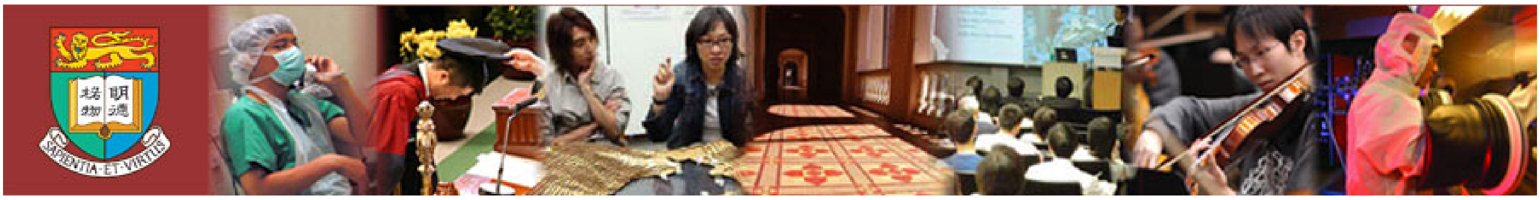

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Semi-Global Leader-Following Consensus of Linear Multi-Agent Systems With Input Saturation via Low Gain Feedback

Housheng Su, Michael Z. Q. Chen, *Member, IEEE*, James Lam, *Fellow, IEEE*, and Zongli Lin, *Fellow, IEEE*

***Abstract*—This paper investigates the problem of leader-following consensus of a linear multi-agent system on a switching network. The input of each agent is subject to saturation. Low gain feedback based distributed consensus protocols are developed. It is established that, under the assumptions that each agent is asymptotically null controllable with bounded controls and that the network is connected or jointly connected, semi-global leader-following consensus of the multi-agent system can be achieved. Numerical examples are presented to illustrate this result.**

***Index Terms*—Consensus, input saturation, leader-following, low gain feedback.**

I. Introduction

In RECENT years, the problem of consensus of multi-agent systems has received increasing interest in different fields including biology, physics, computer science, and control engineering (see, for example, [1] and the references therein). Indeed, the spirit of consensus protocol is inspired by the pioneering models of Reynolds [2] and Vicsek [3]. Other topics that are highly relevant to consensus are synchronization [4]–[14], swarming [15], [16], flocking [17]–[20] and rendezvous [21].

Consensus has been studied for many different agent dynamics including single-integrator kinematics [22]–[25], double-integrator dynamics [26]–[29], high-order-integrator dynamics [30], linear dynamics [31]–[38], and nonlinear dynamics [39]–[41]. The agents described by linear dynamics [31]–[38] can be regarded as a generalization of agents with single-integrator kinematics, double-integrator dynamics and high-order-integrator dynamics. However, only a few works on multi-agent consensus have taken input saturation into account [27], [42]–[44]. In particular, consensus in the presence of agent input saturation is investigated for agents governed by single-integrator kinematics [42] and double-integrator dynamics [27], respectively. A distributed projected consensus algorithm is proposed for the constrained consensus problem [43]. Consensus algorithms without using velocity measurement for double-integrator dynamics subject to input saturation are proposed in [44].

To the best of our knowledge, input saturation has not been taken into account in the consensus of more general agent dynamics and in the leader-following consensus. This paper will address the leader-following consensus of agents described by general linear systems subject to input saturation. By utilizing the low gain design technique [45]–[47], we will design consensus algorithms that achieve semi-global leader-following consensus of such agents on a switching network. The salient features of the algorithms proposed in this paper are as follows. Generally speaking, this paper extends the consensus of linear multi-agent systems in [31]–[38] to the case with input saturation, and extends the consensus with input saturation in [27], [42]–[44] to the consensus of more general agent dynamics. In contrast to the known results on the consensus of linear multi-agent systems in [31], [33]–[38] and the consensus with input saturation in [27], [42]–[44], the proposed consensus algorithms are analyzed on both connected and jointly connected networks. Differently from the analysis method for the consensus on jointly connected networks in [32], our method is less complicated as it does not need the rule of labeling. Besides taking input saturation into account, another distinct feature of our algorithms, in comparison with existing works on consensus of agents whose dynamics are described by linear systems [31]–[38], is that they do not require any knowledge of the interaction network topology, that is, the knowledge of eigenvalues of the coupling matrix of the network.

II. Problem Statement

We consider a group of N agents with general linear dynamics, labeled as $1, 2, \ldots, N$. The motion of each agent is described by

$$\dot{x}_i = Ax_i + B\sigma(u_i), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbf{R}^n$ is the state of agent i, $u_i \in \mathbf{R}^m$ is the control input acting on agent i, and $\sigma : \mathbf{R}^m \to \mathbf{R}^m$ is a saturation function defined as $\sigma(u_i) = [\mathrm{sat}(u_{i1}) \ \ \mathrm{sat}(u_{i2}) \ \ \cdots \ \ \mathrm{sat}(u_{im})]^{\mathrm{T}}$, $\mathrm{sat}(u_{ij}) = \mathrm{sgn}(u_{ij}) \min\{|u_{ij}|, \varpi\}$, for some constant $\varpi > 0$. For notational convenience, we also define $x = [{x_1}^{\mathrm{T}}, {x_2}^{\mathrm{T}}, \ldots, {x_N}^{\mathrm{T}}]^{\mathrm{T}}$

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and $u = [u_1^{\mathrm{T}}, u_2^{\mathrm{T}}, \ldots, u_N^{\mathrm{T}}]^{\mathrm{T}}$. The dynamics of the leader, labeled as $N+1$, is described by

$$\dot{x}_{N+1} = Ax_{N+1}. \quad (2)$$

The problem of semi-global leader-following consensus for the agents and leader described above is the following: For any *a priori* given bounded set $\mathcal{X} \subset \mathbf{R}^n$, construct a control law u_i for each agent i, which use only local information from neighbor agents, such that

$$\lim_{t\to\infty} \|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

as long as $x_i(0) \in \mathcal{X}$ for all $i = 1, 2, \ldots, N, N+1$.

Assumption 1: The pair (A, B) is asymptotically null controllable with bounded controls, that is,

1) (A, B) is stabilizable;
2) All the eigenvalues of A are in the closed left-half s-plane.

Definition 1: A set Q is said to be a positively invariant set with respect to $\dot{s} = f(s)$ if $s(0) \in Q$ implies $s(t) \in Q, \forall t > 0$, where $f : D \to \mathbf{R}^n$ is a locally Lipschitz map from a domain $D \subseteq \mathbf{R}^n$ into $\mathbf{R}^n$.

In this paper, the notation $M > 0$ denotes a positive definite matrix M, and the notation $M \geq 0$ denotes a nonnegative definite matrix M.

III. Main Results

A. Consensus on a Connected Switching Network

We consider the problem of semi-global leader-following consensus when the network is switching. The network consisting of N agents is described by an undirected graph $G(t) = \{V, E(t)\}$. In this graph, the set of vertices $V = \{1, 2, \ldots, N\}$ represents the agents in the group and the time-varying set of edges $E(t) = \{(i,j) \in V \times V : i \sim j\}$, containing unordered pairs of vertices, represents neighboring relations among the agents. Vertices i and j are said to be adjacent if $(i,j) \in E(t)$. We define the adjacency matrix $A(t) = (a_{ij}(t))$ of graph $G(t)$ as $a_{ij}(t) = 1$ if $(i,j) \in E(t)$, and $a_{ij}(t) = 0$ otherwise. The Laplacian of graph $G(t)$ with adjacency matrix $A(t)$ is given by $L(t) = \Delta(A(t)) - A(t)$, where the degree matrix $\Delta(A(t))$ is a diagonal matrix with ith diagonal elements $\sum_{j=1, j\neq i}^{N} a_{ij}(t)$. Denote the eigenvalues of $L(t)$ as $\lambda_1(L(t)) \leq \lambda_2(L(t)) \leq \cdots \leq \lambda_N(L(t))$. Then, $\lambda_1(L(t)) = 0$ with a corresponding eigenvector $\mathbf{1} = [1\ 1\ \cdots\ 1]^{\mathrm{T}} \in \mathbf{R}^N$. Moreover, if $G(t)$ is a connected graph, then $\lambda_2(L(t)) > 0$ [48]. The n-dimensional graph Laplacian is defined as $\hat{L}(t) = L(t) \otimes I_n$, where I_n is the identity matrix of order n and $\otimes$ stands for the Kronecker product.

Let $\bar{G}(t)$ be a graph generated by graph $G(t)$ and the leader, $L_{\delta(t)}$ be the symmetric Laplacian of the undirected graph $G(t)$ consisting of N agents, and matrix $H_{\delta(t)} = \mathrm{diag}\{h_1(t), h_2(t), \ldots, h_N(t)\}$, where $\delta : [0, \infty) \to \Gamma$ is a switching signal whose value at time t is the index of the graph at time t and Γ is finite. If agent i is a neighbor of the leader at time t, then $h_i(t) = 1$; otherwise, $h_i(t) = 0$.

Lemma 1 [49]: Let L be the symmetric Laplacian of an undirected graph G consisting of N agents. Let $\bar{G}$ be the graph consisting of the N agents and a leader and containing a spanning tree with the leader as the root vertex. Then, $L + H > 0$, where $H = \mathrm{diag}\{h_1, h_2, \ldots, h_N\}$.

Lemma 2 [40]: Let L_1 and L_2 be the symmetric Laplacians of graphs G_1 and G_2 consisting of N agents, respectively. Let $\bar{G}_1$ be a graph consisting of the N agents and a leader and containing a spanning tree. Let $\bar{G}_2$ be a graph generated by adding some edge(s) among the N agents into graph $\bar{G}_1$. Then, $\lambda_1(L_2 + H) \geq \lambda_1(L_1 + H) > 0$.

Remark 1: Let $\bar{G}_s$ be a spanning tree consisting of the N agents and a leader, and $\bar{G}_{s1}$ be a graph generated by adding some edge(s) among the N agents into the graph $\bar{G}_s$. Let L_s and L_{s1} be, respectively, the corresponding symmetric Laplacians of G_s and the G_{s1} consisting of the N agents. Then, $\lambda_1(L_s + H) \leq \lambda_1(L_{s1} + H)$. Since the number of the vertices of the spanning tree is finite and fixed, the number of possible spanning trees consisting of the N agents and the leader is finite. Therefore, one can get the minimum value of $\lambda_1(L_s + H)$, that is, $\min\{\lambda_1(L_s + H)\}$, using an exhaustive search method.

Assumption 2: The graph $\bar{G}(t)$ consisting of the N agents and the leader contains a spanning tree rooted at the leader all the time.

Lemma 3 [45]: Let Assumption 1 hold. Then, for each $\varepsilon \in (0, 1]$, there exists a unique matrix $P(\varepsilon) > 0$ that solves the ARE

$$A^{\mathrm{T}} P(\varepsilon) + P(\varepsilon)A - P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon) + \varepsilon I = 0.$$

Moreover, $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$.

The low gain feedback design for the multi-agent system (1) is carried out in two steps.

Low gain based consensus algorithm:

Step 1. Step 1. Solve the parametric algebraic Riccati equation (ARE)

$$A^{\mathrm{T}} P(\varepsilon) + P(\varepsilon)A - 2\gamma P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon) + \varepsilon I = 0, \quad \varepsilon \in (0, 1], \quad (3)$$

where $\gamma \leq \min\{\lambda_1(L_s + H)\}$ is a positive constant. The existence of a unique positive definite solution $P(\varepsilon)$ for ARE (3) is established in Lemma 3.

Step 2. Step 2. Construct a linear feedback law for agent i as

$$u_i = -B^{\mathrm{T}}P(\varepsilon)\sum_{j=1}^{N} a_{ij}(t)(x_i - x_j) - B^{\mathrm{T}}P(\varepsilon)h_i(t)(x_i - x_{N+1}), \quad i = 1, 2, \ldots, N. \quad (4)$$

The fact that $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$, as established in Lemma 3, motivates the term of low gain feedback.

Remark 2: In the above design algorithm, each agent only acquires the state information of its neighbors. Furthermore, no information of the network topology is required. Only the number of agents N is needed to determine the value of γ.

Theorem 1: Consider a multi-agent system of N agents with general linear dynamics (1) and a leader with dynamics (2). Suppose that Assumptions 1 and 2 hold. Then, the agent control inputs u_i as given by (4) achieve semi-global consensus of the multi-agent system. That is, for any *a priori* given bounded set $\mathcal{X} \subset \mathbf{R}^n$, there is an $\varepsilon^* > 0$ such that, for each given $\varepsilon \in (0, \varepsilon^*]$,

$$\lim_{t\to\infty} \|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

as long as $x_i(0) \in \mathcal{X}$ for all $i = 1, 2, \ldots, N, N+1$.

Proof: Denote the difference between the states of agent i and the leader as $\tilde{x}_i = x_i - x_{N+1}$. Then, we have

$$\dot{\tilde{x}}_i = A\tilde{x}_i - B \times \sigma\left(B^{\mathrm{T}}P(\varepsilon)\left(\sum_{j=1}^N a_{ij}(t)(\tilde{x}_i - \tilde{x}_j) + h_i(t)\tilde{x}_i\right)\right), \quad (5)$$

for which let us consider the Lyapunov function

$$V(\tilde{x}) = \sum_{i=1}^N \tilde{x}_i^{\mathrm{T}} P(\varepsilon)\tilde{x}_i, \quad (6)$$

where, for notational convenience, we have defined $\tilde{x} = [\tilde{x}_1^{\mathrm{T}} \;\; x_2^{\mathrm{T}} \;\; \cdots \;\; \tilde{x}_N^{\mathrm{T}}]^{\mathrm{T}}$.

Let $c > 0$ be a constant such that

$$c \geq \sup_{\varepsilon\in(0,1], x_i(0)\in\mathcal{X}, i=1,2,\ldots,N+1} \sum_{i=1}^N \tilde{x}_i^{\mathrm{T}}(0)P(\varepsilon)\tilde{x}_i(0). \quad (7)$$

Such a c exists since $\mathcal{X}$ is bounded and $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$ by Lemma 3.

Let $L_V(c) := \left\{\tilde{x} \in \mathbf{R}^{Nn} : V(\tilde{x}) \leq c\right\}$, and let $\varepsilon^* \in (0, 1]$ be such that, for each $\varepsilon \in (0, \varepsilon^*]$, $\tilde{x} \in L_V(c)$ implies that

$$\left\| B^{\mathrm{T}}P(\varepsilon)\left(\sum_{j=1}^N a_{ij}(t)(x_i - x_j) + h_i(t)(x_i - x_{N+1})\right)\right\|_\infty \leq \varpi, \quad i = 1, 2, \ldots, N, \quad (8)$$

where $\|z\|_\infty = \max_i |z_i|$ for $z \in \mathbf{R}^m$. The existence of such an ε^* is again due to the fact that $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$.

Thus, for any $\varepsilon \in (0, \varepsilon^*]$, the dynamics of (5) remains linear within $L_V(c)$. Consequently, we can evaluate the derivative of V along the trajectories of the agents within the set $L_V(c)$ as

$$\begin{aligned}\dot{V}(\tilde{x}) &= \sum_{i=1}^N \dot{\tilde{x}}_i^{\mathrm{T}} P(\varepsilon)\tilde{x}_i + \sum_{i=1}^N \tilde{x}_i^{\mathrm{T}} P(\varepsilon)\dot{\tilde{x}}_i \\ &= \sum_{i=1}^N \tilde{x}_i^{\mathrm{T}}\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}_i \\ &\quad - \sum_{i=1}^N\sum_{j=1}^N a_{ij}(t)(\tilde{x}_i^{\mathrm{T}} - \tilde{x}_j^{\mathrm{T}})P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)(\tilde{x}_i - \tilde{x}_j) \\ &\quad - 2\sum_{i=1}^N h_i(t)\tilde{x}_i^{\mathrm{T}}P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\tilde{x}_i. \end{aligned} \quad (9)$$

Recalling from [22] the fact that for any $\xi_i \in \mathbf{R}^m$, $i = 1, 2, \ldots, N$,

$$\frac{1}{2}\sum_{i=1}^m\sum_{j=1}^N a_{ij}(t)(\xi_i - \xi_j)^{\mathrm{T}}(\xi_i - \xi_j) = \xi^{\mathrm{T}}(L(t)\otimes I_m)\xi,$$

where $\xi = [\xi_1 \;\; \xi_2 \;\; \cdots \;\; \xi_N]^{\mathrm{T}}$, and using the identity

$$(A\otimes B)(C\otimes D) = AC\otimes BD,$$

we can continue (9) as follows:

$$\begin{aligned}\dot{V}(\tilde{x}) &= \tilde{x}^{\mathrm{T}}\left(I_N\otimes(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x} \\ &\quad - 2\tilde{x}^{\mathrm{T}}\left(I_N\otimes P(\varepsilon)B\right) \\ &\quad \times (L(t)\otimes I_m)\left(I_N\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x} \\ &\quad - 2\tilde{x}^{\mathrm{T}}\left(I_N\otimes P(\varepsilon)B\right)(H(t)\otimes I_m) \\ &\quad \times \left(I_N\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x} \\ &= \tilde{x}^{\mathrm{T}}\left(I_N\otimes\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\right)\tilde{x} \\ &\quad - \tilde{x}^{\mathrm{T}}\big((L(t) + H(t)) \\ &\qquad \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\big)\tilde{x}. \end{aligned} \quad (10)$$

The symmetry of matrix $L(t)+H(t)$ implies that there exists an orthogonal matrix $T(t) \in \mathbf{R}^{N\times N}$ and $\hat{x} = (T(t)\otimes I_n)\tilde{x}$ such that

$$\begin{aligned} L(t)+H(t) &= T^{\mathrm{T}}(t)\mathrm{diag}\{\lambda_1(L(t)+H(t)), \lambda_2(L(t)+H(t)), \\ &\qquad \ldots, \lambda_N(L(t)+H(t))\}T(t) \\ &:= T^{\mathrm{T}}(t)\mathrm{diag}\{\lambda_1(t), \lambda_2(t), \ldots, \lambda_N(t)\}T(t). \end{aligned}$$

Thus, (10) can be further continued as

$$\begin{aligned}\dot{V}(\tilde{x}) &= \tilde{x}^{\mathrm{T}}\left((T^{\mathrm{T}}(t)T(t))\otimes\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\right)\tilde{x} \\ &\quad - \tilde{x}^{\mathrm{T}}(T^{\mathrm{T}}(t)\mathrm{diag}\{\lambda_1(t), \lambda_2(t), \ldots, \lambda_N(t)\} \\ &\quad \times T(t)\otimes\left(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right))\tilde{x} \\ &= \tilde{x}^{\mathrm{T}}(T^{\mathrm{T}}(t)\otimes I_n)\left(I_N\otimes\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\right) \\ &\quad \times (T(t)\otimes I_n)\tilde{x} \\ &\quad - \tilde{x}^{\mathrm{T}}(T^{\mathrm{T}}(t)\otimes I_n)(\mathrm{diag}\{\lambda_1(t), \lambda_2(t), \ldots, \lambda_N(t)\} \\ &\quad \otimes\left(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right))(T(t)\otimes I_n)\tilde{x} \\ &\leq \sum_{i=1}^N \hat{x}_i^{\mathrm{T}}\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon) - 2\gamma P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right)\hat{x}_i \\ &= -\varepsilon\sum_{i=1}^N \tilde{x}_i^{\mathrm{T}}(T^{\mathrm{T}}(t)\otimes I_n)(T(t)\otimes I_n)\tilde{x}_i \\ &< 0, \quad \forall\tilde{x}\in L_V(c)\setminus\{0\}. \end{aligned}$$

This implies that the trajectory $\tilde{x}$ starting from the level set $L_V(c)$ will converge to the origin $\tilde{x} = 0$ asymptotically as time goes to infinity, which in turn implies that

$$\lim_{t\to\infty}\|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N.$$

This completes the proof.

Assumption 3: The graph $\bar{G}$ consisting of the N agents and the leader is fixed and contains a spanning tree rooted at the leader.

The following corollary is a special case of Theorem 1 for the case of a fixed network.

Corollary 1: Consider a multi-agent system of N agents with general linear dynamics (1) and a leader with dynamics (2). Suppose that Assumptions 1 and 3 hold. Then, the agent control inputs u_i as given by (4) achieve semi-global consensus of the multi-agent system. That is, for any *a priori* given bounded set $\mathcal{X} \subset \mathbf{R}^n$, there is an $\varepsilon^* > 0$ such that, for each $\varepsilon \in (0, \varepsilon^*]$,

$$\lim_{t\to\infty} \|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

as long as $x_i(0) \in \mathcal{X}$ for all $i = 1, 2, \ldots, N, N+1$.

B. Consensus on a Jointly Connected Switching Network

It can be seen that Assumption 2 requires the agents to stay in touch with the leader all the time in order to track the leader. It is, however, more practical to require only that the agents get in touch, directly or indirectly, with the leader from time to time.

Assumption 4: There exists an infinite sequence of contiguous, nonempty, and uniformly bounded time-intervals $[t_k, t_{k+1})$, $k = 0, 1, \ldots$, and there are m_k switching topologies in each uniformly bounded time-intervals $[t_k, t_{k+1})$ (that is, there exists a finite sequence of contiguous and nonempty time-subintervals $[t_k^j, t_k^{j+1})$, $j = 0, 1, \ldots, m_k$, with $t_k = t_k^0$, $t_{k+1} = t_k^{m_k}$ and $t_k^{j+1} - t_k^j \geq \tau$ for some constant $\tau > 0$, and the interconnection topology does not change during each of such time-subintervals) such that across each time interval there exists a joint path from the leader to every agent. In other words, the neighboring graph $\bar{G}(t)$ has a jointly spanning tree across each uniformly bounded interval $[t_k, t_{k+1})$, $k = 0, 1, \ldots$, with $t_0 = 0$ and $t_{k+1} - t_k \leq T$ for some constant $T > 0$.

Assumption 5: There exists a $P(\varepsilon) > 0$ which satisfies ARE (3) and $A^{\mathrm{T}}P(\varepsilon) + P(\varepsilon)A \leq 0$.

Remark 3: Assumption 5 requires that A be marginally stable, that is, all the eigenvalues of A are in the closed left-half s-plane, with those on the imaginary axis simple.

Theorem 2: Consider a multi-agent system of N agents with general linear dynamics (1) and a leader with dynamics (2). Suppose that Assumptions 1, 4, and 5 hold. Then, the agent control inputs u_i as given by (4) achieve semi-global consensus of the multi-agent system. That is, for any *a priori* given bounded set $\mathcal{X} \subset \mathbf{R}^n$, there is an $\varepsilon^* > 0$ such that, for each $\varepsilon \in (0, \varepsilon^*]$,

$$\lim_{t\to\infty} \|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

as long as $x_i(0) \in \mathcal{X}$ for all $i = 1, 2, \ldots, N, N+1$.

Proof: Let us again consider the Lyapunov function (6), where the positive definite solution $P(\varepsilon)$ satisfies Assumption 5. Let us choose $\varepsilon^* > 0$ as in the proof of Theorem 1. We will first show that the set $L_V(c) := \left\{\tilde{x} \in \mathbf{R}^{Nn} : V(\tilde{x}) \leq c\right\}$ is positively invariant. The derivative of V along the trajectories of the agents in the set $L_V(c)$ is given by

$$\begin{aligned}\dot{V}(\tilde{x}) = \tilde{x}^{\mathrm{T}}(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)) \\ -(L(t) + H(t)) \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)))\tilde{x}.\end{aligned} \tag{11}$$

Because of the symmetry of matrix $L(t)+H(t) \geq 0$, there exists an orthogonal matrix $T(t) \in \mathbf{R}^{N\times N}$ and $\hat{x} = (T(t) \otimes I_n)\tilde{x}$, such that

$$\begin{aligned}\dot{V}(\tilde{x}) &= \sum_{i=1}^{N} \hat{x}_i^{\mathrm{T}}\bigg(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon) \\ &\qquad - \lambda_i(t)(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\bigg)\hat{x}_i \\ &\leq \sum_{i=1}^{N} \hat{x}_i^{\mathrm{T}}\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\hat{x}_i \\ &\leq 0.\end{aligned} \tag{12}$$

Therefore, the set $L_V(c)$ is positively invariant. From (12), $\lim_{t\to\infty} V(\tilde{x}(t))$ exists. Considering the infinite sequences $V(\tilde{x}(t_k))$, $k = 0, 1, \ldots$ and using Cauchy's convergence criteria, one has that, for any $\vartheta > 0$, there exists a positive number M_ϑ such that $\forall k \geq M_\vartheta$,

$$0 \leq V(\tilde{x}(t_k)) - V(\tilde{x}(t_{k+1})) = -\int_{t_k}^{t_{k+1}} \dot{V}(\tilde{x}(\nu))\mathrm{d}\nu < \vartheta.$$

Therefore,

$$\begin{aligned}\vartheta &> -\int_{t_k}^{t_{k+1}} \dot{V}(\tilde{x}(\nu))\mathrm{d}\nu \\ &= -\int_{t_k^0}^{t_k^1} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\ &\quad + \int_{t_k^0}^{t_k^1} \tilde{x}^{\mathrm{T}}(\nu)\left(\left(L_{\delta(t_k^0)} + H_{\delta(t_k^0)}\right) \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\ &\quad \times \tilde{x}(\nu)\mathrm{d}\nu \\ &\quad - \cdots - \int_{t_k^{m_k-1}}^{t_k^{m_k}} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\ &\quad + \int_{t_k^{m_k-1}}^{t_k^{m_k}} \tilde{x}^{\mathrm{T}}(\nu) \\ &\quad \times \left(\left(L_{\delta\left(t_k^{m_k-1}\right)} + H_{\delta\left(t_k^{m_k-1}\right)}\right) \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\ &\quad \times \tilde{x}(\nu)\mathrm{d}\nu \\ &\geq -\int_{t_k^0}^{t_k^0+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\ &\quad + \int_{t_k^0}^{t_k^0+\tau} \tilde{x}^{\mathrm{T}}(\nu) \\ &\quad \times \left(\left(L_{\delta(t_k^0)} + H_{\delta(t_k^0)}\right) \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\ &\quad \times \tilde{x}(\nu)\mathrm{d}\nu \\ &\quad - \cdots - \int_{t_k^{m_k-1}}^{t_k^{m_k-1}+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\ &\quad + \int_{t_k^{m_k-1}}^{t_k^{m_k-1}+\tau} \tilde{x}^{\mathrm{T}}(\nu) \\ &\quad \times \left(\left(L_{\delta\left(t_k^{m_k-1}\right)} + H_{\delta\left(t_k^{m_k-1}\right)}\right) \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\ &\quad \times \tilde{x}(\nu)\mathrm{d}\nu.\end{aligned}$$

Consequently, for $j = 0, 1, \ldots, m_k - 1$,

$$\begin{aligned}\vartheta &> -\int_{t_k^j}^{t_k^j+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\ &\quad + \int_{t_k^j}^{t_k^j+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(\left(L_{\delta(t_k^j)} + H_{\delta(t_k^j)}\right) \otimes (2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\ &\quad \times \tilde{x}(\nu)\mathrm{d}\nu\end{aligned}$$

which implies that, for $j = 0, 1, \ldots, m_k - 1$,

$$\lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu - \lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(\left(L_{\delta(t_k^j)} + H_{\delta(t_k^j)}\right)\otimes(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \times \tilde{x}(\nu)\mathrm{d}\nu = 0.$$

Thus, one has

$$\lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu - \lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(\left(L_{\delta(t_k^0)} + H_{\delta(t_k^0)}\right)\otimes(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \times \tilde{x}(\nu)\mathrm{d}\nu - \cdots - \lim_{t\to\infty}\int_t^{t+\tau}\tilde{x}^{\mathrm{T}}(\nu) \times \left(\left(L_{\delta\left(t_k^{m_k-1}\right)} + H_{\delta\left(t_k^{m_k-1}\right)}\right)\otimes(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \times \tilde{x}(\nu)\mathrm{d}\nu = 0.$$

By Assumption 4 and Lemma 1, we have

$$L_{\delta(t_k^0)} + \cdots + L_{\delta\left(t_k^{m_k-1}\right)} + H_{\delta(t_k^0)} + \cdots + H_{\delta\left(t_k^{m_k-1}\right)} > 0.$$

By Lemma 2 and Assumption 5,

$$\begin{aligned}
0 =& \lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\
&- \lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(\left(L_{\delta(t_k^0)} + H_{\delta(t_k^0)}\right)\otimes(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\
&\times \tilde{x}(\nu)\mathrm{d}\nu - \cdots - \lim_{t\to\infty}\int_t^{t+\tau}\tilde{x}^{\mathrm{T}}(\nu) \\
&\times \left(\left(L_{\delta\left(t_k^{m_k-1}\right)} + H_{\delta\left(t_k^{m_k-1}\right)}\right)\otimes(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right) \\
&\times \tilde{x}(\nu)\mathrm{d}\nu \\
=& \lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)\left(I_N \otimes (P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon))\right)\tilde{x}(\nu)\mathrm{d}\nu \\
&- \lim_{t\to\infty}\int_t^{t+\tau} \tilde{x}^{\mathrm{T}}(\nu)(T^{\mathrm{T}}(t)\mathrm{diag}\left\{\lambda_1(t), \ldots, \lambda_N(t)\right\}T(t) \\
&\otimes(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)))\tilde{x}(\nu)\mathrm{d}\nu \\
\leq& \lim_{t\to\infty}\int_t^{t+\tau}\sum_{i=1}^{N}\hat{x}_i^{\mathrm{T}}(\nu)\left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\hat{x}_i(\nu)\mathrm{d}_\nu \\
&- \lim_{t\to\infty}\int_t^{t+\tau}\sum_{i=1}^{N}\hat{x}_i^{\mathrm{T}}(\nu)\left(\gamma(2P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon))\right)\hat{x}_i(\nu)\mathrm{d}_\nu \\
=& -\varepsilon\lim_{t\to\infty}\int_t^{t+\tau}\sum_{i=1}^{N}\tilde{x}_i^{\mathrm{T}}(\nu)\tilde{x}_i(\nu)\mathrm{d}\nu \\
\leq& 0,
\end{aligned}$$

which implies that

$$\lim_{t\to\infty}\sum_{i=1}^{N}\tilde{x}_i^{\mathrm{T}}(t)\tilde{x}_i(t) = 0,$$

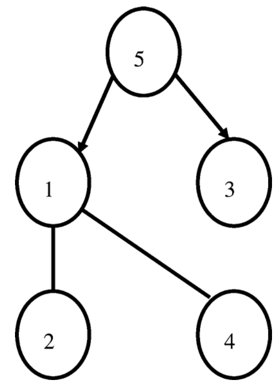

Fig. 1. The interaction network.

that is,

$$\lim_{t\to\infty}\tilde{x}_i(t) = 0.$$

This completes the proof.

IV. Numerical Examples

A. Consensus on Fixed Networks

The simulation is performed with four agents and one leader. The system matrices are chosen as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -4 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

It is straightforward to verify that (A, B) is asymptotically null controllable with bounded controls (ANCBC). The initial states x_{i1}, x_{i2}, x_{i3} and x_{i4} of all agents are randomly chosen from box $[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]$, respectively, and the initial state of the leader is chosen as $[1.5, 1.5, 1.5, 1.5]$. The interaction network is chosen as in Fig. 1. The arrows represent informed agents. The Laplacian L and the matrix H are as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, \; H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The minimum eigenvalue of $L + H$ is 0.2679. Here, we do not use the information of the interaction network topology. Since there are four agents and a leader in the group, using an exhaustive search method, we can obtain the minimum eigenvalue of the possible spanning trees consisting four agents and a leader, $\min\{\lambda_1(L_s + H)\} = 0.1206$. Therefore, we can choose $\gamma = 0.1 \leq \min\{\lambda_1(L_s + H)\} = 0.1206$. For $\varepsilon = 0.1$, $\varepsilon = 0.01$ and $\varepsilon = 0.001$, by using standard numerical software, we obtain positive definite matrices,

$$P(0.1) = \begin{bmatrix} 1.9721 & 0.3313 & 1.3587 & 0.0498 \\ 0.3313 & 6.5031 & 0.4756 & 1.9527 \\ 1.3587 & 0.4756 & 3.4637 & 0.1310 \\ 0.0498 & 1.9527 & 0.1310 & 1.3453 \end{bmatrix},$$

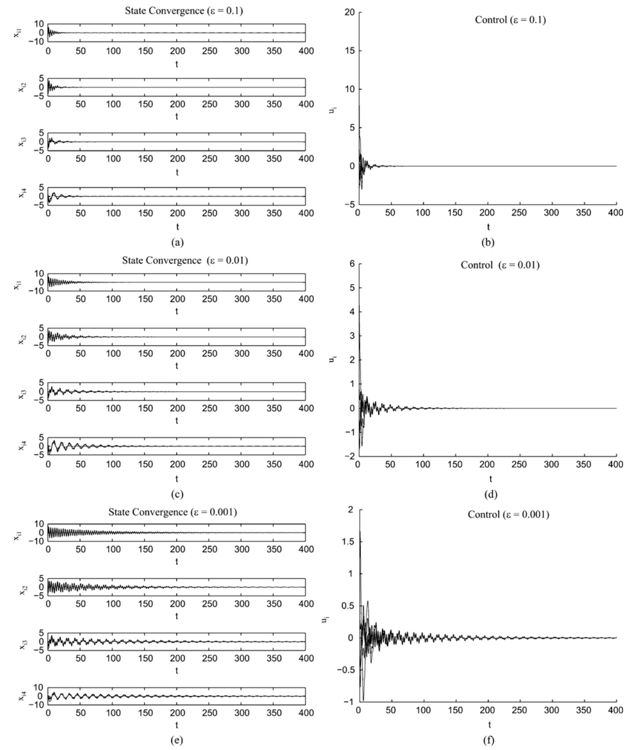

Fig. 2. Leader-following consensus of 4 agents with a leader under control protocol (4).

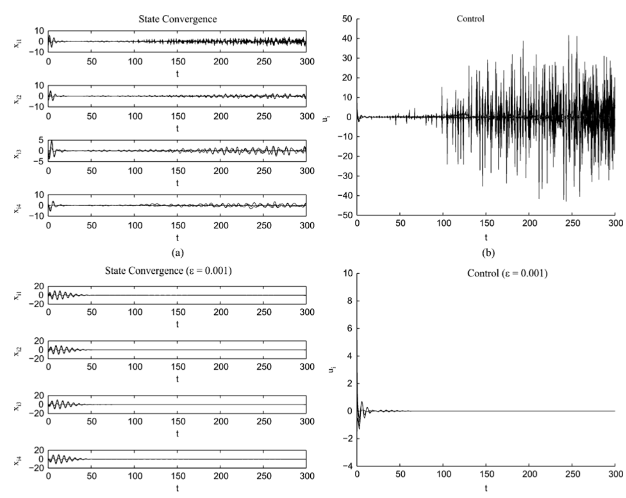

Fig. 3. Leader-following consensus of 4 agents with a leader applying consensus algorithm (13) and consensus algorithm (4).

$$P(0.01)=\begin{bmatrix}0.6184 & 0.0332 & 0.4266 & 0.0050\\ 0.0332 & 2.0461 & 0.0477 & 0.6177\\ 0.4266 & 0.0477 & 1.0879 & 0.0132\\ 0.0050 & 0.6177 & 0.0132 & 0.4261\end{bmatrix}$$

and

$$P(0.001)=\begin{bmatrix}0.1954 & 0.0033 & 0.1348 & 0.0005\\ 0.0033 & 0.6467 & 0.0048 & 0.1954\\ 0.1348 & 0.0048 & 0.3438 & 0.0013\\ 0.0005 & 0.1954 & 0.0013 & 0.1348\end{bmatrix},$$

each of which satisfies condition (3). Fig. 2 shows the consensus of the four agents under control protocol (4) and the leader. Fig. 2(a) and 2(b) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents when $\varepsilon = 0.1$. Fig. 2(c) and 2(d) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents when $\varepsilon = 0.01$. Fig. 2(e) and 2(f) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents when $\varepsilon = 0.001$. It is obvious from Fig. 2 that the control protocol (4) is capable of achieving stable consensus motion, and for the same initial conditions, as the value of ε decreases, the state peaks slowlier to a higher value before convergence while the peak value of each control input decreases. This indicates the semi-global nature of the consensus in the presence of agent input saturation.

In Fig. 3, we compare the consensus algorithm (4) proposed in this work with the consensus algorithm in [32]. The simulation is also performed with four agents and one leader, and the interaction network is also chosen as in Fig. 1. The system matrices are chosen as

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -1 & 0 & -2 & 0\end{bmatrix},\ B=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}.$$

It is easy to see that (A,B) is asymptotically null controllable with bounded controls (ANCBC). The initial states x_{i1}, x_{i2}, x_{i3}, and x_{i4} of all agents are randomly chosen from box $[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]$, respectively, the initial state of the leader is chosen as $[1.5,1.5,1.5,1.5]$, and the constant ϖ in the saturation function is chosen as 10. If we use the consensus algorithm in [32], the control inputs can be written as follows:

$$\begin{aligned} u_i = &-B^{\mathrm{T}}\bar{P}\sum_{j=1}^{N}a_{ij}(t)(x_i-x_j)\\ &-B^{\mathrm{T}}\bar{P}h_i(t)(x_i-x_{N+1}),\quad i=1,2,\ldots,N, \end{aligned}\tag{13}$$

where

$$A^{\mathrm{T}}\bar{P}+\bar{P}A-2\lambda_1(L+H)\bar{P}BB^{\mathrm{T}}\bar{P}+\lambda_1(L+H)I<0. \tag{14}$$

Since $\lambda_1(L+H)=0.2679$, we can use

$$A^{\mathrm{T}}\bar{P}+\bar{P}A-0.5358(L+H)\bar{P}BB^{\mathrm{T}}\bar{P}+0.268I=0$$

to get a $\bar{P}$ to satisfy the inequality (14). For the consensus algorithm (4) proposed in this work, we choose $\varepsilon = 0.001$. By using standard numerical software, we obtain positive definite matrices,

$$\bar{P}=\begin{bmatrix}2.9353 & 1.8843 & 2.6570 & 0.1295\\ 1.8843 & 5.0677 & 3.5245 & 2.7448\\ 2.6570 & 3.5245 & 4.2478 & 1.5199\\ 0.1295 & 2.7448 & 1.5199 & 2.4846\end{bmatrix}$$

and

$$P(0.001)=\begin{bmatrix}1.1975 & 0.1429 & 1.1933 & 0.0005\\ 0.1429 & 1.2355 & 0.2852 & 1.1974\\ 1.1933 & 0.2852 & 1.2227 & 0.1419\\ 0.0005 & 1.1974 & 0.1419 & 1.1931\end{bmatrix}.$$

Fig. 3(a) and 3(b) plot respectively the evolution of the state difference between the four agents and the leader and the control of

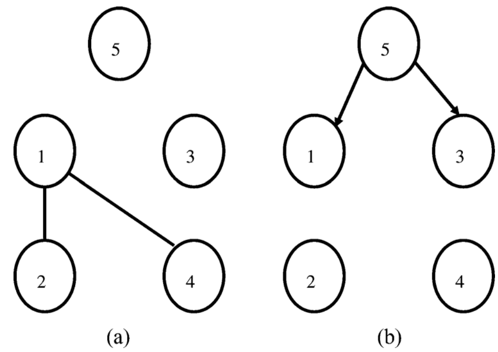

Fig. 4. The interaction networks. (a) $\bar{G}_1$. (b) $\bar{G}_2$.

the four agents by using the consensus algorithm (13). Fig. 3(c) and 3(d) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents by using the consensus algorithm (4). We can see that all agents reach a common state by using the consensus algorithm (4), and the magnitude of the control is less than the constant ϖ. However, due to input saturation, the agents may not achieve state consensus by using the consensus algorithm (13).

B. Consensus on Switching Networks

The simulation is again performed with four agents and one leader. The system matrices are chosen as

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

It is straightforward to verify that (A, B) is asymptotically null controllable with bounded controls (ANCBC). Initial states x_{i1} and x_{i2} of all agents are randomly chosen from box $[-2, 2] \times [-2, 2]$, respectively, and the initial state of the leader is chosen as $[0.5, 0.5]^T$. The two interaction networks, that is, $\bar{G}_1$ and $\bar{G}_2$, are chosen as shown in Fig. 4, and each network is active for half time in each time-interval. Therefore, there is a jointly connected path from the leader to every agent in each time-interval. Since there are four agents and a leader in the group, we can obtain the minimum eigenvalue of the possible spanning trees consisting four agents and a leader, $\min\{\lambda_1(L_s + H)\} = 0.1206$, using an exhaustive search method. Therefore, we can choose $\gamma = 0.1 \leq \min\{\lambda_1(L_s + H)\} = 0.1206$. For $\varepsilon = 0.1$, $\varepsilon = 0.01$ and $\varepsilon = 0.001$, by using standard numerical software, we find obtain positive definite matrices

$$P(0.1) = \begin{bmatrix} 0.05 & 0.000 \\ 0.00 & 0.7071 \end{bmatrix},$$
$$P(0.01) = \begin{bmatrix} 0.005 & 0.0000 \\ 0.000 & 0.2236 \end{bmatrix}$$
$$\text{and}$$
$$P(0.001) = \begin{bmatrix} 0.0005 & 0.0000 \\ 0.0000 & 0.0707 \end{bmatrix},$$

each of which satisfies Assumption 5. Fig. 5 shows the consensus of the four agents under control protocol (4) and the leader. Fig. 5(a) and 5(b) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents when $\varepsilon = 0.1$. Fig. 5(c) and 5(d) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents when $\varepsilon = 0.01$. Fig. 5(e) and 5(f) plot respectively the evolution of the state difference between the four agents and the leader and the control of the four agents when $\varepsilon = 0.001$. It is obvious from Fig. 5 that the control protocol (4) is capable of achieving stable consensus motion, and for the same initial conditions, as the value of ε decreases, the peak value of each control input decreases, indicating the semi-global nature of the consensus in the presence of agent input saturation.

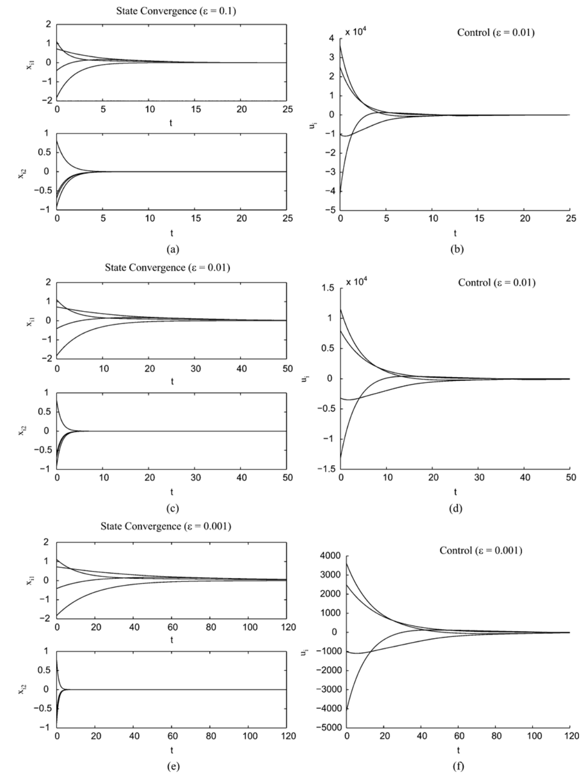

Fig. 5. Leader-following consensus of 4 agents with a leader under consensus algorithm (4) on switching networks.

V. Conclusions

In this paper, we have investigated semi-global leader-following consensus of multi-agent linear systems with input saturation on switching networks. We have used a low gain feedback strategy to design distributed consensus algorithms, without requiring any knowledge of the interaction network topology. Under the assumption that the system is asymptotically null controllable with bounded controls and the networks are connected or jointly connected, all the agents in the group asymptotically synchronize with the leader, both from any *a priori* given bounded set of initial conditions. Finally, numerical examples have been provided for consensus on fixed networks and switching networks to illustrate the effectiveness of the proposed protocols.

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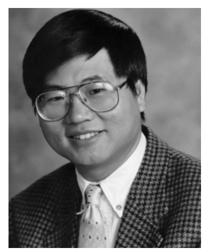

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