Optimal Multistage PMU Placement for Wide-Area Monitoring

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***Abstract*—This paper proposes a novel optimization model to maximize the power system observability by placing phasor measurement units (PMUs) in a multistage manner. The problem is constrained by the financial budgets available at each installation stage. The budget may be spent on purchasing and installing new PMUs and relocating PMUs already installed in the power system. This problem is very difficult to solve when the problem size becomes big. Therefore, a newly developed meta-heuristic, called chemical reaction optimization (CRO), is used to solve this optimal multistage PMU placement problem (OMPP), and numerical studies are carried out on the IEEE 57-bus, 118-bus, and 300-bus systems.**

***Index Terms*—Chemical reaction optimization (CRO), optimal PMU placement, phasor measurement unit (PMU), power system monitoring, smart grid, synchrophasor.**

I. Introduction

PHASOR measurement units (PMU) are measuring devices that offer fast acquisitions of time-synchronized phasor data in a power system [1]. Time tagged by global positioning system (GPS) with a resolution of less than 1 μs, data gathered by PMUs can significantly improve the performance of power system monitoring and control [2]. Hence, PMUs are seen as necessary components in the future power system, namely, the smart grid.

Placing PMUs on every bus of a power system immediately results in a completely observed system [3]. However, since a bus is observed if a PMU is installed on it or on one or some of its neighboring buses, it is neither necessary nor economical to carry out such full installations. As a result, a problem, named optimal PMU placement (OPP) problem, has been raised.

The classical OPP problem finds the minimum number of PMUs required for a completely observable power system. To solve this, researchers have tried using both mathematical programming approaches, such as integer linear programming (ILP) [4] and binary integer linear programming (BILP) [5], and meta-heuristics, such as immunity genetic algorithm (IGA) [6] and nondominated sorting genetic algorithm (NSGA) [7].

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Since PMUs are expensive devices and a power system with considerable size will require huge numbers of PMUs for full observability, the OPP problem was further extended to multistage installations. This means that, instead of solving the optimal PMU placement in one shot, we need to come up with an optimal incremental placement strategy that allows multistage installations of PMUs. This type of OPP problem has been considered in [8], [9] and [10].

Nuqui *et al.* proposed the idea of depth of unobservability in [9]. They used simulated annealing (SA) to solve the optimal multistage PMU placement (OMPP) problem, given the number of stages required, by determining the number of PMUs for each stage such that the depth of unobservability after each stage is lower than that of the previous stage. However, we believe it is more reasonable to let companies determine how many PMUs to install at each stage based on companies' financial budgets, rather than we tell them how many are required. In [8], Dua *et al.* addressed the OMPP problem by calculating the final year PMU placement at the beginning and then schedule PMU placement strategies in intermediate stages in a backward manner. ILP was used to attack the problem. Aminifar *et al.* followed a similar multistage PMU installation approach in [10], and used IGA to attack the problem. All these three papers made the assumption that, once a PMU is installed at a specific position in early stages, it cannot be relocated later. However, after careful investigations, we find this an inappropriate assumption. Relocating some already installed PMUs at certain costs would result in increased system observability in intermediate stages.

In this paper, we propose an optimization model for the OMPP problem that we believe to be more realistic and better fits the multistage installation scenario. The model takes the financial budgets at each installation stage as constraints, and aims to maximize the cumulative observability in intermediate stages. The budget consists of the costs of purchasing and installing new PMUs and those of relocating installed PMUs. A variant of the model that alleviates the trouble of estimating the costs of relocating a PMU is also proposed. In addition, a new metric, *evolvability index (EI)*, is introduced in this paper to measure the quality of a PMU placement strategy in the multistage installation scenario. We shall prove in the next section that OMPP is NP-complete. Therefore, we propose using a newly developed meta-heuristic, named simplified chemical reaction optimization (SCRO) [11], to attack this problem. The major reason for choosing a meta-heuristic algorithm over mathematical programming approaches and graphical analytical approaches is the superiority in computation time for large systems. According to [11], SCRO gives very good results within reasonable computation times for solving the

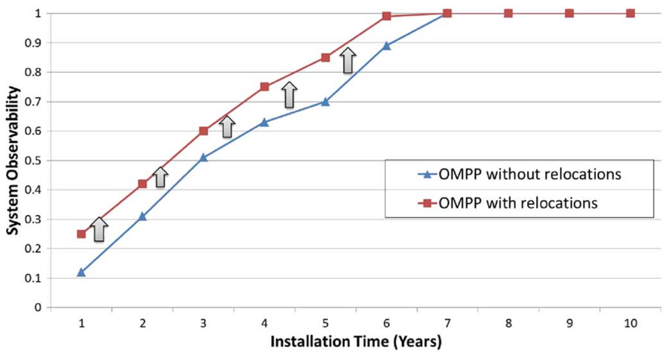

Fig. 1. System observability during staged installation of PMUs.

OPP problem in large power systems, whereas mathematical programming approaches, ILP in particular, may fail to converge even after running for days.

The rest of the paper is organized as follows. Section II will give the formulation and detailed explanation of our optimization model. *EI* is introduced in Section III. The problem solver, SCRO, will be discussed in Section IV. Case studies on the IEEE 57-bus, 118-bus, and 300-bus systems, as well as the numerical results will be given in Section V, and the paper concludes in Section VI.

II. Formulation of Optimal Multistage PMU Placement

When the planned installation duration, consisting of multiple stages, is considerable compared to the expected life time of the PMU network, utilities may be better off to spend some more money to relocate PMUs already installed to achieve higher system observability in intermediate stages. The resulting higher observability is obvious since by allowing PMU relocations, we allow a larger search space for intermediate solutions. To illustrate this, we consider a case where a power company plans a six-year installation plan and the entire system is expected to be in service for 10 years. By allowing careful PMU relocations in intermediate stages, the curve of system observability with respect to number of years could be shifted up, as shown in Fig. 1. It has to be noted that such increased observability comes at the cost of PMU relocations. Therefore, it is up to a system planner to decide whether the benefit of PMU relocations transcends their costs.

Unlike [12], where bad data detection is provided by providing higher measurement redundancy ratio, we focus on maximizing the cumulative system observability. This is because we believe higher observability is more desirable during intermediate stages, while higher measurement redundancy may matter more once the system becomes fully observable.

A. OMPP Formulation

The objective of OMPP is to maximize the cumulative system observability over the planned installation duration, P, subject to the financial budgets allocated to each installation stage. Moreover, the system observability after the final stage, Stage P, should be greater than a threshold value, O_P.

Since no existing work has addressed the cost of relocating PMUs, ε, we propose a simple method to estimate the value of ε in this section. We consider two major factors that contribute to the cost of relocating a PMU, namely, the delivery cost of moving a standard PMU from one substation to another, C_{del}, and the cost of purchasing and installing a PMU on the destination bus, C_{ins}. These two values are bus-dependent and market-dependent. An extra factor, C_{com}, is present to reflect the availability and quality of communication channels at the destination substation. The value of C_{com} is normally set to 0 when the destination substation is equipped with good communication devices such as optical fibers, and is set to a high value when the substation has very limited communication access. In other words

$$\varepsilon = C_{del} + C_{ins} + C_{com}. \tag{1}$$

It can be seen that the value of ε is both bus-dependent and market-dependent, precise evaluation of which requires extensive studies and is beyond the scope of this work. Therefore, in order to simplify our study, we assume ε to be constant in this paper.

Let us assume that if the financial budget at a certain stage cannot be used up within that stage, the residual amount is available for use in the future. Then our proposed OMPP model is formulated as follows:

$$\begin{aligned} \max. \quad & \tilde{O} \\ \text{s.t.} \quad & \begin{cases} \gamma_i \varepsilon + \delta_i c \le \zeta_i + r_i & \forall i \in \{1, 2, \ldots, P\} \\ O(\boldsymbol{\mu}_P) \ge O_P. \end{cases} \end{aligned} \tag{2}$$

In this formulation, c is the cost of purchasing a new PMU. $\boldsymbol{\mu}_i = (\mu_{i,1}, \mu_{i,2}, \ldots, \mu_{i,B})$, $i = 1, \ldots, P$, is the $1 \times B$ PMU installation matrix in a power system with B buses at Stage i. An element $\mu_{i,j}$ equals to 1 if a PMU has been installed on bus j by stage i, and it equals 0 otherwise. The financial budget for stage i is denoted by ζ_i, and the residue money from previous stages is r_i. γ_i is the number of relocated PMUs and δ_i is the number of newly purchased and installed PMUs during stage i. r_i, δ_i, and γ_i can be derived as follows:

$$r_i = \sum_{k=1}^{i-1} (\zeta_k - \gamma_k \varepsilon - \delta_k c) \tag{3}$$

$$\delta_i = \|\boldsymbol{\mu}_i\|_1 - \|\boldsymbol{\mu}_{i-1}\|_1 \tag{4}$$

$$\gamma_i = \frac{1}{2} \left(\|\boldsymbol{\mu}_i - \boldsymbol{\mu}_{i-1}\|_1 - \delta_i \right) \tag{5}$$

where $\|\boldsymbol{\mu}_i\|_1$ is the 1-norm of $\boldsymbol{\mu}_i$.

To understand the calculations of δ_i and γ_i, let us consider a case where $\boldsymbol{\mu}_{i-1} = (0, 0, 1, 0, 1)$ and $\boldsymbol{\mu}_i = (1, 1, 0, 0, 1)$. According to (4), $\delta_i = 1$, which means that one new PMU is installed during stage i. Then $\|\boldsymbol{\mu}_i - \boldsymbol{\mu}_{i-1}\|_1 = 3$ indicates that three elements of $\boldsymbol{\mu}_{i-1}$ have changed either from 0 to 1 or vice versa. Since installing one new PMU introduces one element change (from zero to one) and relocating a PMU causes two changes (one from zero to one and the other from one to zero), the number of relocations during stage i is calculated as $\gamma_i = (3 - \delta_i)/2 = 1$. In other words, for this case, the PMU originally installed on Bus 3 is re-deployed together with a new one, installed on Bus 1 and Bus 2.

The cumulative system observability, $\tilde{O}$, and the observability after stage i, $O(\boldsymbol{\mu}_i)$, are defined as follows:

$$\tilde{O} = \sum_{i=1}^{P} O(\boldsymbol{\mu}_i) \tag{6}$$

$$O(\boldsymbol{\mu}_i) = \frac{\boldsymbol{\rho}_i \boldsymbol{e}_B}{B}. \tag{7}$$

$O(\boldsymbol{\mu}_i)$ is a positive number representing the percentage of observed buses among all buses after stage i and hence is used as a metric of current system observability. The $1 \times B$ matrix, $\boldsymbol{\rho}_i = (\rho_{i,1}, \rho_{i,2}, \ldots, \rho_{i,B})$, is the binary indicator of whether buses are observable by PMUs or not. $\rho_{i,j}$ equals to 1 if bus j is observable at stage i, and it equals to 0 otherwise. It can be easily derived given a system connectivity matrix, $M_{B\times B}$, with elements $m_{p,k}$, $p = 1, \ldots, B$, $k = 1, \ldots, B$, and $m_{p,k} = 1$ if Bus p and k are either directly connected or $p = k$, and $m_{p,k} = 0$ otherwise [11], and PMU installation matrix at stage i, $\boldsymbol{\mu}_i$. $\boldsymbol{e}_B$ is a $B \times 1$ matrix with all elements equal to 1. Let $\boldsymbol{m}_j$ be the $1 \times B$ matrix consisting of the ith row of $M_{B\times B}$, then $\boldsymbol{\rho}_i$ is defined as follows:

$$\rho_{i,j} = \begin{cases} 1 & \text{if } \boldsymbol{m}_j \boldsymbol{\mu}_i^T > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

In our OMPP model given in (2), by maximizing $\tilde{O}$ we maximize the cumulative system observability over time. In situations when full system observability is not strictly demanded, namely, O_P is set to a value less than 1, our model will give a solution with the highest overall system observability. Such kind of flexibility is provided by no other existing work.

It is worth noting that, the total number of PMU relocations is heavily dependent on the value of ε. To consider two extreme cases, when $\varepsilon = 0$, then at the beginning of every stage, the strategy is equivalent to dismantling all installed PMUs from the system and install all available PMUs on an empty system, resulting in highest possible values of $O(\boldsymbol{\mu}_i)$ at every stage. On the other hand, when $\varepsilon = \infty$, the strategy will treat all installed PMUs as non-relocatable. Then our OMPP model will give the same results as calculating the traditional OPP problem but installing the PMU at several stages.

B. Variant of OMPP Formulation

Since the formulations introduced in Section II-A require a relatively good estimation on the costs of PMU relocations, and such estimations may not always be available, we propose a variant of OMPP formulation that does not require explicit evaluations on PMU relocation costs, i.e., ε or $\varepsilon_{j,k}$. Instead, this new formulation only requires a company to decide how many PMUs can be relocated throughout the installation stages, denoted by $\mathcal{K}$. Since ε and $\varepsilon_{j,k}$ are not used, the stage budget, ζ_i, and the cost of purchasing and installing a new PMU, c and c_i, are not needed, either.

Hence, the new OMPP problem formulation becomes

$$\begin{aligned} \max.\ & \tilde{O} \\ \text{s.t.}\ & \begin{cases} \sum_{i=1}^{P} \gamma_i \leq \mathcal{K} \\ O(\boldsymbol{\mu}_P) \geq O_P. \end{cases} \end{aligned} \tag{9}$$

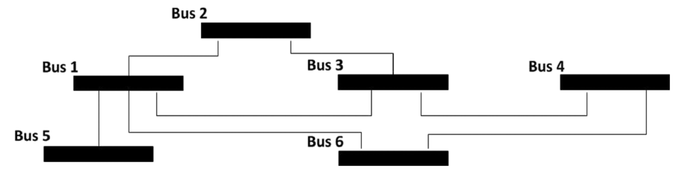

Fig. 2. Six-bus system from [11].

In the above formulation, $\tilde{O}$ is defined in the same way as (6) and (7).

It is worth noting that, despite the fact this formulation makes the computation easier, it does have some drawbacks. The most severe one is that system planners may have difficulty deciding exactly how many relocations they can tolerate. Therefore, it is advised that this formulation is only used to understand how PMU relocations affect multistage installation performances.

C. Modeling Zero Injections

In a power system, zero injection buses are those without generations or loads connected. When zero injection buses are considered in the process of PMU placement, the number of PMUs needed for full system observability can be reduced.

To illustrate this, let us consider a simple six-bus case shown in Fig. 2. Without considering zero injections, two PMU, placed on Bus 1 and 4, will make the system fully observable. However, if Bus 3 is a zero-injection bus and a PMU is placed on Bus 1, the voltage phasors on Bus 1, 2, 3, 5, and 6, denoted by $\boldsymbol{V}_1$, $\boldsymbol{V}_2$, $\boldsymbol{V}_3$, $\boldsymbol{V}_5$, and $\boldsymbol{V}_6$, respectively, are directly measured. Then, according to Ohm's Law, the current phasors between Bus 1 and 3, $\boldsymbol{I}_{1,3}$, and between Bus 2 and 3, $\boldsymbol{I}_{2,3}$, are known as well. Since Bus 3 is a zero-injection bus, by Kirchhoff's Current Law (KCL), we have

$$\boldsymbol{I}_{1,3} + \boldsymbol{I}_{2,3} + \boldsymbol{I}_{3,4} = \boldsymbol{0}. \tag{10}$$

Therefore, the current phasor between Bus 3 and 4, $\boldsymbol{I}_{3,4}$, is solvable, making Bus 4 observable and the entire system fully observable.

To model zero injections, the calculation of $\rho_{i,j}$ in (8) needs to be changed. Let $\boldsymbol{Z} = (Z_1, \ldots, Z_B)$ be the zero-injection indication vector, with $Z_i = 1$ if Bus i is a zero-injection bus, and $Z_i = 0$ otherwise. With zero injections considered, the new $\rho_{i,j}$ should be calculated as follows:

$$\hat{\rho}_{i,j} = \begin{cases} 1 & \text{if } \boldsymbol{m}_j \boldsymbol{\mu}_i^T > 0 \text{ or } Z_i = 1 \text{ and } \boldsymbol{m}_j \boldsymbol{\rho}_i^T \geq \|\boldsymbol{m}_j\|_1 - 1 \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

With $\rho_{i,j}$ replaced by $\hat{\rho}_{i,j}$, the two formulations provided in the previous two subsections will take zero-injections into calculation.

D. NP-Completeness of OMPP

In order to prove a problem is NP-complete, one must show that:

1) the problem is NP; and
2) the problem reduces to a known NP-complete problem.

It is very easy to see that the OMPP problem is NP. For a given B-bus power system, let us assume that at least M PMUs are needed to ensure its full observability. Then, by setting $P = 1, \varepsilon = 0, c \neq 0, \zeta_1 = M \times c$, and $O_P = 1$, problem (2) becomes

$$\begin{aligned} &\max. \quad \frac{\boldsymbol{\rho}_1 \boldsymbol{e}_B}{B} \\ &\text{s.t.} \quad \begin{cases} \|\boldsymbol{\mu}_1\|_1 = M \\ O(\boldsymbol{\mu}_1) \geq 1. \end{cases} \end{aligned} \tag{12}$$

This is equivalent to the OPP problem, which determines the PMU installation locations such that the total number of PMUs required is minimum. Since OPP is NP-complete [13], OMPP is also NP-complete.

III. Evolvability Index of Optimal PMU Placement Solutions

In a power system, solving the OPP problem gives us the minimum number of PMUs required for full system observability. However, it is frequently seen that the minimum number of PMUs for full system observability can be achieved by multiple different placement schemes. As a result, it is necessary to design a metric that can be used to select one superior solution among them. A well-known metric is the *System Observability Redundancy Index (SORI)* proposed in [8]. The value of *SORI* of a particular PMU placement strategy is calculated by summing up the number of PMUs monitoring each bus over all buses in a power system. A higher *SORI* value means the PMU placement strategy offers higher reliability. However, although *SORI* considers reliability as a secondary metric, it does not address the situation where multistage PMU placement is required. As a result, based on the OMPP formulations proposed in Section II, we introduce another metric, called *Evolvability Index (EI)*, as a performance metric for different OPP solutions in the multistage installation scenario in this section.

A. Evolvability Index (EI)

EI is the metric indicating how well different OPP solutions perform given a certain multistage installation plan. In particular, the *EI* value of a particular OPP solution under a given installation plan is determined by the maximum value of $\bar{O}$ that it can achieve. Let us consider a B-bus power system. Let $\boldsymbol{\mu}_P$ denote an OPP solution and $\boldsymbol{l}$ represent a P-stage installation plan, the ith element of which, l_i, denotes the number of PMU to be installed during stage i. Then the *EI* of $\boldsymbol{\mu}_P$ with respect to $\boldsymbol{l}$ is defined as follows:

$$EI(\boldsymbol{\mu}_P, \boldsymbol{l}) = \max. \sum_{i \in \boldsymbol{l}} O(\boldsymbol{\mu}_i) \tag{13}$$

such that, $\forall i \in \{1, 2, \ldots, P\}$

$$\|\boldsymbol{\mu}_i - \boldsymbol{\mu}_{i-1}\|_1 = l_i \tag{14}$$

$$\|\boldsymbol{\mu}_i\|_1 - \|\boldsymbol{\mu}_{i-1}\|_1 = l_i. \tag{15}$$

In other words, given $\boldsymbol{l}$, to compute the *EI* value of $\boldsymbol{\mu}_P$ is to determine $(P-1)$ intermediate PMU installation matrices, $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \ldots, \boldsymbol{\mu}_{P-1}$, such that the cumulative system observability is maximized. We let $\boldsymbol{\mu}_0 = \mathbf{0}$, meaning that no PMU were installed before stage 1.

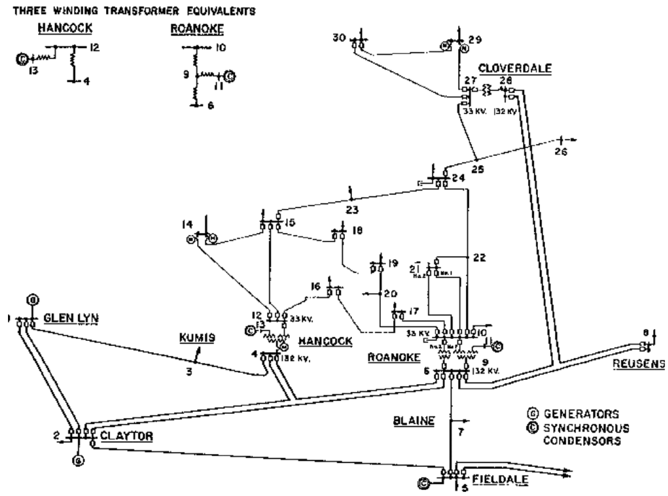

Fig. 3. IEEE 30-bus system [19].

As can be seen, calculating *EI* is equivalent to solving a special case of the OMPP problem in (2), with the financial budget on stage i, $\zeta_i = (l_i \times c), \forall i = \{1, \ldots, P\}, O_P = 1$, and $\varepsilon = 0$. However, the primary purpose of introducing *EI* is to propose a novel metric for comparing different OPP solutions without considering PMU relocations.

B. IEEE 30-Bus System Example

To help readers further understand the concept of *EI*, we now give an illustration using the IEEE 30-bus system, shown in Fig. 3.

After solving the OPP problem on this system, we obtain four different solutions, denoted by $\mathcal{P}_i$, $i = 1, 2, 3, 4$, which can achieve full system observability with the minimum number of PMUs, namely, 10 PMUs. The four solutions are:

- $\mathcal{P}_1 = \{3, 5, 8, 9, 10, 12, 15, 18, 25, 30\}$
- $\mathcal{P}_2 = \{3, 6, 7, 9, 10, 12, 19, 24, 25, 30\}$
- $\mathcal{P}_3 = \{1, 6, 7, 9, 10, 12, 19, 23, 25, 27\}$
- $\mathcal{P}_4 = \{1, 2, 6, 9, 10, 12, 18, 24, 26, 27\}$

An element, k, in $\mathcal{P}_i$ denotes that the ith solution requires a PMU to be installed on Bus-k.

With the above solutions available, we decide to place the PMUs in a three-stage manner, with three PMUs installed in the first stage, three more in the second, and four more in the last. In other words, $\boldsymbol{l} = (3, 3, 4)$. Then the *EI* values of each OPP solution, $EI(\mathcal{P}_i, \boldsymbol{l})$, can be calculated according to (13). The results are shown in Table I. We notice that $EI(\mathcal{P}_4, \boldsymbol{l}) = EI(\mathcal{P}_3, \boldsymbol{l}) > EI(\mathcal{P}_2, \boldsymbol{l}) > EI(\mathcal{P}_1, \boldsymbol{l})$. This means that $\mathcal{P}_4$ and $\mathcal{P}_3$ are better than the other two solutions in terms of *EI*, and a manager should choose either one of the two for this particular multistage installation plan.

IV. Solving OMPP With Chemical Reaction Optimization (CRO)

Chemical reaction optimization (CRO) is a metaheuristic algorithm developed by Lam and Li [14]. Although it is relatively new, CRO has been successfully applied in solving many NP-hard optimization problems, including, task scheduling in grid computing [15], spectrum allocation in cognitive radio

TABLE I
EI OF THE FOUR OPP SOLUTIONS FOR IEEE 30-BUS SYSTEM

$\mathcal{P}_1 = \{3, 5, 8, 9, 10, 12, 15, 18, 25, 30\}$

$\mathcal{P}_{1,1} = \{10, 12, 15\}$	$EI(\mathcal{P}_1, \boldsymbol{l}) = 2.233$
$\mathcal{P}_{1,2} = \{3, 9, 25\}$	
$\mathcal{P}_{1,3} = \{5, 8, 18, 30\}$	

$\mathcal{P}_2 = \{3, 6, 7, 9, 10, 12, 19, 24, 25, 30\}$

$\mathcal{P}_{2,1} = \{6, 12, 19\}$	$EI(\mathcal{P}_2, \boldsymbol{l}) = 2.3$
$\mathcal{P}_{2,2} = \{9, 25, 30\}$	
$\mathcal{P}_{2,3} = \{3, 7, 10, 24\}$	

$\mathcal{P}_3 = \{1, 6, 7, 9, 10, 12, 19, 23, 25, 27\}$

$\mathcal{P}_{3,1} = \{10, 12, 27\}$	$EI(\mathcal{P}_3, \boldsymbol{l}) = 2.433$
$\mathcal{P}_{3,2} = \{6, 19, 23\}$	
$\mathcal{P}_{3,3} = \{1, 7, 9, 25\}$	

$\mathcal{P}_4 = \{1, 2, 6, 9, 10, 12, 18, 24, 26, 27\}$

$\mathcal{P}_{4,1} = \{10, 12, 27\}$	$EI(\mathcal{P}_4, \boldsymbol{l}) = 2.433$
$\mathcal{P}_{4,2} = \{1, 18, 24\}$	
$\mathcal{P}_{4,3} = \{2, 6, 9, 26\}$	

[16], and repeat identification in multiple DNA sequences [17]. Interested readers may refer to [18] for the state-of-the-art developments of CRO.

As discussed in earlier sections, we decided to use CRO to attack the OMPP problem because of its superiority in computation time. As a matter of fact, the authors first tried ILP approach to attack the problem, however, the program failed to converge after several tens hours of running when a relatively large-scale system is being tested.

A. Simplified Version of CRO

In this paper, we adopt a simplified version of CRO named SCRO, which was first introduced in [11]. Note that the details of the canonical CRO can be found in both [14] and [11]. Since [11] has demonstrated that the performance of SCRO is superior to that of canonical CRO in OPP, this work only focuses on applying SCRO to OMPP. Before illustrating SCRO, let us first review the basic concepts in CRO.

CRO is inspired by the phenomenon that in a chemical reaction the final products are generated from reactants by a sequence of intermediate collisions. Moreover, in a microscopic view, molecules will finally reach the lowest free energy, which gives them the most stable state. Similarly, the main agent in CRO is the molecule with several attributes. In particular, the molecular structure is considered as a possible solution in the optimization problems. For a specific molecule ϖ, the potential energy (PE) is viewed as the objective function value, while the kinetic energy (KE) reflects the activity degree. For example, a new molecule ϖ' produced from ϖ would be accepted if $PE_{\varpi'}+KE_{\varpi'} \geq PE_{\varpi}$. Thus, the larger the value of KE, the more likely a new molecule is accepted. Moreover, the number of hits, the minimum hit number, and the minimum value are another three attributes of a molecule, and they are used to trigger the energy regain in SCRO.

In both CRO and SCRO, the reaction is supposed to happen in a closed vessel. Furthermore, there is an central energy buffer $(buffer)$, which can exchange energy with inside molecules. Hence, according to the law of energy conservation, the total energy of the system consisting of all PEs, KEs, and $buffer$ will

TABLE II
SCHEMATIC PROCEDURE OF THE SCRO ALGORITHM

Stag 1:	1.1: Initialize a molecule, calculate its *PE*, and configure other attributes
	1.2: Set the system parameters, i.e., *buffer*, α, and *KELossRate*
Stag 2:	While the stopping criterion is not satisfied, do:
	2.1: Examine whether the mechanism of energy regain is triggered, and if so, go to 2.2; otherwise, go to 2.3
	2.2: $KE_{\varpi} = KE_{\varpi}+$*buffer*, and then set *buffer* $= 0$
	2.3: Try on-wall ineffective collision. If it succeeds, update the molecule and *buffer*
Step 3:	Output the best solution;

not be changed during the process. In SCRO, only the on-wall ineffective collision, which is one of the four elementary reactions of the canonical CRO, is implemented. Particularly, let ϖ' and ϖ be the new and original molecules, respectively. Then, ϖ' will substitute ϖ if $PE_{\varpi'} + KE_{\varpi'} \geq PE_{\varpi}$. The kinetic energy for ϖ' is calculated by

$$KE_{\varpi'} = (PE_{\varpi'} + KE_{\varpi'} - PE_{\varpi}) \times \theta \tag{16}$$

where θ is randomly selected from $[KELossRate, 1]$. $KELossRate$, predefined by the user, is the maximum percentage of new KE absorbed by $buffer$ at one time. Accordingly, $buffer$ will be updated on each successful collision by

$$buffer = buffer + (PE_{\varpi'} + KE_{\varpi'} - PE_{\varpi}) \times (1-\theta). \tag{17}$$

Moreover, a parameter α is introduced as the criterion to trigger the energy regain. In other words, when (number of hits—the minimum hit number $> \alpha$), the mechanism of energy regain takes place, and $buffer$ releases its entire energy to the molecule's KE. By doing this, the molecule is able to escape from the local optima where it may be trapped.

Table II gives the schematic procedure of SCRO. It can be divided into three stages. In stage 1, we initialize a molecule and configure its attributes. Meanwhile, the system parameters are also set at this stage. Then, SCRO enters into the iteration stage until the stopping criterion is met. In each iteration, it first checks whether the condition of energy regain is satisfied. If so, the molecule absorbs $buffer$ into its KE. Otherwise, the algorithm attempts the on-wall ineffective collision. The best solution as well as its objective function value are output in the final stage.

B. Implementation of SCRO on OMPP

In order to apply SCRO to OMPP, the key part is to design the pattern of the solution (i.e., molecular structure) and the operator for on-wall ineffective collision. Since OMPP involves deciding multiple stages of installation of PMUs, we employ $[\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \cdots, \boldsymbol{\mu}_N]$, where $\boldsymbol{\mu}_i = (\mu_{i,1}, \mu_{i,2}, \ldots, \mu_{i,B})$ for $i = 1, 2, \cdots, N$, as the solution vector. In this way, the solution space becomes very huge. For example, in the case of IEEE 118-bus system with six stages of installation, the solution space has a size of 2^{708}. Therefore, we only generate feasible solutions in the course of SCRO, which can greatly improve the efficiency

TABLE III
PARAMETERS FOR SCRO

	Parameter	Value
SCRO	*KELossRate*	0.8
	α	100*B
	Initial *KE*	B−initial *PE*
	Stopping criterion	1000*B iterations

in the exploration of the solution space. In this paper, we adopt the one-resource change and pair-wise exchange operators as introduced in [15].

Parameters for SCRO are listed in Table III. Note that initial KE is set to (B—initial PE), which enables the molecule to jump out of the local optimum after each energy regain. Moreover, we code SCRO with C++ and the following simulation is conducted on a PC with an Intel Core Duo 2.66-Hz CPU and 2 GB of RAM.

V. NUMERICAL STUDY

In this section, we carry out numerical studies applying our OMPP optimization models introduced in Section II-A. Both (9) and (2) are studied on the IEEE 57-bus system, the IEEE 118-bus system, and the IEEE 300-bus system.

In order to apply our proposed OMPP models, the user input data required are as follows:

1) The connectivity matrix of the power system, $M_{B\times B}$. The matrices used in our study are derived from the three systems.
2) The financial budget for PMU installation in each stage, ζ_i.
3) The number of installation stages, P. In our study, the value of P is set to be 4 for the IEEE 57-bus system and 6 for the IEEE 118-bus and 300-bus systems.
4) The system observability requirement after multistage installations, O_P. We set O_P to be 1 for all cases in our simulations. In other words, we require a fully observable system after P installations.
5) The expected cost of purchasing and installing a PMU, c. We use the value of US\$11 535, which is the price obtained from a PMU vendor from the U.S.
6) The expected cost of relocating PMUs, ε. In each case, ε is set to US\$1000.

A. OMPP Model Study

1) IEEE 57-Bus System: According to [11], without zero injections, at least 17 PMUs are needed to make the 57-bus system fully observable. Therefore, the total financial budget should be greater than or equal to $17 \times \$11\,535 = \$196\,095$ so as to make the installation feasible. In our study, the financial budget at each stage is set as \$46,601, \$46,601, \$46,601, and \$58,252. The total financial budget is around \$1,960 more than the minimum budget, and this is the financial budget for PMU relocations. For the case where zero injections are considered, at least 14 PMUs are needed. Following the same philosophy, we set the financial budget at each stage as \$34 951, \$34 951, \$46 601, and \$46 601, leaving \$1615 as the PMU relocation budget.

TABLE IV
RESULTS OF OMPP FOR THE 57-BUS SYSTEM WITHOUT ZERO INJECTIONS

Stage # (i)	1	2	3	4
# of new PMUs (δ_i)	4	4	4	5
# of PMU relocations (γ_i)	0	0	0	1
New installations	4 9 38 56	1 24 29 32	20 46 50 53	27 30 36 41 57
Relocated PMUs	–	–	–	56→15
System installation status	4 9 38 56	1 4 9 24 29 32 38 56	1 4 9 20 24 29 32 38 46 50 53 56	1 4 9 15 20 24 27 29 30 32 36 38 41 46 50 53 57
Observability	0.403509	0.701754	0.877193	1

TABLE V
RESULTS OF OMPP FOR THE 57-BUS SYSTEM WITH ZERO INJECTIONS

Stage # (i)	1	2	3	4
# of new PMUs (δ_i)	3	3	4	4
# of PMU relocations (γ_i)	0	0	1	0
New PMU installations	36 38 41	1 22 32	27 47 50 52	19 45 55 57
Relocated PMUs	–	–	38→25	–
System installation status	36 38 41	1 22 32 36 38 41	1 22 25 27 32 36 41 47 50 52	1 19 22 25 27 32 36 41 45 47 50 52 55 57
Observability	0.315789	0.561404	0.824561	1

At the first stage, four new PMUs will be placed at Buses 4, 9, 38, and 56. After this first stage, system observability will be 0.4035, indicating that around 40% of all buses in the system are observable. At the second stage, four new PMUs will be installed at Buses 1, 24, 29, and 32. After this stage, the system observability will increase to 0.7018. The third stage comes with four new PMUs being installed on Buses 20, 46, 50, and 53. At the final stage, five new PMUs will be installed on Buses 27, 30, 36, 41, and 57, and the PMUs previously installed on Bus 56 will be moved to Bus 15. By the end of the installation process, Buses 1, 4, 9, 15, 20, 24, 27, 29, 30, 32, 36, 38, 41, 46, 50, 53, and 57 will have PMUs installed, and the system observability becomes 1. The full installation process is illustrated in Table IV. This scenario with $\varepsilon = 1000$ has $\tilde{O} = 2.982$. Similarly, the four-stage PMU installation considering zero injections are given in Table V. In this case, $\tilde{O} = 2.702$.

In order to see how our proposed OMPP model performs compared to existing work, we carry out this multistage installation (without zero injections) again using the method proposed in [8] and compared the results with our OMPP. The resulting multistage placement from [8] yields $\tilde{O} = 2.684$, which is less than the result of our OMPP model. To be complete, the scenario with maximum *EI*, which takes $\tilde{O} = 2.965$, is added for comparison as well. Fig. 4 plots the change of system observability with respect to staging. For the case considering zero injections, as shown in Fig. 5, similar conclusions can be drawn.

TABLE VI
RESULTS OF OMPP FOR THE 118-BUS SYSTEM WITHOUT ZERO INJECTIONS

Stage # (i)	1	2	3	4	5	6
# of new PMUs (δ_i)	5	5	5	5	6	6
# of PMU relocations (γ_i)	0	0	1	0	0	2
New PMU installations	12 37 49 80 92	17 23 59 85 105	56 62 71 77 110	28 34 45 52 68	10 20 25 63 75 114	21 40 86 90 94 102
Relocated PMUs	–	–	59→5	–	–	20→2 92→15
System installation status	12 37 49 80 92	12 17 23 37 49 59 80 85 92 105	5 12 17 23 37 49 56 62 71 77 80 85 92 105 110	5 12 17 23 28 34 37 45 49 52 56 62 68 71 77 80 85 92 105 110	5 10 12 17 20 23 25 28 34 37 45 49 52 56 62 63 68 71 75 77 80 85 92 105 110 114	2 5 10 12 15 17 21 23 25 28 34 37 40 45 49 52 56 62 63 68 71 75 77 80 85 86 90 94 102 105 110 114
Observability	0.338983	0.576271	0.737288	0.847458	0.940678	1

TABLE VII
RESULTS OF OMPP FOR THE 118-BUS SYSTEM WITH ZERO INJECTIONS

Stage # (i)	1	2	3	4	5	6
# of new PMUs (δ_i)	4	5	5	5	5	5
# of PMU relocations (γ_i)	0	0	0	2	0	1
New PMU installations	17 37 49 80	12 23 59 92 105	65 71 77 85 110	34 56 62 64 68	45 53 75 94 115	30 41 86 91 102
Relocated PMUs	–	–	–	59→21 65→28	–	92→11
System installation status	17 37 49 80	12 17 23 37 49 59 80 92 105	12 17 23 37 49 59 65 71 77 80 85 92 105 110	12 17 21 23 28 34 37 49 56 62 64 68 71 77 80 85 92 105 110	12 17 21 23 28 34 37 45 49 53 56 62 64 68 71 75 77 80 85 92 94 105 110 115	11 12 17 21 23 28 30 34 37 41 45 49 53 56 62 64 68 71 75 77 80 85 86 91 94 102 105 110 115
Observability	0.29661	0.567797	0.754237	0.864407	0.940678	1

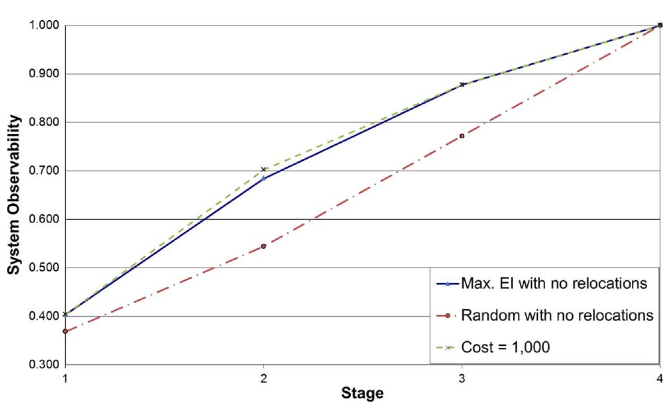

Fig. 4. System observability changes during multistage PMU installations in IEEE 57-bus system without zero injections.

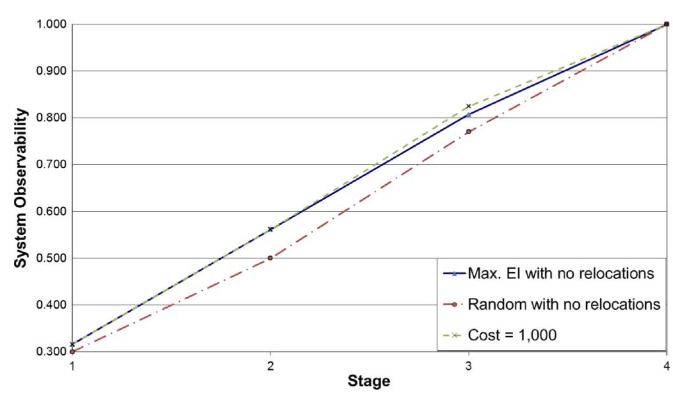

Fig. 5. System observability changes during multistage PMU installations in IEEE 57-bus system with zero injections.

Therefore, in this 57-bus case without zero injections, with our proposed OMPP, the system observability during PMU installation process is significantly increased at the cost of no more than 1% of the total financial budget.

2) IEEE 118-Bus System: According to [11], at least 32 PMUs are required for full observability in the IEEE 118-bus system without zero injections. Using a similar philosophy as the 57-bus system, we planned a six-stage installation with stage bugets set to \$58 252, \$58 252, \$58 252, \$58 252, \$69 902, and \$69 902. In total, \$3691 is reserved for PMU relocations. For the case with zero injections, 29 PMUs are needed, and we set stage budgets to \$46 601, \$58 252, \$58 252, \$58 252, \$58 252, and \$58 252. Total PMU relocation budget is \$3345.

Table VI shows the OMPP results without zero injections, and Table VII shows the results with zero injections. For the no-zero-injection case, maximum *EI* multistage installation results in $\tilde{O} = 4.398$, the method proposed in [8] gives $\tilde{O} = 4.195$, and our OMPP model gives the best result with $\tilde{O} = 4.441$. The comparison can be found in Figs. 6 and 7.

3) IEEE 300-Bus System: The IEEE 300-bus system is the most complicated system we will look into in this study. Based on the method used in [11], we calculate the minimum number of PMUs required for full observability to be 87. We fulfill

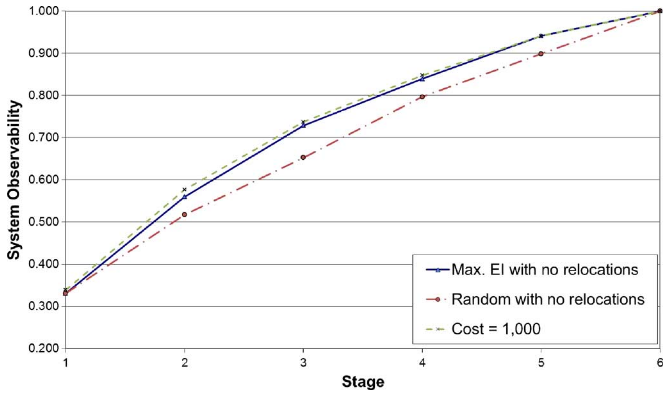

Fig. 6. System observability changes during multistage PMU installations in IEEE 118-bus system without zero injections.

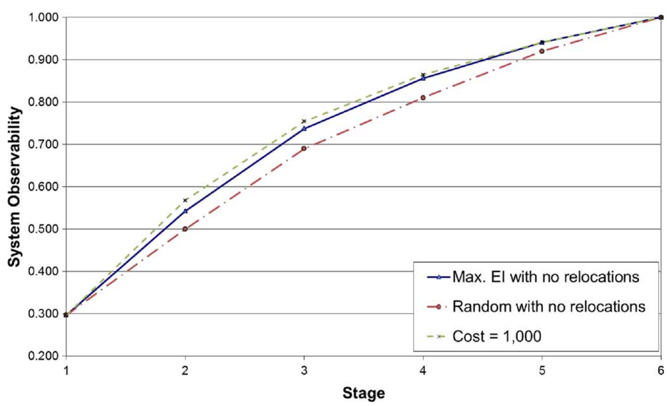

Fig. 7. System observability changes during multistage PMU installations in IEEE 118-bus system with zero injections.

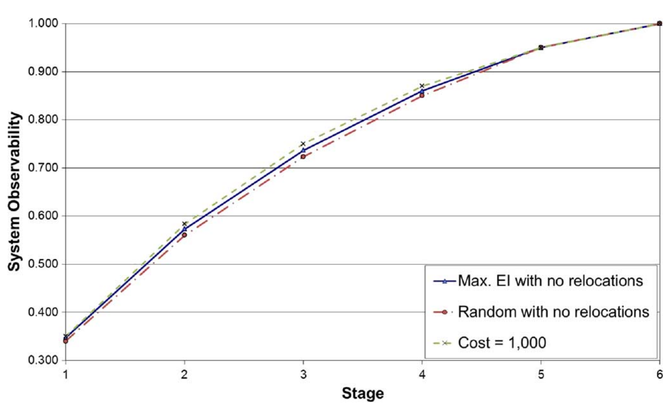

Fig. 8. System observability changes during multistage PMU installations in IEEE 300-bus system without zero injections.

the six-stage PMU installation plan by allowing \$163 105, \$163 105, \$163 105, \$174 755, \$174 755, and \$174 755 to be used for PMU installations and relocations in each stage. The total PMU relocation budget is \$10 035. Due to the lack of information, we do not consider zero injections on the 300-bus system.

As shown in Fig. 8, our OMPP model gives the best results, with $\tilde{O} = 4.503$. The maximum *EI* scenario has $\tilde{O} = 4.467$. The worse result, with $\tilde{O} = 4.423$, is generated using the method proposed in [8].

B. Results of the Variant of OMPP

The variant of OMPP is an alternative way to plan multistage PMU installations, which do not require explicit financial budget information. In other words, we look at the system observability given a fixed number of PMU relocations, i.e., $\mathcal{K}$. As discussed in Section II-B, although this OMPP variant may be problematic for real-world uses, it serves as a good tool to investigate how PMU relocations affect multistage PMU installation performances.

For simplicity, in the following cases, the number of new PMUs installed at each stage are the same as illustrated in Section V-A.

1) IEEE 57-Bus System: In this case, results shown in Table IV and Table V indicates that the cumulative system observability can be improved by allowing one PMU relocations. However, when we allow two and three PMU relocations by setting $\mathcal{K} = 2$ and $\mathcal{K} = 3$, respectively, we do not see an increase in the resulting $\tilde{O}$ value. Therefore, we conclude that, the optimal $\tilde{O}$ can be achieved by allowing at most two PMU relocations.

2) IEEE 118-Bus System: Considering the scenario with $\mathcal{K} = 1$, Table VIII and Table IX gives the full installation results for cases without and with zero injections, respectively. Without zero injections, $\mathcal{K} = 1$ gives $\tilde{O} = 4.432$, which is less than 4.441 shown in Table VI. Similarly, with zero injections, $\mathcal{K} = 1$ gives $\tilde{O} = 4.398$, which is less than 4.424 shown in Table VII. This implies that, allowing only one PMU relocation is not optimal for this multistage installation on the 118-bus system.

3) IEEE 300-Bus System: This case is very complicated, and it requires at least nine PMU relocations to maximize the system observability at each stage. Table X displays the full processes of system observability changes with $\mathcal{K}$ from 0 to 9. It shows that the value of $\tilde{O}$ increases as PMU relocation number increases. Due to lack of information, zero injections are not considered in the 300-bus system.

C. Discussions

In addition to the study results shown, there are two additional matters that we need to discuss here.

First, there is an implicit assumption that the extra money needed for PMU relocations should be less than the price of a single PMU. In other words

$$c \times K \times (1 + 1\%) \leq c. \tag{18}$$

We made this assumption because of the simple fact that more PMUs will always make the system better, and relocations are meaningful only when the extra money available is not sufficient to purchase one more PMU.

Second, as can be demonstrated from Table X, $\tilde{O}$ increases if more PMU relocations are allowed. In other words, given the same financial budget, lower ε value could possibly lead to higher overall multistage installation performances. From the results in Section V-B, one can infer that, given all other parameters fixed, there exist ε_l and ε_u such that the value of $\tilde{O}$ increases as ε decreases only if $\varepsilon \in [\varepsilon_l, \varepsilon_u]$. There also exist $\tilde{O}_{min}$ and $\tilde{O}_{max}$ such that, for any $\varepsilon \in [\varepsilon_u, \infty]$, $\tilde{O}$ remains its minimum value, $\tilde{O}_{min}$, and for any $\varepsilon \in [0, \varepsilon_u]$, $\tilde{O}$ remains its maximum value, $\tilde{O}_{max}$.

TABLE VIII
RESULTS OF THE VARIANT OF OMPP FOR THE 118-BUS SYSTEM WITHOUT ZERO INJECTIONS, $\mathcal{K} = 1$

Stage # (i)	1	2	3	4	5	6
# of new PMUs (δ_i)	5	5	5	5	6	6
# of PMU relocations (γ_i)	0	0	0	0	0	1
New PMU installations	12 37 49 80 92	17 23 56 85 105	5 64 71 77 110	34 45 62 115 116	10 20 29 53 75 86	15 30 41 90 94 102
Relocated PMUs	–	–	–	–	–	92→1
System installation status	12 37 49 80 92	12 17 23 37 49 56 80 85 92 105	5 12 17 23 37 49 56 64 71 77 80 85 92 105 110	5 12 17 23 34 37 45 49 56 62 64 71 77 80 85 92 105 110 115 116	5 10 12 17 20 23 29 34 37 45 49 53 56 62 64 71 75 77 80 85 86 92 105 110 115 116	1 5 10 12 15 17 20 23 29 30 34 37 41 45 49 53 56 62 64 71 75 77 80 85 86 90 94 102 105 110 115 116
Observability	0.338983	0.567797	0.737288	0.847458	0.940678	1

TABLE IX
RESULTS OF THE VARIANT OF OMPP FOR THE 118-BUS SYSTEM WITH ZERO INJECTIONS, $\mathcal{K} = 1$

Stage # (i)	1	2	3	4	5	6
# of new PMUs (δ_i)	4	5	5	5	5	5
# of PMU relocations (γ_i)	0	0	0	1	0	0
New PMU installations	17 37 49 80	12 59 71 85 105	23 68 77 94 110	34 45 56 62 102	53 63 75 91 115	11 20 30 42 86
Relocated PMUs	–	–	59→28	–	–	–
System installation status	17 37 49 80	12 17 37 49 59 71 80 85 105	12 17 23 37 49 59 68 71 77 80 85 94 105 110	12 17 23 28 34 37 45 49 56 62 68 71 77 80 85 94 102 105 110	12 17 23 28 34 37 45 49 53 56 62 63 68 71 75 77 80 85 91 94 102 105 110 115	11 12 17 20 23 28 30 34 37 42 45 49 53 56 62 63 68 71 75 77 80 85 86 91 94 102 105 110 115
Observability	0.29661	0.559322	0.745763	0.855932	0.940678	1

TABLE X
SYSTEM OBSERVABILITY CHANGES DURING MULTISTAGE PMU INSTALLATIONS IN IEEE 300-BUS SYSTEM

$\mathcal{K}$	$O(\boldsymbol{\mu}_i)$: stage # (i)						$\tilde{O}$
	1	2	3	4	5	6	
0	0.347	0.573	0.737	0.860	0.950	1.000	4.467
1	0.347	0.570	0.740	0.863	0.950	1.000	4.470
2	0.347	0.577	0.740	0.860	0.950	1.000	4.473
3	0.347	0.580	0.747	0.863	0.950	1.000	4.487
4	0.347	0.580	0.747	0.863	0.950	1.000	4.487
5	0.347	0.580	0.747	0.863	0.950	1.000	4.487
6	0.350	0.583	0.747	0.867	0.950	1.000	4.497
7	0.350	0.583	0.747	0.867	0.950	1.000	4.497
8	0.350	0.583	0.750	0.867	0.950	1.000	4.500
9	0.350	0.583	0.747	0.870	0.950	1.000	4.503

The values of $\tilde{O}_{min}$ and $\tilde{O}_{max}$ can be obtained directly by solving the problem again using the variant of OMPP, as in Section V-B. However, finding the values of ε_l and ε_u are non-trivial and cannot be obtained by the variant of OMPP. To understand this, let us consider the situation when the cost of PMU relocations are so high that only one relocation is allowed. Since the relocation budgets at each stage are cumulative, the worst case is that this one relocation is only allowed at the final stage, until when enough money can be cumulated. However, if the cost is slightly reduced but still not sufficient for two relocations, we can use it in earlier stages rather than the last one. This will give us different results which cannot be obtained by solving (9) with $\mathcal{K} = 1$. As a result, calculating the values of ε_l and ε_u remain a problem to solve in our future work.

VI. CONCLUSION

In this paper, we have proposed a novel OMPP model that takes the relocations of installed PMUs into account. Our proposed OMPP model is more realistic since it accounts for the financial constraints at each stage. Since mathematical programming methods cannot solve the OMPP problem within acceptable time scale, we solved the problem using a newly developed meta-heuristic, called SCRO, and carried out numerical studies on the IEEE 57-bus system, IEEE 118-bus system, and IEEE 300-bus system. The studies showed that our model gives better performance than the existing ones in terms of cumulative system observability during the installation process. It is worth noting that our improved performance is at the expense of a slight increase in financial budget. However, we believe that the extra money needed is marginal compared with the observability gain in intermediate stages, and if a company does not want to spend this extra money, our model will work at least as good as the existing ones. In addition, we also proposed a new metric, *EI*, to sift through multiple OPP solutions in this paper, and demonstrated how *EI* could be used by an example on the IEEE 30-bus system.

As mentioned in Section V-C, our future work includes studying how to determine the values of ε_l and ε_u. Moreover, as discussed in Section II, we also will carry out case studies to evaluate PMU relocation costs, ε, and then provide a new OMPP model to accommodate bus-and market-dependent ε in the future.

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