots, m_i, m_i > 1. \end{cases}$$  \tag{12}

With these support nodes, the univariate piecewise linear inter-
polation basis functions are given as

$$a^1_i = 1 \quad \text{for } i = 1, \text{ and}$$  \tag{13}

$$a^j_i = \begin{cases} 1 - (m_i - 1) \cdot |x - x^j|, & \text{if } |x - x^j| < 1/(m_i - 1) \\ 0, & \text{otherwise} \end{cases}$$  \tag{14}

for $i > 1$ and $j = 1, \ldots, m_i$.

The definition of 1-D basis function can be directly stretched to the $d$-dimensional case. The multivariate Lagrange polynomials and the piecewise multilinear basis functions can be con-
structed using the tensor-products as

$$a^j_i := a^j_{i_1} \otimes \cdots \otimes a^j_{i_d} = \prod_{l=1}^{d} a^j_{i_l}$$  \tag{15}

with $j$ denoting the multi-index $(j_1, \ldots, j_d) \in \mathbb{N}^d$, $j_l = 1, \ldots, m^\Delta_{i_l}$, and $l = 1, \ldots, d$. 
C. Adaptive Strategy

Referring to (7) again, we note that the multivariate hierarchical surpluses tend to decay quickly with increasing interpolation level for slow-varying functions of random inputs. However, for nonsmooth functions, the adaptive sparse grid technique is more superior. It places more support points around the discontinuities while other smooth regions are treated normally.

The basic idea of the adaptive strategy is to use the hierarchical surplus \( v^2 \) as the error indicator to judge whether or not the local refinement is required for the present grid point \( X \). If the condition \( \| v^2 \| \geq \varepsilon \) is satisfied, we generate \( 2d \) sons (For a \( d \)-dimension stochastic space, there are \( 2d \) sons for a grid point since each univariate grid point has two sons.). Then, we move to the next grid point, and repeat the previous operation until all grid points at current level have been scanned. We keep these new generated sons and simultaneously remove redundant ones since some of them are same. It is noted that the number of the grid points is increased linearly with the growth of stochastic dimension rather than a \( O(2^d) \) scheme.

To show the efficiency of this ASGC method, the following function on \([0, 2]^2\)

\[
f(\xi_1, \xi_2) = \frac{1}{|0.25 - (\xi_1 - 1)^2 - (\xi_2 - 1)^2| + 0.25}
\]

(16) is considered. \( \xi_1 \) and \( \xi_2 \) are two random inputs. From (19), we know that there is a fast-varying along a circular line: \( \{(\xi_1, \xi_2) | (\xi_1 - 1)^2 + (\xi_2 - 1)^2 = 0.25\} \). Around this curve, the grid points would be locally refined. In the following, different error indicators \( \varepsilon = 1.0e - 1 \), \( 1.0e - 2 \), \( 1.0e - 3 \) are employed as the thresholds. To evaluate the convergence property with different error thresholds, we randomly generated 10 000 points in \([0, 2]^2\) (Here, we assume that the random inputs \( \xi_1 \) and \( \xi_2 \) have uniform distributions), and the \( L^\infty \) norm is computed. Namely,

\[
e = \max\{|f(\xi_k) - A_{q, d}(f)(\xi_k)|, k = 1, \ldots, 10 \ 000\}
\]

(17) where \( A_{q, d} \) denotes the interpolation value from the ASGC algorithm with the piecewise linear basis functions. Fig. 1 shows the \( L^\infty \) error with respect to the required number of collocation points for different thresholds. It is noted that more collocation points are needed for smaller thresholds, but a higher accuracy is achieved. Besides, much fewer points are required for the ASGC than the conventional sparse grid collocation (CSGC) method (no adaptivity), e.g., there are 32 597 points for \( \varepsilon = 1.0e - 3 \) with the ASGC and the corresponding interpolation level is 21, while 262 144 points are needed with the CSGC. In Fig. 2, the distribution of collocation points as the interpolation level evolves is presented. As expected, the support points are locally refined along the singularity line. For further comparison, the exact value of function and the interpolated one are shown in Fig. 3. Good agreements are observed.

D. Statistical Data and Complexity Analysis

To obtain the statistical data, including the mean value, variance, pdf, and cdf, one method is through the MC simulation, another is based on the previous interpolation function \( A_{q, d} \). For the MC method, \( N_{MC} \) points are randomly generated according to the pdfs of the uncertainty inputs, then the mean value and variance of an arbitrary stochastic observable denoted as \( V(X) \) can be calculated by

\[
E(V) = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} V_k
\]

(18) and

\[
\text{Var}(V) = E(V^2) - [E(V)]^2.
\]

(19)

To calculate the pdf, we divide the range between the minimum and maximum values of \( V_k, k = 1, \ldots, N_{MC} \), into \( N_{bin} \) equal intervals, and the number of \( V \in [Z_k, Z_{k+1}] \), \( Z_k = V_{min} + (k-1)(V_{max} - V_{min})/N_{bin} \), defined as \( N^k \) is counted, and then the pdf in this interval denoted as \( p^k \) is proportional to \( N^k / N_{MC} \). As for the cdf, the total number of \( V \in (-\infty, Z_k] \)
defined as $N^{k,C}_V$ is counted, and the cdf can be defined as $C^{k}_V \propto N^{k,C}_V/N_{MC}$.

For the second method, the values of stochastic observable $V(X)$ are calculated by the previous obtained surrogate interpolation function $A_{q,d}$. Next, we repeat the procedure in the MC simulation process, then all statistical information can be obtained.

To compare the computational complexity of MC and ASGC methods, we indicate the CPU time of single realization of the deterministic simulation, the evaluation of $V_k$ using the surrogate interpolation function as $T_{run}$ and $T_{app}$, respectively. For the MC method, the total CPU cost is $N_{MC} T_{run}$, while the total CPU cost of the ASGC method is $N_{ASGC} T_{run} + N_{MC} T_{app}$. Since $N_{ASGC} \ll N_{MC}$ and $T_{app} \ll T_{run}$, the CPU cost of the AGSC method is much smaller than the MC simulation.

III. HYBRID FULL-WAVE EM-CIRCUIT SOLVER

To analyze the hybrid EM-circuit systems, a DGTD-MNA solver [27], [28] developed by us is applied. Wherein the DGTD is responsible for the distributive part governed by the Maxwell’s equations, while the MNA approach is used to model the circuit networks based on the Kirchoff’s current law [32]. To achieve the coupling from the circuit subsystem to the EM subsystem, a surface port current density $J^{CKT}_{i}$ is introduced over the lumped port, while to facilitate the coupling from the EM to the circuit domain, a lumped port voltage $V^{CKT}_{i}$ is introduced that is equal to the line integral of the electric field along the reference potential direction. Due to the local operations of DGTD, the EM-circuit coupling matrix is also local, which is different to the finite-element method-based circuit solver. The dimension of the coupling matrix is $n^{(1)}_e + n^{(2)}_e + \ldots + n^{(f)}_e + N^{CKT}$ with $f$ denoting the number of lumped ports, and $n^{(1)}_e, \ldots, n^{(f)}_e$ representing the number of basis functions for electric field used for EM-circuit coupling, and $N^{CKT}$ denoting the number of unknowns in the circuit network.

When nonlinear lumped circuits such as the power amplifier, diodes, oscillators, etc., are involved, the Newton-Raphson method is used to solve the above coupling matrix equation.

IV. HYBRID DGTD-BI ALGORITHM

As a differential method-based numerical solver, the DGTD must be combined with artificial boundary to truncate the computational domain for the open-space problems. In DGTD, the solution is assumed to be piecewise constant and the information exchange between neighboring elements is achieved by the numerical flux denoted as $F(E^-, H^-, E^+, H^+)$. In fact, the numerical flux can be decomposed into the outgoing part denoted by $F^-$ that is a function of the fields in current element (denoted by $E^-$ and $H^-$) and the incoming part denoted by $F^+$ that is a function of the fields from its neighboring elements (denoted by $E^+$ and $H^+$). At the truncation boundary, the field values $(E^+, H^+)$ for the calculation of the incoming flux $F^+$ are not available. To get the field values $(E^+, H^+)$ accurately at the truncation boundary, a Huygens’ surface enclosing the scatterer is defined. Based on the equivalent electric and magnetic
TABLE I
CALCULATED MEAN VALUE AND VARIANCE OF THE REAL AND IMAGINARY PARTS OF THE INPUT IMPEDANCE BY ASGC METHOD, AND MC SIMULATION

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<tr>
<td>$\text{mean}$</td>
<td>$\text{variance}$</td>
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<tr>
<td>$E(Z_{\text{in}}^{\text{Re}})$</td>
<td>$E(Z_{\text{in}}^{\text{Im}})$</td>
<td>$E(Z_{\text{in}}^{\text{Re}})$</td>
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<tr>
<td>599.486</td>
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<tr>
<td>599.584</td>
<td>5.564e-5</td>
<td>327.553</td>
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<td>599.617</td>
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Fig. 5. Pdfs of the real (a) and imaginary (b) parts of the input impedance $Z_{\text{in}}$ by ASGC method and MC simulation.

Fig. 6. Cdfs of the real (a) and imaginary (b) parts of the input impedance $Z_{\text{in}}$ by the ASGC method and the MC simulation.

currents calculated by the DGTD, the field values ($\mathbf{E}^+, \mathbf{H}^+$) used for the incoming flux evaluation can be obtained according to the Huygens’ principle [33], [36]. In the TDBI process, only forward matrix-vector products are involved, thus it is free of matrix inversion. This method is mathematically rigorous and the truncation boundary can be conformal to the scatterers. For disconnected scatterers, each of them can be truncated by the local mesh. Therefore, the computational domain can be as small as possible.

V. NUMERICAL RESULTS

This section presents several typical engineering examples to illustrate the applicability and accuracy of the algorithm. We always use piecewise multilinear functions as the interpolation functions except specific illustration. Also, the relative hierarchical surplus $||w_j^i/f(x_j^i)||$ is employed as the error indicator instead of $||w_j^i||$ for the following numerical benchmarks since the relative difference is a good measure for the data deviation.

A. Parallel Plate Waveguide Loaded By R, L, and C Components

In this example, a 0.25 m parallel plate waveguide loaded with a linear $RLC$ circuit network is analyzed by the developed EM-circuit solver [27], [28]. The top and bottom surfaces of this waveguide are the perfectly electric conductor, while the two side surfaces are the perfectly magnetic conductor. In this way, the transverse electromagnetic mode can propagate in this structure. The near and far ends of the waveguide are truncated by the absorbing boundary conditions serving as matched loads, and a plane wave is launched at the near end serving as the excitation. The exact equivalent circuit model is shown in Fig. 4. We consider $R_1$, $L_1$, and $C_1$ as three random parameters.
with uniform distributions: \( R_1 \in [295, 305] \Omega, L_1 \in [2, 9] \) nH, and \( C_1 \in [10, 20] \) pF. The stochastic observable is the input impedance \( Z_{in} \) at 356.72 MHz. The mean and variance of \( Z_{in} \) for error thresholds \( \varepsilon = 1.0 \) and \( 1.0 \) are listed in Table II. For comparison, the results by 100 000 MC realizations are also provided, which is based on the analytical expression of the input impedance using the equivalent circuit model in Fig. 4. Also, the pdf and cdf of real and imaginary parts of \( Z_{in} \) are presented in Figs. 5 and 6. To calculate the pdf and cdf, the number of intervals is 20 and the corresponding intervals in Fig. 5(a) and (b) are 9.08 and 1.88, respectively. Very good agreements between the ASGC method and the MC simulation are noted.

B. DC Operation of A MESFET Power Amplifier

To further verify the proposed stochastic simulation method, this part a power amplifier is investigated. The microstrip interconnects and the equivalent circuit model of the power amplifier are shown in Fig. 7. The dc biasing voltages for the Gate (\( V_{GG} \)) and the Drain (\( V_{DD} \)) are applied at port I and IV, respectively. This power amplifier has two nonlinear components: a voltage-controlled current source defined as

\[
I_{ds} = \tanh(V_{ds})(A_0 + A_1 V_{gs} - A_2 V_{gs}^2 - A_3 V_{gs}^3) \tag{20}
\]

and a voltage-controlled capacitor defined by

\[
C_{gs} = \begin{cases} 
\frac{3}{\sqrt{1-V_c/0.7}} \text{ pF} & V_c < 0.35 \text{V} \\
3\sqrt{2}(0.5 + V_c/0.7) \text{ pF} & V_c \geq 0.35 \text{V}. \tag{21}
\end{cases}
\]

Firstly, the impact of three parasitic resistors \( R_0 \), \( R_d \), and \( R_s \) on the dc operating property are investigated with \( V_{GG} = -0.81 \) V and \( V_{DD} = 18.96 \) V. We assume that all three random inputs follow the Gaussian distribution within the ranges: \( R_g \in [0.1, 0.9] \) \( \Omega \), \( R_d \in [0.1, 0.9] \) \( \Omega \), and \( R_s \in [0.1, 1.3] \) \( \Omega \), while \( L_g = 0.05 \) nH, \( L_d = 0.05 \) nH, and \( L_s = 0.1 \) nH. Four different sets of values \( (\mu_s^i, \sigma_s^i) \) listed in Table II are considered. Here the dc voltage drop via the Drain to Source \( V_{DS} \) is considered as the stochastic observable. For this example, both piecewise linear and Lagrange basis functions are applied. The calculated mean and variance with error threshold \( \varepsilon = 1.0 \) are shown in Table II. The numbers of required collocation points for the piecewise linear and the Lagrange polynomial basis functions are 65 with interpolation depth \( L_s^{16} = 5 \) and 29 with interpolation depth \( L_s^{16} = 3 \), respectively. The numbers of CGSC corresponding to interpolation depth \( L_s^{16} = 5 \) and \( L_s^{16} = 3 \) are 441 and 62, respectively. The pdfs and cdfs of \( V_{DS} \) for these four sets are shown in Fig. 8. To calculate the pdf and cdf, the number of intervals are five for all four sets and the intervals for set 1, 2, 3, and 4 are 0.07, 0.17, 0.23, and 0.296, respectively. In Fig. 9, the mean and variance of the Drain to Source drop versus different bias \( V_{DD} \) are presented. The results using piecewise linear and Lagrange polynomial basis functions agree with each other very well.

Next, a scenario with seven random inputs (\( V_{GG}, V_{DD}, R_c, R_0, R_g, R_d, R_s \)) are considered. For dc characterization, the influences of inductor and capacitor are ignored since inductor is equivalent to short circuit circuit while capacitor corresponds to open circuit. The above seven random variables are supposed to They are to follow uniform distributions within the ranges: \( V_{GG} \in [-0.825, -0.775] \) V, \( V_{DD} \in [18.75, 19.25] \) V, \( R_c \in [45, 55] \) \( \Omega \), \( R_0 \in [45, 55] \) \( \Omega \), \( R_g \in [0.1, 0.9] \) \( \Omega \), \( R_d \in [0.1, 0.9] \) \( \Omega \), and \( R_s \in [0.1, 1.3] \) \( \Omega \). The number of support points and the interpolation level by the piecewise linear basis with different error thresholds are shown in Table III. For comparison, the number of support points with CGSC method (\( N_{CGSC}^{C} \)) is also presented. Notably, the number of support points for high interpolation level case are significantly reduced with ASGC method.

C. Scattering From Spheres

Finally, the ASGC method is employed to study the impacts of material uncertainties on the scattering from a dielectric sphere.
The stochastic observable of interest for this example is the RCS, while the relative dielectric permittivity $\varepsilon_r$ is considered as the random input following uniform distribution in $[2.25, 2.75]$. The radius of the sphere and the frequency of interest are $0.125$ m and $1.0036$ GHz, respectively. The error threshold $\varepsilon$ in this example is set to be $5.0\epsilon - 3$. To calculate the RCS, we use our proposed DGTD-BI algorithm [36], in which the radiation condition is enforced mathematically exact and the resulting computation domain is as small as possible since the truncation boundary is now allowed to be conformal to the scatterer’s shape and to be located very close to its surface. For the excitation, a $x$-polarized plane wave propagating along the positive $z$-axis is employed.

In the beginning, the impact of random variables on the RCS in the $xoz$ plane with sampling resolution $\Delta \theta = 2.0^\circ$ (resulting in 91 sampling points) is investigated. In this case, the local hierarchical surplus [see (7)] is a vector comprising of 91 elements instead of a scalar. Thus, the root-mean-squared (RMS) error is

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$N^{\text{in}}$</th>
<th>$L^{\text{in}}$</th>
<th>$N^{\text{CSGC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0e-1</td>
<td>63</td>
<td>2</td>
<td>221</td>
</tr>
<tr>
<td>1.0e-2</td>
<td>209</td>
<td>4</td>
<td>8801</td>
</tr>
</tbody>
</table>

TABLE III

NUMBER OF SUPPORT POINTS ($N^{\text{in}}$) AND INTERPOLATION DEPTH ($L^{\text{in}}$) FOR PIECEWISE LINEAR BASIS FUNCTIONS WITH DIFFERENT ERROR THRESHOLDS AS WELL AS THE COMPARISON FROM THE CSGC METHOD
employ as the error indicator for adaptivity. Namely,

$$\text{RMS}_{w_i} = \sqrt{\frac{(w_{i,1}^t)^2 + \cdots + (w_{i,M}^t)^2}{M \int f(x_i^t)^2}}$$

with $M = 91$.

Then, the mean value and variance of RCS in the $xoz$ plane calculated by both the piecewise linear and the Lagrange polynomial basis functions are presented in Fig. 10(a). It is noted that the results from these two different basis functions agree with each other very well. For this simulation, the number of collocations and interpolation level using piecewise linear basis function are 17 and 4, respectively; while only nine collocation points are required for Lagrange polynomial basis function and the corresponding interpolation level is 3.

Next, the RCS in the $yoz$ plane are quantified with same sampling rules as above. The computed mean value and variance of RCS are shown in Fig. 10(b), also very good agreements between these two different basis functions are observed. As presented in Fig. 10(b), the variation of permittivity has significant influence on the RCS located at deeps such as $\theta = 150^\circ$ since the variance at these places are relative larger than others. For this simulation, the number of collocation points using piecewise linear and Lagrange polynomial basis function are 45 and 11, respectively, and the corresponding interpolation levels are 7 and 4.

To verify the accuracy of the ASGC and DGTD-BI algorithm, the reference by the MC simulation is proposed. For the MC simulation, instead of using full wave solution, the analytical formula derived from Mie theory [37], [38] is employed, which expand the scattered field by spherical wave functions. The support nodes for MC method is generated according to the distribution of random variables. For this example, the total number of support nodes is 1000. The calculated mean and variance by the MC simulation are also shown in Fig. 10. Apparently, good agreements are observed, which proves the accuracy of both ASGC and DGTD-BI method. Last, the pdf and cdf of the forward bistatic RCS are presented in Figs. 11 (a) and (b), respectively. Again, very good agreements between the two different basis functions are observed.

VI. CONCLUSION

In this paper, a stochastic simulation algorithm built upon the ASGC method is applied to characterize the EM-circuit systems and the scattering from a dielectric sphere. Via the Smolyak’s construction algorithm and the adaptive strategy guided by the local hierarchical surpluses, the sharp variations and discontinuities of stochastic observables can be efficiently quantified with only much fewer collocation points. The use of different basis functions makes this algorithm more flexible to handle different stochastic systems. To verify the effectiveness, robustness and flexibility of this stochastic simulation method, microwave circuits with microstrip interconnects, RLC resonant circuits and nonlinear power amplifiers are benchmarked. Furthermore, the far-field scattering of a dielectric sphere is also investigated. All numerical results demonstrate the excellent performance of this ASGC method.

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Ping Li (S’12) received the Postgraduate Engineering Fellowship for outstanding academic performance in 2010. He has been working toward the Ph.D. degree with the center of Electromagnetics and Optics, University of Hong Kong (HKU), Hong Kong, since 2010. He was awarded the HKU Postgraduate Engineering Fellowship for outstanding academic performance in 2010.

He has authored more than ten journal papers on IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, IEEE TRANSACTIONS ON ELECTROMAGNETIC AND COMPUTABILITY, IEEE TRANSACTIONS ON COMPONENTS, PACKAGING, AND MANUFACTURING TECHNOLOGIES, etc. He was the Reviewer of IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS, Proceedings of IEEE, Journal of Electromagnetic Waves and Applications, and Journal of Applied Computational Electromagnetics. His research interests include the near-field to far-field transformation techniques, phaseless equivalent source reconstruction methods, discontinuous Galerkin time-domain method, and uncertainty quantification for large scale electromagnetic systems.

Mr. Li’s paper was selected as the Finalist paper in 29th International Review of Progress in Applied Computational Electromagnetics and 2014 International Symposium on Electromagnetic Compatibility, and he received the Best Student Paper Award in 12th International Workshop on Finite Elements for Microwave Engineering. He is listed in Marquis Who’s Who in the World, 32nd Edition, 2015.

Li Jun Jiang (S’01–M’04–SM’13) received the B.S. degree in Electrical Engineering from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 1993, the M.S. degree from the Tsinghua University, Beijing, China, in 1996, and the Ph.D degree from the University of Illinois at Urbana-Champaign, Champaign, IL, USA, in 2004.

From 1996 to 1999, he was an Application Engineer with the Hewlett-Packard Company. Since 2004, he has been the Postdoctoral Researcher, the Research Staff Member, and the Senior Engineer at IBM T.J. Watson Research Center, Yorktown Heights, NY, USA. Since the end of 2009, he is also an Associate Professor with the Department of Electrical and Electronic Engineering, University of Hong Kong, Hong Kong. He is the Associate Editor of IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, the Associate Editor of Progress in Electromagnetics Research, the Associate Guest Editor of the PROCEEDINGS OF IEEE Special Issue in 2011–2012. He is also the Senior Visiting Professor at Tsinghua University, Beijing, China, since June 2013.

Dr. Jiang received the IEEE MTT Graduate Fellowship Award in 2003 and the Y.T. Lo Outstanding Research Award in 2004. He is an IEEE AP-S Member, an IEEE MTT-S Member, an IEEE EMC-S Member, an ACES Member, and a Member of the Chinese Computational Electromagnetics Society. He was the Semiconductor Research Cooperation (SRC) Industrial Liaison for several academic projects. He was the Scientific Consultant to Hong Kong ASTRID (Hong Kong Applied Science and Technology Research Institute Company Limited) in 2010–2011, the Panelist of the Expert Review Panel of Hong Kong R&D Centre for Logistics and Supply Chain Management Enabling Technologies since January 1st, 2013. He was the TPC Chair of the 7th International Conference on Nanophotonic/RF in 2010, the 3rd Conference on Advances in Optoelectronics, and Cross-Nano Optics, the TPC Co-Chair of the 12th International Workshop on Finite Elements for Microwave Engineering, the Co-Chair of 2013 International Workshop on Pulsed Electromagnetic Field at Delft, the Netherlands, the General Chair of 2014 IEEE 14th HK AP/MTT Postgraduate Conference. He was the elected TPC Member of IEEE Electrical Performance of Electronic Packaging since 2014, the TPC Member of IEEE Electrical Design of Advanced Packaging and Systems (EDAPS) since 2010, the TPC Member of IEEE International Conference on Microwave Technology Computational Electromagnetics, the Scientific Committee Member of 2010 IEEE SMEE, the special session organizers of IEEE EDAPS, IEEE Electromagnetic Compatibility Society (EMC-S), Automation, Control, Energy and Systems, AsiaPacific Radio Science Conference, PIER, and also a Reviewer of HKU Computational Science and Engineering Workshops in 2010-2012, the TC-9 and TC-10 member of IEEE EMC-S since 2011, and Session Chairs of many international conferences. He was also the Reviewer of IEEE Transactions on several topics, and other primarily electromagnetic and microwave related journals. He has been working collaboratively with many international researchers.