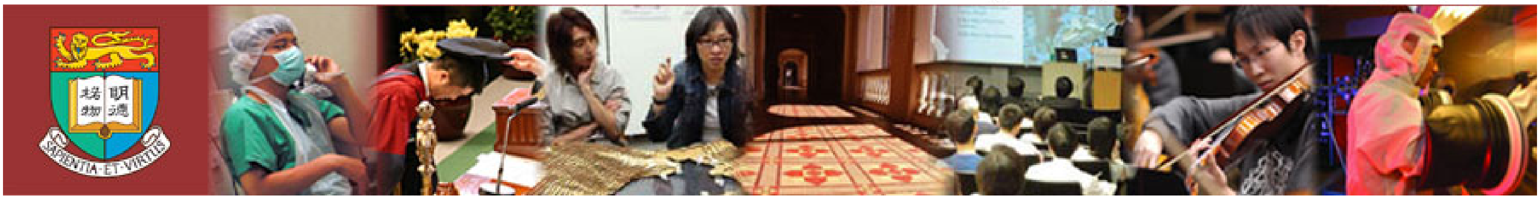

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One Dimensional Multiple Periodic Composite Right/Left Handed (CRLH) Structures

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***Abstract*—The wave-propagation characteristics in a one-dimensional multiple-periodic (MP) Composite right/left-handed (CRLH) structures is presented, where it is found that an increase in the number of unit cells in each supercell leads to new passbands and stopbands. To understand this phenomenon, the network parameters are employed for the theoretical analysis. Detailed dispersion characteristics and the relation between passbands and sub-periodicities are investigated using both analytical and full-wave results, and the reasons for their emergence is qualitatively discussed. Besides, its application to multi-band leaky-wave radiators is also suggested.**

I. INTRODUCTION

Composite Right/Left-Handed (CRLH) materials, due its unique properties, is playing a significant role in today's electromagnetics and microwave engineering field, where it is often used as the host medium for wave propagation and radiation. From the circuit point of view, the unit cell of a CRLH Transmission Line (TL) can be represented as a series LC resonant tank and a shunt LC resonance. The CRLH TL is formed by cascading these basic unit cells to form the uniform structure, which has been used for various guided and radiative wave applications as discussed in [1], [2], including its detailed circuit model and dispersion analysis.

To date, the existing work on the CRLH periodic structures is chiefly focused on single periodic unit cells both in one dimension and in multi-dimensions. Recently, in [3], a one dimensional double-periodic CRLH antenna is proposed. In this work, the concept of multiple periodicities is generalized for the case of CRLH structures, and its effect on the wave-guiding properties is investigated. Furthermore, its application towards leaky-wave antennas is also suggested.

II. TRANSMISSION-LINE ANALYSIS OF MP-CRLH STRUCTURES

The general configuration of a MP-CRLH structure along with its equivalent circuit model is shown in Fig. 1. The basic supercell is composed of several different CRLH unit cells, which is repeated periodically to construct the overall transmission line structure. Each unit cell comprise series inductance L_R cascading with a capacitance C_L and a shunt inductance L_L in parallel to the capacitance C_R.

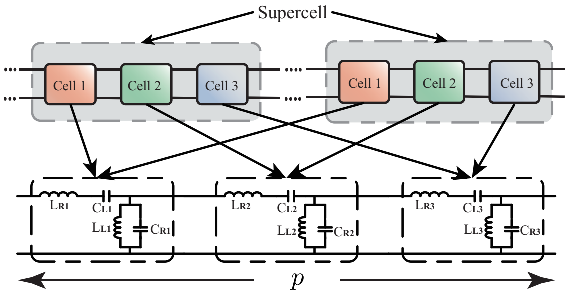

Fig. 1. Multiple-periodic (MP) Composite Right/Left-Handed (CRLH) transmission line (TL) structure and its equivalent circuit model.

The transmission characteristics of this MP-CRLH structure can be obtained by cascading ABCD matrices of the individual unit cells involved, using standard network analysis. For instance, the overall ABCD matrix of a supercell consisting of N unit cells can be written as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{n=1}^{N} \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix}, \tag{1}$$

where n refers to the n^{th} unit cell. For each CRLH unit cell, its ABCD matrix is given in terms of series impedance $Z_{se,n}$ and shunt admittance $Y_{sh,n}$ as

$$\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} = \begin{bmatrix} 1 & Z_{se,n} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{sh,n} & 1 \end{bmatrix}, \tag{2}$$

$$Z_{se,n} = j\left(\omega L_{R,n} - \frac{1}{\omega C_{L,n}}\right) \tag{3a}$$

$$Y_{sh,n} = j\left(\omega C_{R,n} - \frac{1}{\omega L_{L,n}}\right). \tag{3b}$$

Applying the periodic boundary conditions (Bloch-Floquet analysis) across the super-cell, the voltages and currents at the two ports of one supercell are given by $V_{N+1} = V_N e^{-\gamma p}$ and $I_{N+1} = I_N e^{-\gamma p}$, where $\gamma = \alpha + j\beta$ is propagation constant and p is super-cell size. Finally, the dispersion relation of a general MP-CRLH TL can be obtained using $\cosh(\gamma p) = (A+D)/2$.

III. RESULTS

A. Theoretical Results

Fig. 2 shows a typical analytical dispersion diagram for number of super-cells consisting of one-, two- and three-unit cells. The circuit parameters are arbitrarily chosen in this case to demonstrate the multi-band phenomenon in MP-CRLH structures. As can be seen, as the number of unit-cells

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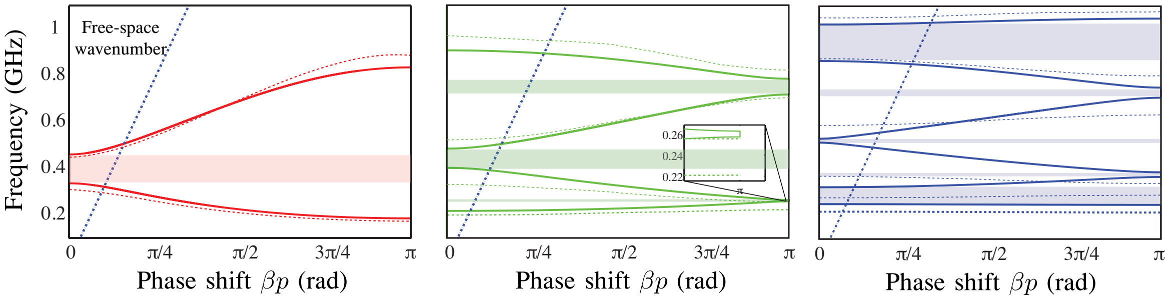

(a) (b) (c)

Fig. 2. Dispersion characteristics of MP-CRLH structures computed using analytical (Bloch-Floquet) [solid curve] and full-wave (FEM-HFSS) [dashed curve] analysis for the case of a) single-periodicity, b) double-periodicity, and c) triple-periodicity. The values of lumped components used in analytical results are: $C_{L,i} \in \{9\ 5\ 2\}$ pF, $L_{L,i} \in \{12\ 12\ 12\}$ nH, $C_{R,i} \in \{10\ 8\ 6\}$ pF, and $L_{R,i} \in \{25\ 24\ 23.5\}$ nH.

increases, the number of stop bands progressively increases as well. For instance, while a single unit cell case [Fig. 2(a)] has one stop band, the three unit cell case [Fig. 2(c)] has now five stop bands. Therefore, for a general configuration of N unit-cells, the maximum number of stopbands that can be created is $2N-1$. It should be noted, that by controlling the individual circuit elements of the unit-cells, the bandgaps can be engineered. Furthermore, the various passbands alternate between left-hand (anti-parallel phase and group velocities) and right-hand (parallel phase and group velocities) frequency bands. In addition, it can also be seen from Fig. 2, that various dispersion bands cross the fast-wave region, thereby making them suitable for multi-band leaky-wave radiation applications.

The creation of new passbands can be understood from following different aspects. From direct inspection of the dispersion curves of Fig. 2, the overall dispersion of the MP-CRLH structure can be seen as a mixture of the each unit cell. The appearance of passbands and bandgaps is related to constructive and destructive interferences of the waves on the structure. As a result, based on this property, this structure can be used for multi-resonance multi-band leaky wave radiation applications.

B. Full-wave Simulation Results

To confirm the multi-band behaviour of the MP-CRLH structure, the metal-insulator-metal (MIM) implementation of a CRLH unit-cell is simulated in FEM-HFSS, as shown in Fig. 3, in stripline technology, to realize the circuit elements of Fig. 2. Different physical dimensions were used for each unit-cell to obtain different LC values in CRLH unit cells, to construct a super-cell. In particular, $L_{\text{bp}}/2$ and $L_{\text{bp}}/4$ were used for the second and third unit cells for multiple-periodic cases, respectively. The corresponding dispersion diagrams are shown and compared with the analytical ones in Fig. 2, and a good agreement is seen between the two, thereby confirming the creation of new bandgaps in the MP-CRLH structures.

IV. CONCLUSION

Dispersion analysis of a one dimensional MP-CRLH structure has been presented using both analytical and full-wave analyses, for different cases of sub-periodicities. The emergence of new passbands and stopbands has been shown as the sub-cell periodicity of the supercell is increased, and this phenomenon has been qualitatively discussed. This phenomenon can find applications in various multi-band multi-resonant leaky-wave and wave-guiding components.

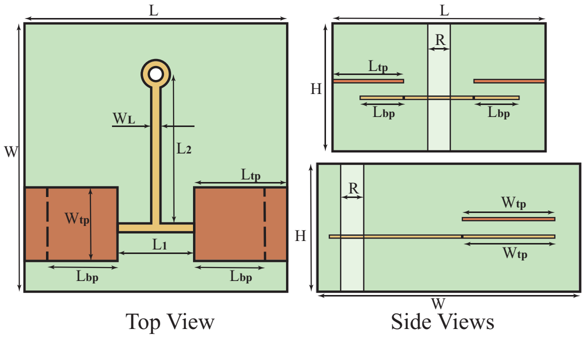

Fig. 3. The metal-insulator-metal (MIM) implementation of the individual CRLH cell in stripline technology, used in FEM-HFSS analysis. The physical parameters are: $L = 37.048$ mm, $W = 50$ mm, $H = 3.302$ m, $W_{\text{tp}} = 12.7$ mm, $L_{\text{tp}} = 14.024$ mm, $L_{\text{bp}} = 12.5$ mm, $L_1 = 9$ mm, $L_2 = 23.016$ mm, $W_L = 1.016$ mm and $R = 2.048$ mm.

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