Forecasting Exchange Rates

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Degree Thesis
International Business
2010

BACHELOR'S DEGREE THESIS	
Arcada Polytechnic	
Degree Programme:	International Business
Identification number:	8253
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Title:	Forecasting Exchange Rates
Supervisor (Arcada):	Karis Badal Durbo
Commissioned by:	

Abstract:
The work of this thesis primarily revolves around the concept of forecasting the daily exchange rates of the European Euro valued in United States Dollars. Forecasting is a relatively important issue in business operations, however it is also one of the most problematic. With the uncertainty of the future, forecasts are difficult to assess. The aim of this thesis is to successfully forecast the future exchange rates of the European Euro in terms of United States Dollars for the month of May 2010, and determine whether the forecasting models properly work when applied to exchange rates, why or why not, and their measure of accuracy. The methodology of this thesis revolves extensively around quantitative research, largely including a probability and forecasting approach. Throughout a period of three months, actual exchange rate values were collected and recorded to form a data set. The exchange rate data was then used in the application of a variety of mathematical forecasting models to forecast the daily exchange rates for a future, one-month period. Upon measuring the accuracy of the forecasts, the forecasted exchange rates contained very little error. Therefore, the forecasts are considered to be successful, and the hypothesis that exchange rates could be determined with the aid of a mathematical forecasting model is accepted. Though it is very difficult to consistently estimate exchange rates successfully, the work of this thesis shows there is always a greater probability of benefiting from a forecast.

Keywords:	Economics, Exchange Rates, Forecasting, Mathematics, Quantitative
Number of pages:	88
Language:	English
Date of acceptance:	25/11/2010

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FORMULAS

ABBREVIATIONS

- **EUR/EURO:** European Euro
- **FX:** Foreign Exchange
- **MAD:** Mean Absolute Deviation
- **MAPE:** Mean Absolute Percent Error
- **MSE:** Mean Squared Error
- **PPP:** Purchasing Power Parity
- **RMSE:** Root Mean Squared Error
- **U.S:** United States
- **USD:** United States Dollar

DEFINITIONS

- **Actual Data** – Any number of values that have been historically recorded.
- **Data** – A collection of organized information from which conclusions may be drawn.
- **Equation** – Two mathematical models expressed as being equal to each other.
- **Expected Value** – The average value that an experiment is expected to produce if it is repeated a large amount of times.
- **Forecasting** – The science of predicting future events, which may involve taking historical data and projecting them into the future with the application of a mathematical model.
- **Formula** – A systematical process of a mathematical model.
- **Model** – A system of numbers and/or variables that represent a real-life situation.
- **Probability** – The likelihood or chance that an event will occur.
- **Sample** – A limited quantity of something that is intended to represent the whole.
- **Sample Size** – The number of observations that is to be included within a defined sample.
- **Trend** – A general direction of movement.

1 INTRODUCTION

1.1 Description and Motivation for Topic

The work of this thesis primarily revolves around the concept of forecasting the daily exchange rates of the European Euro (*EUR*) valued in United States Dollars (*USD*). Throughout a period of three months, actual exchange rate values were collected and recorded to form a data set. The exchange rate data was then used in the application of a variety of forecasting models to forecast the daily exchange rates for a future, one-month period. As the month approached, the actual exchange rates were collected and recorded. Lastly, the actual exchange rates were compared with the forecasted exchange rates to form an analysis that measured the forecasting accuracy of the various models.

The thesis has been largely motivated by an operations management course and a statistics course completed by the author. With a curiosity to perform a forecast upon completion of the operations management course, the thesis topic was developed instantaneously.

1.2 Aim of Research

Forecasting is a relatively important issue in business operations, however it also happens to be one of the most problematic of issues. Because the future is always uncertain, forecasts are difficult to assess; yet they are rarely ever avoidable in effective business planning. The aim of this thesis is to successfully forecast the future exchange rates of the *EUR* in terms of *USD* for the month of May 2010. Furthermore, the author expects to determine whether the forecasting models properly work when applied to exchange rates, why or why not, and their measures of accuracy.

1.3 Hypotheses

1. Future exchange rates can be determined with the aid of a mathematical forecasting model.
2. Future exchange rates cannot be determined with the aid of a mathematical forecasting model.

1.4 Description of Methods

The methodology of this thesis revolves extensively around quantitative research, and largely includes a probability and forecasting approach. The forecasting system that is implemented involves the determining of an appropriate sample size, a collection of the necessary data, the application of relevant forecasting models, and an analysis of the results. Additionally, a variety of mathematical models that are commonly used in forecasting are applied to the sample of collected exchange rate data. These models include a naive approach, a moving average, exponential smoothing, and linear regression.

1.5 Limitations

It is evident that forecasting exchange rates is no simple task. Therefore, a variety of limitations are present within the work of this thesis. Foremost, there are approximately one hundred eighty-two currencies in existence throughout the world. The foreign exchange rate data that is used in this thesis only includes two currencies, which happen to be one of the most common pairs in currency trading. Due to the lack of randomness, a biased selection of variables exist; making it unknown whether or not the same results would be achieved when using a different, unbiased, pair of currencies. Furthermore, there are numerous forecasting models available, however, this thesis only includes models that are familiar to the author. Therefore, it is uncertain whether these models are actually efficient in forecasting exchange rates. Additionally, it is imperative to bear in mind that the forecasting models used in this thesis do not support the probability of natural disasters or political/economical factors occurring. Lastly, since exchange rates do not typically follow an identifiable trend, it is difficult to conclude whether or not the same results would be achieved at a different period of time.

1.6 Technical frame of reference

A majority of the theories that this thesis is based upon have been referenced from four significant works of literature. Lind, Marchal, and Mason's "Statistical Techniques in Business & Economics" (2002) was a fundamental element in interpreting the theories of correlation, linear regression, indexes, and dispersion. Mendenhall, Reinmuth, and Beaver's "Statistics for Management and Economics" (1993) was essential in supporting the theories of the measures of central tendency, as well as the measures of forecast accuracy. Heizer and Render's "Operations Management" (2004) was the core foundation for explaining the theory of forecasting. Lastly, Henderson's "Currency Strategy" (2002) and Rosenberg's "Exchange-Rate Determination" (2003) provided the framework for the context of exchange rates. Additional text was referenced from relevant internet sites that contained theories pertaining to forecasting and exchange rates.

2 DESCRIPTIVE MEASURES

2.1 Numerical Descriptive Methods

"When presented with a set of quantitative data, most people have difficulty making any sense out of it" (Mendenhall, Reinmuth, & Beaver, 1993, p. 16). An efficient way of illustrating the significances of a data set is to present the data's numerical descriptive methods. Descriptive methods signify the first approach to organizing, summarizing, and presenting a set of data informatively. Descriptive methods are particularly useful in quickly gathering an initial description of data because they provide a generalization of the center and spread of the set of data. Numerical descriptive measures can be divided into two categories: measures of central tendency and measures of variability. The measures of central tendency identify the center of the data, while the measures of variability describe how closely the data values are located to the data's center.

(Mendenhall, Reinmuth, & Beaver, 1993),(Lind, Marchal, & Mason, 2002)

2.2 Measures of Central Tendency

A measure of central tendency is a descriptive method that is used to locate the center of distribution of a set of data. Measures of central tendency summarize the data by determining a single numerical value to represent the data set. There are numerous measures of central tendency, and presented below are a few of the most common.

(Mendenhall, Reinmuth, & Beaver, 1993), (Lind, Marchal, & Mason, 2002)

2.2.1 Mean

The arithmetic mean, also referred to as simply the mean, is a very useful and quite common measure of central tendency. The mean is used in mathematics to measure the average of a set of data. The arithmetic mean of a data set can be expressed as the following formula:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (1)$$

Where $\bar{x}$ = the symbol used to denote the mean

n = the number of measurements in the sample, and

$\sum_{i=1}^{n} x_i$ = the summation of the data values.

The mean is a delicate measure of central tendency, however, as it is easily influenced by extreme data values. Therefore, it is important to note that the arithmetic mean may not always be accurately representative of the data set.

(Mendenhall, Reinmuth, & Beaver, 1993)

2.2.2 Median

The median is a second example of a measure of central tendency. When the data is arranged in order of ascending value, the median is the value that divides the data into two equal parts. The median can easily be identified with an odd-quantity set of data, however when there is an even-quantity set of data the median must be found by calculating the average of the two middle values.

(Mendenhall, Reinmuth, & Beaver, 1993)

2.2.3 Mode

A third example that can also be used as a measure of central tendency is the mode. The mode is found by identifying the data value that has the greatest frequency of occurrence. Though the mode is not used as often as the mean or median, it can be useful in cases where the frequency is considered to be a significant part of the evaluation.

(Mendenhall, Reinmuth, & Beaver, 1993)

2.3 Measures of Variability

Once the center of distribution for a data set has been identified, the next part of the process is to provide a measure of the variability. Variation is a very important property of data, as its measure is necessary in visualizing the distribution of frequencies.

Dispersion is a term used in statistics to describe the variation within a data set. While the mean or median identify the average value for a set of data, the measure of dispersion determines the distance from the data's average to the minimum and maximum values. Dispersion allows the forecaster to determine whether the average is, or is not, representative of the data according to how close the data values are situated to the measure of central tendency. A large measure of dispersion indicates a wide variation in data values, while a small measure of dispersion indicates little variation. Therefore, a small dispersion measure would indicate that the mean or median is representative of the data.

(Mendenhall, Reinmuth, & Beaver, 1993), (Lind, Marchal, & Mason, 2002)

2.3.1 Range

The simplest way to measure the dispersion of a data set is by using the range. The range can be determined by calculating the difference between the highest and lowest values within a data set. The following exemplifies this as an equation:

$$\text{Range} = \text{highest value} - \text{lowest value} \qquad (2)$$

(Lind, Marchal, & Mason, 2002), (Mendenhall, Reinmuth, & Beaver, 1993)

2.3.2 Variance

The variance can be defined as the arithmetic mean of the squared deviations from the mean of a set of data, in which the deviation is the difference between the mean of a data set and its data values. The variance is a measure that is used to describe a data set's variation from the mean. By squaring the deviations, it becomes impossible for the variance to be negative in value. Additionally, the variance will only be zero if the values of the data set are equivalent.

Population Variance

The formula for the population variance is:

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \mu\right)^2}{n} \qquad (3)$$

Where σ^2 = the symbol used to denote the population variance
n = the number of items in the sample

$\sum_{i=1}^{n}$ = the summation of n items, and

$(x_i - \mu)^2$ = the squared value of the mean subtracted from an item in the sample.

Sample Variance

The formula for the sample variance is:

$$\sigma_s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \tag{4}$$

Where σ_s^2 = the symbol used to denote the sample variance

n = the number of items in the sample

$\sum_{i=1}^{n}$ = the summation of n items, and

$(x_i - \bar{x})^2$ = the squared value of the mean subtracted from an item in the sample.

Similar to the other measures of variability and the measures of central tendency, the variance is a descriptive method that can be used to compare the variability in two or more sets of data. It is often difficult, however, to interpret the significance of the variance in one set of data due to its squared value.

(Lind, Marchal, & Mason, 2002)

2.3.3 Standard Deviation

A simpler way to interpret the variance is with the measure of the standard deviation. The standard deviation is defined as the square root of the variance. By un-squaring the variance, the value is transformed into the same unit of measurement as the set of data. In the previous section we mentioned it is impossible for the variance to be negative in value. This is essential for the standard deviation formula to work, as the square root does not work with negative numbers.

Population Standard Deviation

The formula for the population standard deviation is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \mu\right)^2}{n}} \tag{5}$$

Where σ = the symbol used to denote the population standard deviation

n = the number of items in the sample

$\sum_{i=1}^{n}$ = the summation of n items, and

$\left(x_i - \mu\right)^2$ = the squared value of the mean subtracted from an item in the sample.

Sample Standard Deviation

The formula for the sample standard deviation is:

$$\sigma_s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}} \tag{6}$$

Where σ_s = the symbol used to denote the sample standard deviation

n = the number of items in the sample

$\sum_{i=1}^{n}$ = the summation of n items, and

$\left(x_i - \bar{x}\right)^2$ = the squared value of the mean subtracted from an item in the sample.

(Lind, Marchal, & Mason, 2002)

2.4 Simple Index Numbers

A simple index number, also known as just an index number, refers to a number for a defined period of time that expresses a change in value relative to a base period. The change in value of the indexed number is for a single variable and can be represented by the following equation:

$$P = \left(\frac{p_t}{p_0}\right) * 100 \tag{7}$$

Where p_t = the amount at any given period, and

p_0 = the base-period amount.

The use of index numbers conveniently expresses the change within a diverse set of data by showing the ratios of one time period to another. Additionally, by converting the data into an index, it is easier to observe any visible trends within the data set.

(Lind, Marchal, & Mason, 2002)

2.5 Example Calculations of Numerical Descriptive Methods

In addition to individually explaining each of the basic numerical descriptive methods, a simple example of each method is solved below to demonstrate the application of these formulas.

2.5.1 Data

The following table provides a fictive set of data to be used in solving the examples.

Table 1. Sample data used for solving the numerical descriptive methods.

Sample Data
X
11
25
38
25
6

2.5.2 Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^{5} x_i}{5}$$

$\bar{x} = (x_5 + x_4 + x_3 + x_2 + x_1) / 5$

$\bar{x} = (6 + 25 + 38 + 25 + 11) / 5$

$\bar{x} = 105 / 5$

$\bar{x} = \mathbf{21}$

2.5.3 Median

Since there is no equation for finding the median, the median is found by simply identifying the number in the sample that is located directly in the middle. In this case the median is **38**.

2.5.4 Mode

Since there is no equation for finding the mode, the mode is found by simply identifying the number in the sample that has the greatest frequency of occurrence. In this case the mode is **25**.

2.5.5 Range

Range = highest value – lowest value

Before the range can be calculated, the data must first be arranged in order from smallest to largest.

Data$_{\text{(smallest to largest)}}$ = 38, 25, 25, 11, 6

The formula for finding the range is then calculated as follows:

Range = $x_5 - x_1$

Range = 38 – 6

Range = **32**

2.5.6 Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{5} \left[x_i - \left(\sum_{i=1}^{5} x_i \Big/ 5 \right) \right]^2}{5}$$

$$\sigma^2 = \frac{\sum_{i=1}^{5} \left[x_i - \left({x_5 + x_4 + x_3 + x_2 + x_1} \Big/ {5} \right) \right]^2}{5}$$

$$\sigma^2 = \frac{\sum_{i=1}^{5}\left[x_i - \left(6+25+38+25+11 \middle/ 5\right)\right]^2}{5}$$

$$\sigma^2 = \frac{\sum_{i=1}^{5}\left[x_i - \left(105 \middle/ 5\right)\right]^2}{5}$$

$$\sigma^2 = \frac{\sum_{i=1}^{5}\left(x_i - 21\right)^2}{5}$$

$\sigma^2 = [(x_5 - 21)^2 + (x_4 - 21)^2 + (x_3 - 21)^2 + (x_2 - 21)^2 + (x_1 - 21)^2] / 5$

$\sigma^2 = [(6 - 21)^2 + (25 - 21)^2 + (38 - 21)^2 + (25 - 21)^2 + (11 - 21)^2] / 5$

$\sigma^2 = [(-15)^2 + (4)^2 + (17)^2 + (4)^2 + (-10)^2] / 5$

$\sigma^2 = (225 + 16 + 289 + 16 + 100) / 5$

$\sigma^2 = 646 / 5$

$\sigma^2 = \mathbf{129.2}$

2.5.9 Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \mu\right)^2}{n}}$$

$$\sigma = \sqrt{\sigma^2}$$

$$\sigma = \sqrt{129.2}$$

$\sigma = \mathbf{11.367}$

2.5.8 Sample Variance

$$\sigma_s^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}$$

$$\sigma_s^2 = \frac{\sum_{i=1}^{5}\left[x_i - \left(\sum_{i=1}^{5} x_i \middle/ 5\right)\right]^2}{5-1}$$

$$\sigma_s^2 = \frac{\sum_{i=1}^{5}\left[x_i - \left(x_5 + x_4 + x_3 + x_2 + x_1 \big/ 5\right)\right]^2}{4}$$

$$\sigma_s^2 = \frac{\sum_{i=1}^{5}\left[x_i - \left(6 + 25 + 38 + 25 + 11 \big/ 5\right)\right]^2}{4}$$

$$\sigma_s^2 = \frac{\sum_{i=1}^{5}\left[x_i - \left(105 \big/ 5\right)\right]^2}{4}$$

$$\sigma_s^2 = \frac{\sum_{i=1}^{5}\left(x_i - 21\right)^2}{4}$$

$\sigma_s^2 = [(x_5 - 21)^2 + (x_4 - 21)^2 + (x_3 - 21)^2 + (x_2 - 21)^2 + (x_1 - 21)^2] / 4$

$\sigma_s^2 = [(6 - 21)^2 + (25 - 21)^2 + (38 - 21)^2 + (25 - 21)^2 + (11 - 21)^2] / 4$

$\sigma_s^2 = [(-15)^2 + (4)^2 + (17)^2 + (4)^2 + (-10)^2] / 4$

$\sigma_s^2 = (225 + 16 + 289 + 16 + 100) / 4$

$\sigma_s^2 = 646 / 4$

$\sigma_s^2 = \mathbf{161.5}$

2.5.10 Sample Standard Deviation

$$\sigma_s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n - 1}}$$

$$\sigma_s = \sqrt{\sigma_s^2}$$

$$\sigma_s = \sqrt{161.5}$$

$\sigma_s = \mathbf{12.708}$

2.5.8 Simple Index Numbers

$$P = \left(\frac{p_t}{p_0}\right) * 100$$

$P_1 = (P_1 / P_1) * 100$

$P_1 = (11 / 11) * 100$

$P_1 = 1 * 100$

$P_1 = \mathbf{100}$

$P_2 = (P_2 / P_1) * 100$
$P_2 = (25 / 11) * 100$
$P_2 = 2.272 * 100$
$P_2 = \mathbf{227.2}$

$P_3 = (P_3 / P_1) * 100$
$P_3 = (38 / 11) * 100$
$P_3 = 3.454 * 100$
$P_3 = \mathbf{345.4}$

$P_4 = (P_4 / P_1) * 100$
$P_4 = (25 / 11) * 100$
$P_4 = 2.272 * 100$
$P_4 = \mathbf{227.2}$

$P_5 = (P_5 / P_1) * 100$
$P_5 = (6 / 11) * 100$
$P_5 = 0.545 * 100$
$P_5 = \mathbf{54.5}$

3 BACKGROUND OF FORECASTING

Forecasting is best described as the art and science of predicting a future event. It is the estimating of a future outcome of a random process. One of the main assumptions of forecasting is that the results achieved from the random process will provide accurate and reliable evidence of the future. However, limits do exist in what can be expected from a forecast. Though a forecast can provide a great amount of insight, it is rarely ever perfect. There are numerous unpredictable circumstances that can affect the actual outcome of a forecast. Nevertheless, businesses cannot afford to avoid the process of forecasting, as effective planning in both the short and long run often depend greatly on a forecast.

(Heizer & Render, 2004), (Mendenhall, Reinmuth, & Beaver, 1993)

3.1 Forecasting Approaches

There are two distinct approaches to making a forecast. One of the approaches, known as quantitative forecasting, is an arithmetical method and involves the implementation of a mathematical model to a set of data. It typically relies on historical data and does not include emotional or intuitional factors, like qualitative forecasting. Qualitative models are intended for situations where essential historical data is not available to the forecaster.

(Heizer & Render, 2004), (Armstrong, 2001), (Mendenhall, Reinmuth, & Beaver, 1993)

3.1.1 Quantitative Forecasting

Quantitative forecasting largely involves the use of historical data, which is projected into the future with the use of one ore more mathematical models. It may, however, in some cases also be a subjective or intuitive prediction based on qualitative forecasting, which focuses on subjective inputs that have been obtained from various sources like historical analogies or personal judgment. In such cases, quantitative forecasting becomes a combination of these two approaches and uses a mathematical model that is adjusted by the forecaster's intuition.

Two subcategories of quantitative forecasting are time-series models and econometric models. The main difference between these two types of models is that while econometric models include the use of supporting variables, time series models do not.

(Heizer & Render, 2004)

3.1.2 Econometric Forecasting Models

An econometric model can be defined as one or more equations that demonstrate the relationship amongst a variety of variables. The variables used in econometric forecasting are often a combination of a time series variable and any number of economic variables. An econometric model consists of one dependent variable and any number of independent variables. Econometric models are probabilistic models, meaning that they are calculated by estimating the probability that the dependent variable will reoccur based on the impact of the independent variables. While econometric models forecast by relating independent variables to a dependent variable, time series models forecast based on a dependent variable's history.

(Mendenhall, Reinmuth, & Beaver, 1993)

3.1.3 Time-series Forecasting Models

"Time-series models predict on the assumption that the future is a function of the past" (Heizer & Render, 2004, p. 107). Since time is often an important factor in decision-making, time-series forecasts were developed to utilize time in the foundation of the forecast. Therefore, time-series forecasts are based on a sequence of historical data points that are measured over any length of time, including daily, monthly or yearly. When a variable is measured over time, it allows for the visibility of possible trends. Once a trend has been identified, the most recent data is projected into the future according to the movement of the identified trend.

The analyses of time series forecasts are largely influential, and often essential, to the formulating of future estimations, predictions, and many other major decisions that pertain to long-term planning. A few examples of the time-series models that have been implemented in the forecasting of this thesis include: naive approach, moving average, exponential smoothing, and trend projection.

(Mendenhall, Reinmuth, & Beaver, 1993), (Lind, Marchal, & Mason, 2002), (Heizer & Render, 2004), (Shim, 2000)

3.2 Choosing a Forecasting Model

Though there are many forecasting methods, there is no such thing as one superior method for use in all forecasting scenarios. “What works best in one firm under one set of conditions may be a complete disaster in another organization; or even in a different department of the same firm” (Heizer & Render, 2004, p. 104). Additionally, though it is relatively simple to apply many of the widely used methods, using a more complex forecasting model may improve the accuracy of the forecast. Nevertheless, these more complex methods often involve a meticulous data collection procedure and/or the application of a computer program, which can become quite costly.

One of the most important parts of forecasting is being able to properly match a forecasting model to the data that is being forecasted. It is the responsibility of the forecaster to select a forecasting model that makes best use of the data that is available and fits within the forecaster’s time and budget. Choosing a model is often intuitive and becomes recognizable to the forecaster through literary knowledge and experience from trial and error.

During the process of selecting an appropriate forecasting model, the forecaster must review an assortment of criteria as part of the decision-making. The criterion includes the various types of forecasts and the time horizons of the forecasting period.

(Heizer & Render, 2004)

3.2.1 Types of Forecasts

The type of forecast depends on how the forecaster intends to use the information that is received through the forecasting of the data. There are three main types of forecasts that are commonly used in the planning of the future:

1. Economic forecasts: These types of forecasts address the business cycle.
2. Technological forecasts: These are concerned with the rate of technological progress.
3. Demand forecasts: Project the demand for a company’s products or services.

(Heizer & Render, 2004)

3.2.2 Time Horizons

Another consideration when selecting a forecasting model is the time horizon for the forecasting period. Once the use of the forecast has been determined, the forecast is categorized by its time horizon. "Some models are more accurate for short-term time horizons and others are more reliable for long-term horizons" (Mendenhall, Reinmuth, & Beaver, 1993, p. 667). There are three kinds of time horizons:

1. Short-ranged forecasts: May have a time span of up to one year but are normally less than three months.
2. Medium-ranged forecasts: Generally span from three months to three years.
3. Long-ranged forecasts: Any length of time lasting three years or longer.

With the constant change in demand patterns, long-term forecasts are normally used as a planning model for product lines and capital investment decisions, while short-term forecasts usually involve forecasting sales, price changes, and customer demand.

(Mendenhall, Reinmuth, & Beaver, 1993)

3.3 Forecasting System

The forecasting system is an exemplary guide for successfully completing a forecast. There are seven steps that form the forecasting system, and together they present a systematic way of initiating, designing, and implementing a forecasting project. The steps of the forecasting system have been applied throughout the forecasting within this thesis. The seven steps that form the forecasting system are:

1. Determine the use of the forecast.
2. Select the items to be forecasted.
3. Determine the time horizon of the forecast.
4. Select the forecasting model(s).
5. Gather the data needed to make the forecast.
6. Make the forecast.
7. Validate and implement the results.

(Heizer & Render, 2004)

3.4 Importance of Forecasting

Forecasting plays an important role in the success of many companies around the world. Inventory must be ordered without confidence of what sales will be, new equipment must be purchased with uncertainty of the final products' demand, and investments are commonly made without knowing what profits will be. Thus, it is important for forecasters to consistently make estimates of what will happen in the future. The projection of a good estimate is the core purpose of forecasting, as it is essential to efficient service and manufacturing operations. "A forecast is the only estimate of demand until actual demand becomes known" (Heizer & Render, 2004, p. 105). Therefore, effective planning in both the short and long run depends on forecasts, making it the drive behind many critical decisions.

Due to the uncertainty of what the future may hold, it is very unlikely for a forecast to be completely accurate. Oftentimes, the further a forecast is projected into the future, the more apprehensive its outcome becomes. A majority of the results produced by a forecast are in some way influenced by one or more unexpected factors. A forecast, long or short-term, is rarely able to offset the effects of each and every possible influencing factor. Still, forecasting is often inevitable due to the probability of benefitting from the forecast, compared to not forecasting at all.

It is important to consider forecasting as a practice that can consistently be perfected. As the forecaster gains experience, they become more capable of adjusting their methods according to changes within the forecast's environment. It is possible for even the worst forecasters to occasionally produce good forecasts, while even the best forecasters sometimes miss completely.

(Heizer & Render, 2004), (Mendenhall, Reinmuth, & Beaver, 1993)

4 FORECASTING MODELS

4.1 Naive Approach

The naive approach is the simplest way to make a forecast because there are no actual calculations involved in its determination. The approach is to assume that the demand in the next period will be the same as the demand in the most recent period.

$$F_t = A_{t-1} \qquad (8)$$

Where F_t = the new forecast, and

A_{t-1} = the previous period's actual value.

(Heizer & Render, 2004), (Shim, 2000)

4.2 Simple Moving Averages

A simple moving average, also referred to as moving average, is an approach that bases its forecast on an average of a fixed number of previous cycles within a data set. As time moves forward, the earliest data is excluded from the average to include the most recent cycle's actual data, creating a continuous inflow of current data and smoothes fluctuations. Therefore, by following this process, any irregular variations that may be present in the set of data are eliminated. This allows for any visible long-term trends within the set of data to be presented. The formula for a simple moving average is:

$$F_t = \frac{\sum_{i=1}^{n} A_i}{n} \qquad (9)$$

Where F_t = the new forecast

n = the number of previous periods in the sample, and

$\sum_{i=1}^{n} A_i$ = the summation of the actual values for the previous n periods.

Moving averages are useful when the forecaster can assume that the data will move steadily with time. When the data exhibits extreme, random movement a moving average forecasting model is likely to provide inaccurate results.

(Lind, Marchal, & Mason, 2002), (Heizer & Render, 2004), (Shim, 2000), (Mendenhall, Reinmuth, & Beaver, 1993)

4.3 Exponential Smoothing

An exponential smoothing model can be described as a combination of a naive approach and a moving-average model, with an added smoothing constant. Exponential smoothing is used to reveal possible trends and seasonal and cyclic effects within a set of data. Similar to the other time-series models, exponential smoothing assumes that past trends and cycles will continue into the future. Though the exponential smoothing formula forecasts by extending these patterns, it involves very little record keeping and only uses one period of previous data.

Naturally, a pattern does not always occur exactly the same as it did previously. Thus, a smoothing constant is included to reduce the random fluctuations that may exist within a pattern. The formula for exponential smoothing is as follows:

$$F_t = F_{t-1} + \alpha\left(A_{t-1} - F_{t-1}\right) \tag{10}$$

Where F_t = the new forecast
F_{t-1} = the previous period's forecast
α = the smoothing constant ($0 \leq \alpha \leq 1$), and
A_{t-1} = the previous period's actual value.

(Mendenhall, Reinmuth, & Beaver, 1993), (Heizer & Render, 2004), (Shim, 2000)

4.3.1 Smoothing Constant

The symbol α, read as alpha, is used to represent a smoothing constant. A smoothing constant is a number that is used as a weight in forecasting to make adjustments to the previous forecast's error. The smoothing constant may range between *0* and *1* and is chosen by the forecaster. Deciding on the value of a smoothing constant for a set of data is no simple task. No formula exists for finding the best value. Thus, the smoothing constant is often found through trial and error. A high smoothing constant is used to give more weight to recent data while a low smoothing constant gives more weight to past data. When the smoothing constant is *1* the forecast relies solely on the most recent data period and assumes that the new forecast is equal to the previous period's actual value, making it identical to a naive approach.

A second smoothing constant, β, is used in addition to α when forecasting with trend adjustment. Pronounced as beta, this second smoothing constant is used to weight a measure of trend before it is added to an exponentially smoothed forecast to develop a forecast that accounts for trend. Similar to α, the value of β is chosen by the forecaster and must be a number that ranges between *0* and *1*. Choosing a proper value for both smoothing constants is crucial and can often make the difference between an accurate forecast and an inaccurate forecast.

(Mendenhall, Reinmuth, & Beaver, 1993), (Heizer & Render, 2004)

4.4 Exponential Smoothing with Trend Adjustment

Exponential smoothing with trend adjustment is a modified exponential smoothing model that is able to respond to trends. In a simple exponential smoothing model, a lag between the forecast and actual amount is often present. A trend-adjusted exponential smoothing model is applied to reduce any lag and improve the forecast. The formula for exponential smoothing with trend adjustment is:

$$FIT_t = F_t + T_t \tag{11}$$

Where FIT = the abbreviation for forecasting with trend
F = the exponentially smoothed forecast, and
T = the exponentially smoothed trend.

The exponentially smoothed forecast and trend can be found by the following formulas:

$$F_t = \alpha(A_{t-1}) + (1-\alpha)(F_{t-1} + T_{t-1}) \tag{12}$$

$$T_t = \beta(F_t - F_{t-1}) + (1-\beta)(T_{t-1}) \tag{13}$$

Where F_t = the new forecast
α = the smoothing constant for the average ($0 \leq \alpha \leq 1$)
A_{t-1} = the previous period's actual value
F_{t-1} = the previous period's forecast

T_{t-1} = the previous period's trend

T_t = the new trend, and

β = the smoothing constant for the trend ($0 \leq \beta \leq 1$).

(Heizer & Render, 2004)

4.5 Linear Regression

Linear regression can be described as a forecasting model that is used to express a direct relationship between two or more variables. Unlike the previous mentioned forecasting models, linear regression is an econometrics model; meaning that the dependent variable is predicted based on a selected value of one or more independent variables. Furthermore, the linear regression model uses the same formula that is used to graph a line. By arranging the set of data in a linear form, the general movement of the data is displayed. This allows better accuracy in measuring the trend, which allows for a more accurate forecast. Once the slope and the y-intercept of the line have been determined, the line is extended to forecast for a future period of time. The formula for linear regression is written as follows:

$$\hat{y} = a + bx \tag{14}$$

Where $\hat{y}$ = the predicted value of the y variable for a selected value of x

a = the y-intercept

b = the slope of the line, and

x = any value of the independent variable.

The y-intercept and the slope of the linear regression model can be calculated using the following formulas:

$$b = \frac{\sum_{i=1}^{n} x_i y_i - (n)(\bar{x})(\bar{y})}{\sum_{i=1}^{n} x_i^2 - (n)(\bar{x})^2} \tag{15}$$

$$a = \bar{y} - b\bar{x} \tag{16}$$

Where a = the y-intercept

$\bar{y}$ = the average of the y values

b = the slope of the regression line

$\bar{x}$ = the average of the x values

x = a value of the independent variable

y = a value of the dependent variable, and

n = the number of items in the sample.

(Heizer & Render, 2004),(Swift, 1997), (Lind, Marchal, & Mason, 2002)

4.6.1 Assumptions Underlying Linear Regression

In the application of a linear regression model, there are certain assumptions that must be met in order for the model to properly forecast. For each value of the dependent variable, there must be a value for the independent variable(s). This means that the amount of observations must be the same for all variables. Additionally, the values for each independent variable must not be biased towards other values of the same variable. In other words, the values for an independent variable cannot be dependent on one another. This assumption is particularly important when the set of data is collected over a period of time, as the forecast error for a time period can often be correlated with another period of time.

(Lind, Marchal, & Mason, 2002)

4.6.2 The Standard Error of Estimate

Since it is uncommon for a set of data to lie exactly on a linear regression line, there is some degree of error present in the estimating of the dependent variable. The standard error of estimate is used to measure this error and describe how precise or inaccurate the forecast is, according to the dispersion of the observed values from the linear regression line. The standard error of estimate is noted as $S_{y,x}$, and can be determined with the following equation:

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}} \tag{17}$$

Where $S_{y,x}$ = the symbol used to denote the standard error of estimate for x and y

y = the values of the dependent variable

a = the y-intercept

b = the slope of the regression line

x = the values of the independent variable, and
n = the number of items in the sample.

(Lind, Marchal, & Mason, 2002)

4.6.3 Correlation Analysis

Correlation analysis can be described as the precise study of a relationship between two variables, in which a group of techniques are applied to measure the strength of the association between variables. The first technique that is applied to the data is commonly a scatter diagram. The dependent variable, the variable that is being predicted, is scaled on the vertical axis as the independent variable, the variable that provides the basis for estimation, is scaled on the horizontal axis. After evaluation, if a general relationship is identifiable, the coefficient of correlation is applied to determine the strength and direction of the relationship.

The coefficient of correlation is a formula used to establish a numerical measure of a relationship's strength. Referred to as *r,* the coefficient can assume any value ranging between *-1* and *+1*. An exact measure of either a positive or negative *1* signifies a perfect correlation, meaning that the dependent variable directly relates to the independent variable. The stronger the relationship, the more linear the scatter plot will be. A positive or negative coefficient will indicate the direction of the relationship's movement, with a negative coefficient representing a negative direction and a positive coefficient representing a positive direction. Conversely, if there is absolutely no relationship the coefficient will be *0*. The coefficient of correlation formula can be indentified as follows:

$$r = \frac{n\sum_{i=1}^{n}(x_i * y_i) - \left(\sum_{i=1}^{n} x_i * \sum_{i=1}^{n} y_i\right)}{\sqrt{\left[n\sum_{i=1}^{n} x^2 - \left(\sum_{i=1}^{n} x\right)^2\right]\left[n\sum_{i=1}^{n} y^2 - \left(\sum_{i=1}^{n} y\right)^2\right]}} \qquad (18)$$

Where r = the symbol used to denote the correlation ($-1 \le r \le 1$)
n = the number of paired samples
$\sum_{i=1}^{n} x_i * y_i$ = the summation of the products of the x values and y values

$\sum_{i=1}^{n} x_i$ = the summation of the *x* variables

$\sum_{i=1}^{n} y_i$ = the summation of the *y* variables

$\sum_{i=1}^{n} x^2$ = the summation of the *x* variables squared

$\left(\sum_{i=1}^{n} x\right)^2$ = the squared summation of the *x* variables

$\sum_{i=1}^{n} y^2$ = the summation of the *y* variables squared, and

$\left(\sum_{i=1}^{n} y\right)^2$ = the squared summation of the *y* variables.

If there is a strong relationship between two variables, we are easily tempted to assume that an increase or decrease in one variable will cause a change in the other variable. This, however, is not what the measure of the correlation interprets. A strong coefficient of correlation can only conclude that a relationship between the two variables exists.

(Lind, Marchal, & Mason, 2002)**,** (Heizer & Render, 2004)

5 MEASURING ERROR

5.1 Measures of Forecast Accuracy

As previously mentioned, it is quite rare for a forecast to be entirely perfect. Measuring the amount of error that exists within a forecast is very important in the interpreting of its results. Measures of forecast accuracy are used to summarize the overall precision of the application of a forecasting model. The closer a forecast is to its actual value, the more accurate the forecasting model is. Therefore, through examination of a forecast's error, the quality of the model applied can be evaluated.

(Mendenhall, Reinmuth, & Beaver, 1993)

5.2 Mean Absolute Deviation

The mean absolute deviation is a relatively simple formula that can be used to measure the overall error of a forecasting model. Also known as the *MAD*, the mean absolute deviation is computed by the following formula:

$$MAD = \frac{\sum_{i=1}^{n} |A_i - F_i|}{n} \tag{19}$$

Where *MAD* = the abbreviation for Mean Absolute Deviation

n = the number of periods

$\sum_{i=1}^{n}$ = the summation of n periods, and

$|A_i - F_i|$ = the absolute value of the forecast error for n periods.

(Heizer & Render, 2004), (Mendenhall, Reinmuth, & Beaver, 1993)

5.3 Mean Squared Error

The mean squared error, also referred to as the *MSE*, is another way to measure the overall accuracy of a forecasting model by a forecast's error. The main difference between the *MSE* and the *MAD* formula is that the *MSE* penalizes extreme errors more heavily than the formula of the *MAD*, causing the extreme deviations to be accentuated.

The formula for the *MSE* is:

$$MSE = \frac{\sum_{i=1}^{n} \left(\left|A_i - F_i\right|\right)^2}{n} \qquad (20)$$

Where *MSE* = the abbreviation for Mean Squared Error

n = the number of periods

$\sum_{i=1}^{n}$ = the summation of n periods, and

$\left(\left|A_i - F_i\right|\right)^2$ = the squared absolute value of the forecast error for n periods.

(Heizer & Render, 2004), (Mendenhall, Reinmuth, & Beaver, 1993)

5.4 Root Mean Square Error

The root mean squared error, or *RMSE*, is simply the square root of the *MSE*. The formula for the *RMSE* is as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(\left|A_i - F_i\right|\right)^2}{n}} \qquad (21)$$

Where *RMSE* = the abbreviation for Root Mean Squared Error

n = the number of periods

$\sum_{i=1}^{n}$ = the summation of n periods, and

$\left(\left|A_i - F_i\right|\right)^2$ = the squared absolute value of the forecast error for n periods.

(Heizer & Render, 2004), (Mendenhall, Reinmuth, & Beaver, 1993)

5.5 Mean Absolute Percentage Error

One disadvantage in using the *MAD* and *MSE* in measuring forecast accuracy is that their values are dependent on the magnitude of the item that is forecasted. When a forecasted item is measured in large terms, the *MAD* and *MSE* values tend to be very large. By using the mean absolute percentage error, this problem can be avoided. Conversely, the *MAPE* is not as effective when a forecasted item is measured in extremely small terms.

The mean absolute deviation, also known as the MAPE, can be calculated by the following formula:

$$MAPE = \frac{(100) * \left(\sum_{i=1}^{n} \frac{|A_i - F_i|}{A_i} \right)}{n} \tag{22}$$

Where *MAPE* = the abbreviation for Mean Absolute Percentage Error

n = the number of periods

$\sum_{i=1}^{n}$ = the summation of n periods,

$|A_i - F_i|$ = the absolute value of the forecast errors for n periods, and

A_i = the actual values for n periods.

Since the *MAPE* is measured as a percentage it is a very simple to interpret its results. The application of the *MAPE* is particularly useful in comparing the performance of a model for a variety of time-series.

(Mendenhall, Reinmuth, & Beaver, 1993), (Heizer & Render, 2004)

6 EXCHANGE RATES

6.1 Background of Foreign Exchange Rates

Around the world, numerous forms of money are in existence. Commonly referred to as a currency, each form of money serves a universal purpose – to be exchanged in return for goods and/or services. Ultimately, however, currencies serve as much more than simply a medium of exchange. Often varying by country, a currency is generally representative of a country's market value. Therefore, by converting two or more currencies into a single form, it is possible to evaluate a country's worth. This process of measuring two or more currencies in the form of a single country's currency is known as a foreign exchange rate.

6.2 Forecasting Exchange Rates

Accurately estimating exchange rates is often critical in the financial world. Unfortunately, it is very difficult to consistently estimate exchange rates successfully. A variety of reasons exist as to why forecasts for exchange rates tend to deviate from their actual values. The forecaster's expectations for a currency's future may move in the opposite direction of the exchange rate's actual value. Conversely, the forecaster's expectations may be correct, however, unexpected factors may influence the exchange rate's movement to suddenly change. Lastly, the chosen forecasting model may not be as efficient in forecasting a currency as compared to an alternative forecasting model.

(Rosenberg, 2003)

6.2.1 Influencing Factors

The value of a currency is constantly affected and influenced by numerous factors, with many of them often occurring simultaneously. Since the amount of impact that a factor may have on a currency can vary drastically, the forecasting of exchange rates is often complicated. Due to their effect on the value of an exchange rate, however, the influencing factors can be difficult to avoid. Some of the most common influencing factors include:

- Gross domestic product
- Interest rates

- Government budget deficits/surpluses
- Monetary policies
- Inflation
- Current-account balance
- Political stability
- Natural disasters

(Rosenberg, 2003)

6.2.2 Economical Forecasting Models

A variety of exchange rate forecasting models that predict on the assumption that foreign exchange rates revert to an ideal equilibrium level exists in economics. The models provide a framework for forecasting exchange rates by alerting the forecaster with important changes in the economy and, in turn, how those may affect the exchange rates over a medium or long-term time horizon. The models, however, do not inform the forecaster of when such events are likely to occur. Therefore, the models are quite limited in their ability to forecast exchange rates. A few of the most common economical forecasting models include purchasing power parity, the monetary approach, the interest rate approach, and the balance of payments approach.

(Henderson, 2002)

Purchasing Price Parity

Purchasing power parity, often abbreviated as *PPP*, is one of the most familiar exchange rate models in currency analysis and is commonly referred to as the law of one price. The main concept of the *PPP* model is that by having no barriers in free trading, the price of a good will be valued the same throughout the world. As a result, the exchange rate will move towards a long-term equilibrium level. In order for the purchasing power parity model to work properly, there must not be any trading barriers or transaction costs. The purchasing power parity is calculated by the following formula:

$$E = \frac{P_t}{P_0} \tag{23}$$

Where E = the *PPP* long-term equilibrium exchange rate value,
P_t = the price level of the term currency, and
P_0 = the price level of the base currency.

(Henderson, 2002)

Monetary Approach

The monetary approach is an economical model that believes exchange rates can be determined by balancing the supply and demand for money. With the monetary approach it is believed that a change in the supply of money will eventually be offset by a similar change in the demand for money, thus restoring balance. The monetary approach's concept is as follows:

Change in money supply $\rightarrow$ Change in price $\rightarrow$ Change in exchange rate (24)

(Henderson, 2002)

Interest Rate Approach

The interest rate approach forecasts exchange rates similarly to the purchasing power parity. The interest rate approach consists of three main theories and assumes that the interest rate of a currency will be equalized when it is converted back to its base currency. The interest rate approach is explained by the following:

1. The difference in spot and forward rates = the different in interest rates
2. The difference in interest rates = the difference in expected inflation rates
3. The difference in expected inflation rates = the expected change in the spot exchange rate

(Henderson, 2002)

Balance of Payments Approach

The main theory of the balance of payments approach is that a change in national income will affect current and capital accounts, causing a predictable fluctuation in the exchange rates to restore the balance of payments equilibrium level. The balance of payments approach is as follows:

Change in national income $\rightarrow$ Change in current account balance $\rightarrow$ Monetary reaction $\rightarrow$ Reversal of current account balance change $\rightarrow$ Balance of payments equilibrium restored (25)

(Henderson, 2002)

6.2.3 Time Horizons

Evidently, there have been many forecasting models that have performed poorly in depicting the trend of exchange rates over a short-term horizon. Oftentimes the forecasting models have been more successful in forecasting exchange rates over a medium-ranged time horizon, and exceptionally well with long-ranged horizons. With the uncertainty of the future, however, it is quite risky to depend on long-term forecasts of exchange rates. Therefore, forecasting exchange rates over a short-range time horizon, though more difficult, is optimal.

Estimating exchange rates accurately in the short-term is often more complex than in the medium or long-term due to the greater amount of variability that exists in daily and weekly exchange rates. The unpredictable behavior that exists in daily and weekly exchange rates often causes an obscurity in identifying trends within the movement of exchange rates.

(Rosenberg, 2003)

6.3 Limitations

There are many forecasting models in economics that can be applied to exchange rate data. A majority of the economical models depend on influencing factors, however, rather than previous exchange rate data to make their forecasts. This is due to the perception that exchange rates move independently and do not relate to past exchange rate values. Though the economical forecasting models rely on influencing factors, they still revolve around the notion of an existing equilibrium level. The concept of an equilibrium level existing in exchange rates has rarely been evident in real life with the continuous fluctuation that is often present in exchange rates. While the concepts of the economical forecasting models have been useful and are logically constructed, the results of the actual outcome are often quite vague. The equilibrium models may successfully forecast a long-term exchange rate, however, the models are unable to forecast when the exchange rate will actually occur, and how the exchange rate fluctuates throughout the process. Furthermore, the models are often unsuccessful over short-term time horizons.

Due to the difficulty in forecasting short-term exchange rates using economical forecasting models, economists have often ignored them and dismissed them as speculative, concluding that short-term exchange rates cannot be forecasted. Though economics has been unsuccessful in forecasting short-term exchange rates, there have been successes with the use of other methods like technical analysis and capital flow analysis. These other methods are still not entirely accurate, however, they have been measurably better than the economical forecasting models, providing optimism that exchange rates may also be forecasted over short-term time horizons.

(Henderson, 2002)

7 DATA

Throughout a period of three months, the daily historical exchange rates for the European Euro, measured in United States Dollars, were documented. The collection of the data took place during the month of April 2010 where the historic exchange rates for the months of January, February, and March were collected. The data was collected through the Internet at *x-rates.com*, which records historic exchange rates from sources such as the International Monetary Fund, the European Central Bank, Bank of Canada, and the Federal Reserve Bank of New York. Since the website does not provide historic exchange rates for weekends or recognized holidays they were omitted from the data set. In total, the set of data includes sixty-three samples, with the first sample beginning on January 4th, 2010 and the last sample ending on March 31st, 2010. The following table displays the set of exchange rate data that will be applied to the forecasting models.

Table 2. Exchange rate data

Data			
Sample	**Date**	**Euro**	**USD**
1	4-Jan	1	1.4389
2	5-Jan	1	1.4442
3	6-Jan	1	1.4350
4	7-Jan	1	1.4304
5	8-Jan	1	1.4273
6	11-Jan	1	1.4528
7	12-Jan	1	1.4481
8	13-Jan	1	1.4563
9	14-Jan	1	1.4486
10	15-Jan	1	1.4374
11	18-Jan	1	1.4369
12	19-Jan	1	1.4279
13	20-Jan	1	1.4132
14	21-Jan	1	1.4064
15	22-Jan	1	1.4135
16	25-Jan	1	1.4151
17	26-Jan	1	1.4085
18	27-Jan	1	1.4072
19	28-Jan	1	1.3999
20	29-Jan	1	1.3966
21	1-Feb	1	1.3913
22	2-Feb	1	1.3937
23	3-Feb	1	1.3984

24	4-Feb	1	1.3847
25	5-Feb	1	1.3691
26	8-Feb	1	1.3675
27	9-Feb	1	1.3760
28	10-Feb	1	1.3740
29	11-Feb	1	1.3718
30	12-Feb	1	1.3572
31	15-Feb	1	1.3607
32	16-Feb	1	1.3649
33	17-Feb	1	1.3726
34	18-Feb	1	1.3567
35	19-Feb	1	1.3519
36	22-Feb	1	1.3626
37	23-Feb	1	1.3577
38	24-Feb	1	1.3547
39	25-Feb	1	1.3489
40	26-Feb	1	1.3570
41	1-Mar	1	1.3525
42	2-Mar	1	1.3548
43	3-Mar	1	1.3641
44	4-Mar	1	1.3668
45	5-Mar	1	1.3582
46	8-Mar	1	1.3662
47	9-Mar	1	1.3557
48	10-Mar	1	1.3610
49	11-Mar	1	1.3657
50	12-Mar	1	1.3765
51	15-Mar	1	1.3705
52	16-Mar	1	1.3723
53	17-Mar	1	1.3756
54	18-Mar	1	1.3660
55	19-Mar	1	1.3548
56	22-Mar	1	1.3471
57	23-Mar	1	1.3519
58	24-Mar	1	1.3338
59	25-Mar	1	1.3356
60	26-Mar	1	1.3353
61	29-Mar	1	1.3471
62	30-Mar	1	1.3482
63	31-Mar	1	1.3479

8 CALCULATIONS FOR FORECASTING THE EXCHANGE RATES

The following calculations present the author's forecasted exchange rates for the month of May 2010, based on the author's set of data. The forecasts include the application of a naive approach, a moving average, exponential smoothing, exponential smoothing with trend adjustment, and a linear regression model. The forecasts that were made using the linear regression model were calculated first, and were conducted on April 26th, 2010. The remaining models were applied simultaneously and began on April 30th, 2010 and concluded on May 30th, 2010.

8.1 Naive Approach

Since a naive approach bases its forecast on the previous period's actual value, the only sample that is needed from the data set is March 31st, 2010. The forecast for May 3rd, 2010 is then calculated as follows:

$F_t = A_{t-1}$

$F_{(\text{May 3rd})} = A_{(\text{May 3rd} - 1)}$

$F_{(\text{May 3rd})} = A_{(\text{March 31st})}$

$F_{(\text{May 3rd})} = \mathbf{1.3479}$

The table below displays the forecasts for May using a naive approach.

Table 3. Forecasts for May 2010 using a naive approach.

May Forecasts, Naive Approach				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
63	31-Mar	1	-	1.3479
85	3-May	1	**1.3479**	1.3238
86	4-May	1	**1.3238**	1.3089
87	5-May	1	**1.3089**	1.2924
88	6-May	1	**1.2924**	1.2727
89	7-May	1	**1.2727**	1.2746
90	10-May	1	**1.2746**	1.2969
91	11-May	1	**1.2969**	1.2698
92	12-May	1	**1.2698**	1.2686
93	13-May	1	**1.2686**	1.2587
94	14-May	1	**1.2587**	1.2492
95	17-May	1	**1.2492**	1.2349

96	18-May	1	**1.2349**	1.2428
97	19-May	1	**1.2428**	1.2270
98	20-May	1	**1.2270**	1.2334
99	21-May	1	**1.2334**	1.2497
100	24-May	1	**1.2497**	1.2360
101	25-May	1	**1.2360**	1.2223
102	26-May	1	**1.2223**	1.2309
103	27-May	1	**1.2309**	1.2255
104	28-May	1	**1.2255**	1.2384
105	31-May	1	**1.2384**	1.2307

8.2 Moving Average

A time period of seven days has been chosen by the author to be used in this moving average calculation to forecast the exchange rates for May 2010. Therefore, the seven most recent samples from the data set are used to forecast May 3rd, 2010. The forecast for May 3rd is then calculated in the following manner:

$$F_t = \frac{\sum_{i=1}^{n} A_i}{n}$$

$$F_{(\text{May 3rd})} = \frac{\sum_{i=1}^{7} A_i}{7}$$

$F_{(\text{May 3rd})} = (A_7 + A_6 + A_5 + A_4 + A_3 + A_2 + A_1) / 7$

$F_{(\text{May 3rd})} = (A_{(\text{March 31st})} + A_{(\text{March 30th})} + A_{(\text{March 29th})} + A_{(\text{March 26th})} + A_{(\text{March 25th})} + A_{(\text{March 24th})} + A_{(\text{March 23rd})}) / 7$

$F_{(\text{May 3rd})} = (1.3479 + 1.3482 + 1.3471 + 1.3353 + 1.3356 + 1.3338 + 1.3519) / 7$

$F_{(\text{May 3rd})} = 9.3998 / 7$

$F_{(\text{May 3rd})} =$ **1.3428**

The table below displays the forecasts for May using a moving average.

Table 4. Forecasts for May 2010 using a 7-day moving average.

May Forecasts, 7-Day Moving Average				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
57	23-Mar	1	-	1.3519
58	24-Mar	1	-	1.3338
59	25-Mar	1	-	1.3356
60	26-Mar	1	-	1.3353

61	29-Mar	1	-	1.3471
62	30-Mar	1	-	1.3482
63	31-Mar	1	-	1.3479
85	3-May	1	**1.3428**	1.3238
86	4-May	1	**1.3388**	1.3089
87	5-May	1	**1.3353**	1.2924
88	6-May	1	**1.3291**	1.2727
89	7-May	1	**1.3201**	1.2746
90	10-May	1	**1.3098**	1.2969
91	11-May	1	**1.3025**	1.2698
92	12-May	1	**1.2913**	1.2686
93	13-May	1	**1.2834**	1.2587
94	14-May	1	**1.2762**	1.2492
95	17-May	1	**1.2701**	1.2349
96	18-May	1	**1.2647**	1.2428
97	19-May	1	**1.2601**	1.2270
98	20-May	1	**1.2501**	1.2334
99	21-May	1	**1.2449**	1.2497
100	24-May	1	**1.2422**	1.2360
101	25-May	1	**1.2390**	1.2223
102	26-May	1	**1.2352**	1.2309
103	27-May	1	**1.2346**	1.2255
104	28-May	1	**1.2321**	1.2384
105	31-May	1	**1.2337**	1.2307

8.3 Exponential Smoothing

The author conducted a trial and error method in order to find the best suitable value of alpha. Each value of alpha ranging from *0.1* to *0.9* was tried in the exponential smoothing formula with the application of the exchange rate data. The best value of alpha was *0.9* since it had the smallest forecast error and, as a result, was chosen as the smoothing constant. The forecasts that were made using the other values of alpha can be viewed in the appendix. To forecast May 2010 as accurately as possible with exponential smoothing, the author first used a naive approach to forecast for March 30th and began exponential smoothing with March 31st to set a proper rhythm for forecasting the entire month of May. The forecast for May 3rd using exponential smoothing and a smoothing constant of *0.9* is demonstrated as follows:

$$F_t = F_{t-1} + \alpha\left(A_{t-1} - F_{t-1}\right)$$

$F_{(\text{May 3rd})} = F_{(\text{May 3rd} - 1)} + 0.9 * (A_{(\text{May 3rd} - 1)} - F_{(\text{May 3rd} - 1)})$

$F_{(\text{May 3rd})} = F_{(\text{March 31st})} + 0.9 * (A_{(\text{March 31st})} - F_{(\text{March 31st})})$

$F_{(\text{May 3rd})} = 1.3481 + 0.9 * (1.3479 - 1.3481)$

$F_{(\text{May 3rd})} = 1.3481 + 0.9 * (-0.0002)$

$F_{(\text{May 3rd})} = 1.3481 - 0.00018$

$F_{(\text{May 3rd})} = 1.34792$, which is rounded to **1.3479** to keep a maximum of four decimals.

The following table displays the forecasts for May using exponential smoothing.

Table 5. Forecasts for May 2010 using exponential smoothing.

May Forecasts, Exponential Smoothing				
α = 0.9				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3481	1.3479
85	3-May	1	**1.3479**	1.3238
86	4-May	1	**1.3262**	1.3089
87	5-May	1	**1.3106**	1.2924
88	6-May	1	**1.2942**	1.2727
89	7-May	1	**1.2749**	1.2746
90	10-May	1	**1.2746**	1.2969
91	11-May	1	**1.2947**	1.2698
92	12-May	1	**1.2723**	1.2686
93	13-May	1	**1.2690**	1.2587
94	14-May	1	**1.2597**	1.2492
95	17-May	1	**1.2503**	1.2349
96	18-May	1	**1.2364**	1.2428
97	19-May	1	**1.2422**	1.2270
98	20-May	1	**1.2285**	1.2334
99	21-May	1	**1.2329**	1.2497
100	24-May	1	**1.2480**	1.2360
101	25-May	1	**1.2372**	1.2223
102	26-May	1	**1.2238**	1.2309
103	27-May	1	**1.2302**	1.2255
104	28-May	1	**1.2260**	1.2384
105	31-May	1	**1.2372**	1.2307

8.4 Exponential Smoothing With Trend Adjustment

Similar to the procedure used in choosing the best value of alpha in the exponential smoothing model, the author conducted a trial and error method in order to find the best suitable combination of alpha and beta. Each value of alpha and beta ranging from *0.1* to *0.9* was tried in the trend adjustment formula with the set of exchange rate data. The best value of alpha and beta was *0.5* since it had the smallest forecast error and, as a result, was chosen as the smoothing constant. The forecasts that were made using the other values of alpha and beta can be viewed in the appendix. The same method that was used to forecast May 2010 with exponential smoothing was used to forecast with trend adjustment in order to forecast as accurately as possible. The author first used a naive approach to forecast for March 30th and began exponential smoothing with trend with March 31st to set a proper rhythm for forecasting the entire month of May. The forecast for May 3rd using exponential smoothing with trend adjustment and a smoothing constant of *0.5* is calculated as follows:

$$FIT_t = F_t + T_t$$

$$F_t = \alpha(A_{t-1}) + (1-\alpha)(F_{t-1} + T_{t-1})$$

$F_{(\text{May 3rd})} = (0.5 * A_{(\text{May 3rd} - 1)}) + (1\text{-}0.5) * (F_{(\text{May 3rd} - 1)} + T_{(\text{May 3rd} - 1)})$

$F_{(\text{May 3rd})} = (0.5 * A_{(\text{March 31st})}) + 0.5 * (F_{(\text{March 31st})} + T_{(\text{March 31st})})$

$F_{(\text{May 3rd})} = (0.5 * 1.3479) + 0.5 * (1.34815 + 0.0010)$

$F_{(\text{May 3rd})} = 0.67395 + 0.5 * 1.34915$

$F_{(\text{May 3rd})} = 0.67395 + 0.674575$

$F_{(\text{May 3rd})} = 1.348525$, which rounds to 1.3485 to keep a maximum of four decimals.

$$T_t = \beta(F_t - F_{t-1}) + (1-\beta)(T_{t-1})$$

$T_{(\text{May 3rd})} = 0.5 * (F_{(\text{May 3rd})} - F_{(\text{May 3rd} - 1)}) + (1\text{-}0.5) * T_{(\text{May 3rd} - 1)}$

$T_{(\text{May 3rd})} = 0.5 * (F_{(\text{May 3rd})} - F_{(\text{March 31st})}) + 0.5 * T_{(\text{March 31st})}$

$T_{(\text{May 3rd})} = 0.5 * (1.3485 - 1.34815) + 0.5 * 0.0010$

$T_{(\text{May 3rd})} = (0.5 * 0.00035) + 0.0005$

$T_{(\text{May 3rd})} = 0.000175 + 0.0005$

$T_{(\text{May 3rd})} = 0.000675$, which rounds to 0.0007 to keep a maximum of four decimals.

Once the values of F_t and T_t are determined, they are applied to the original trend adjustment formula:

$FIT_{(\text{May 3rd})} = F_{(\text{May 3rd})} + T_{(\text{May 3rd})}$

$FIT_{(\text{May 3rd})} = 1.3485 + 0.0007$

$FIT_{(\text{May 3rd})} =$ **1.3492**

The following table displays the forecasts for May using exponential smoothing with trend adjustment.

Table 6. Forecasts for May 2010 using exponential smoothing with trend adjustment.

May Forecasts, Trend Adjustment						
α = 0.5 **β = 0.5**						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	***FIT***	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3482	0.0010	-	1.3479
85	3-May	1	1.3485	0.0007	**1.3492**	1.3238
86	4-May	1	1.3365	-0.0057	**1.3309**	1.3089
87	5-May	1	1.3199	-0.0111	**1.3087**	1.2924
88	6-May	1	1.3006	-0.0152	**1.2853**	1.2727
89	7-May	1	1.2790	-0.0184	**1.2606**	1.2746
90	10-May	1	1.2676	-0.0149	**1.2527**	1.2969
91	11-May	1	1.2748	-0.0039	**1.2710**	1.2698
92	12-May	1	1.2704	-0.0041	**1.2662**	1.2686
93	13-May	1	1.2674	-0.0036	**1.2639**	1.2587
94	14-May	1	1.2613	-0.0048	**1.2564**	1.2492
95	17-May	1	1.2528	-0.0067	**1.2462**	1.2349
96	18-May	1	1.2405	-0.0095	**1.2311**	1.2428
97	19-May	1	1.2369	-0.0065	**1.2304**	1.2270
98	20-May	1	1.2287	-0.0074	**1.2213**	1.2334
99	21-May	1	1.2274	-0.0044	**1.2230**	1.2497
100	24-May	1	1.2363	0.0023	**1.2387**	1.2360
101	25-May	1	1.2373	0.0016	**1.2390**	1.2223
102	26-May	1	1.2306	-0.0025	**1.2281**	1.2309
103	27-May	1	1.2295	-0.0018	**1.2277**	1.2255
104	28-May	1	1.2266	-0.0024	**1.2242**	1.2384
105	31-May	1	1.2313	0.0012	**1.2325**	1.2307

8.5 Linear Regression

To forecast the month of May using a linear regression model, the author applied all sixty-three samples of exchange rate data to each day's forecast for May 2010. Since the linear regression model is capable of forecasting over a long-ranged time horizon the author felt it was not necessary to use a short-ranged time horizon, as used in the other forecasting models, and opted for a medium time horizon instead. This allowed the author to produce the linear regression forecasts for May further in advance, which allocated more time throughout the month of May to produce the forecasts using the other models.

The forecast for May 3rd, 2010 using linear regression is calculated by the following:

$$y^t = a + bx$$

b is calculated in the following manner:

$$b = \frac{\sum_{i=1}^{n} x_i y_i - (n)(\bar{x})(\bar{y})}{\sum_{i=1}^{n} x_i^2 - (n)(\bar{x})^2}$$

$$b = \frac{\sum_{i=1}^{63} x_i y_i - (63)\left(\sum_{i=1}^{63} x_i \Big/ 63\right)\left(\sum_{i=1}^{63} y_i \Big/ 63\right)}{\sum_{i=1}^{63} x_i^2 - (63)\left(\sum_{i=1}^{63} x_i \Big/ 63\right)^2}$$

$$b = \frac{(x_{63} * y_{63})...+(x_1 * y_1) - \left[63*\left(x_{63}...+x_1 \big/ 63\right)*\left(y_{63}...+y_1 \big/ 63\right)\right]}{(x_{63}^2...+x_1^2) - \left[63*\left(x_{63}...+x_1 \big/ 63\right)^2\right]}$$

$$b = \frac{(x_{March31st} * y_{March31st})...+(x_{Jan4th} * y_{Jan4th}) - \left[63*\left(x_{March31st}...+x_{Jan4th} \big/ 63\right)*\left(y_{March31st}...+y_{Jan4th} \big/ 63\right)\right]}{(x_{March31st}^2...+x_{Jan4th}^2) - \left[63*\left(x_{March31st}...+x_{Jan4th} \big/ 63\right)^2\right]}$$

$$b = \frac{(63*1.3479)...+(1*1.4389) - \left[63*\left(63...+1 \big/ 63\right)*\left(1.3479...+1.4389 \big/ 63\right)\right]}{(3969...+1) - \left[63*\left(63...+1 \big/ 63\right)^2\right]}$$

$$b = \frac{(84.9177)...+(1.4389) - \left[63*\left(2016 \big/ 63\right)*\left(87.1232 \big/ 63\right)\right]}{85344 - \left[63*\left(2016 \big/ 63\right)^2\right]}$$

$$b = \frac{2753.2859 - (63*32*1.3829)}{85344 - (63*32^2)}$$

$$b = \frac{2753.2859 - 2787.9264}{85344 - (63*1024)}$$

b = -34.6405 / (85344 – 64512)

b = -34.6405 / 20832

b = -0.001663, which rounds to -0.0017 to keep a maximum of four decimals. However, when substituting b in the equation for finding the value of a, the unrounded version is used to keep the results as accurate as possible.

Once the value of b is determined, a is then calculated as follows:

$$a = \bar{y} - b\bar{x}$$

$$a = \left(\sum_{i=1}^{63} y_i \Big/ 63 \right) - (-0.001663) * \left(\sum_{i=1}^{63} x_i \Big/ 63 \right)$$

$$a = \left(y_{63} \ldots + y_1 \big/ 63 \right) + 0.001663 * \left(x_{63} \ldots + x_1 \big/ 63 \right)$$

$$a = \left(y_{(March 31st)} \ldots + y_{(Jan 4th)} \big/ 63 \right) + 0.001663 * \left(x_{(March 31st)} \ldots + x_{(Jan 4th)} \big/ 63 \right)$$

$$a = \left(1.3479 \ldots + 1.4389 \big/ 63 \right) + 0.001663 * \left(63 \ldots + 1 \big/ 63 \right)$$

$$a = \left(87.1232 \big/ 63 \right) + 0.001663 * \left(2016 \big/ 63 \right)$$

a = 1.3829 + (0.001663 * 32)

a = 1.3829 + 0.053216

a = 1.436116, which is rounded to 1.4361 to keep a maximum of four decimals.

The values of a and b are then applied to the original formula:

$Y^{(May\ 3rd)} = a + bx$

$Y^{(May\ 3rd)}$ = 1.4361 + (-0.001663 * 85)

$Y^{(May\ 3rd)}$ = 1.4361 – 0.141355

$Y^{(May\ 3rd)}$ = 1.294745, which rounds to **1.2947** to keep a maximum of four decimals.

The following table displays the forecasts for May 2010 using linear regression.

Table 7. Forecasts for May 2010 using linear regression.

May Forecasts, Linear Regression						
Sample (*x*)	**Date**	**Euro**	***b***	***a***	**USD Forecast (y^t)**	**Actual (*y*)**
85	3-May	1	-0.0017	1.4361	**1.2947**	1.3238
86	4-May	1	-0.0017	1.4361	**1.2931**	1.3089
87	5-May	1	-0.0017	1.4361	**1.2914**	1.2924
88	6-May	1	-0.0017	1.4361	**1.2897**	1.2727
89	7-May	1	-0.0017	1.4361	**1.2881**	1.2746
90	10-May	1	-0.0017	1.4361	**1.2864**	1.2969
91	11-May	1	-0.0017	1.4361	**1.2848**	1.2698
92	12-May	1	-0.0017	1.4361	**1.2831**	1.2686
93	13-May	1	-0.0017	1.4361	**1.2814**	1.2587
94	14-May	1	-0.0017	1.4361	**1.2798**	1.2492
95	17-May	1	-0.0017	1.4361	**1.2781**	1.2349
96	18-May	1	-0.0017	1.4361	**1.2764**	1.2428
97	19-May	1	-0.0017	1.4361	**1.2748**	1.2270
98	20-May	1	-0.0017	1.4361	**1.2731**	1.2334
99	21-May	1	-0.0017	1.4361	**1.2714**	1.2497
100	24-May	1	-0.0017	1.4361	**1.2698**	1.2360
101	25-May	1	-0.0017	1.4361	**1.2681**	1.2223
102	26-May	1	-0.0017	1.4361	**1.2665**	1.2309
103	27-May	1	-0.0017	1.4361	**1.2648**	1.2255
104	28-May	1	-0.0017	1.4361	**1.2631**	1.2384
105	31-May	1	-0.0017	1.4361	**1.2615**	1.2307

9 CALCULATIONS FOR MEASURING ERROR

The following calculations measure the forecast accuracy of the forecasting models that were applied to the set of exchange rate data. The measures include the mean absolute deviation, the mean squared error, the root mean squared error, and the mean absolute percentage error. An example of each measure of forecast accuracy is calculated using the forecasts for May 2010 produced by a naive approach.

9.1 Mean Absolute Deviation

The mean absolute deviation of the forecasts using a naive approach is calculated in the following manner:

$$MAD = \frac{\sum_{i=85}^{n} |A_i - F_i|}{n}$$

$$MAD = \frac{\sum_{i=85}^{21} |A_i - F_i|}{21}$$

$MAD = (|A_{105} - F_{105}| \dots + |A_{85} - F_{85}|) / 21$

$MAD = (|A_{(\text{May 31st})} - F_{(\text{May 31st})}| \dots + |A_{(\text{May 3rd})} - F_{(\text{May 3rd})}|) / 21$

$MAD = (|1.2307 - 1.2384| \dots + |1.3238 - 1.3479|) / 21$

$MAD = (|-0.0077| \dots + |-0.0241|) / 21$

$MAD = (0.0077 \dots + 0.0241) / 21$

$MAD = 0.2698 / 21$

$MAD = 0.012847$, which is rounded to **0.0128** to keep a maximum of four decimals.

The following tables display the mean absolute deviations of the forecasts for May 2010 using a naive approach, a moving average, exponential smoothing, exponential smoothing with trend adjustment, and linear regression.

Table 8. Mean absolute deviation for the forecasts using a naive approach.

MAD For Naive Approach							
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**Σ|*A-F*|**	***n***	***MAD***
85	3-May	1.3238	1.3479	0.0241			
86	4-May	1.3089	1.3238	0.0149			
87	5-May	1.2924	1.3089	0.0165			
88	6-May	1.2727	1.2924	0.0197			
89	7-May	1.2746	1.2727	0.0019			
90	10-May	1.2969	1.2746	0.0223			
91	11-May	1.2698	1.2969	0.0271			
92	12-May	1.2686	1.2698	0.0012			
93	13-May	1.2587	1.2686	0.0099			
94	14-May	1.2492	1.2587	0.0095			
95	17-May	1.2349	1.2492	0.0143			
96	18-May	1.2428	1.2349	0.0079			
97	19-May	1.2270	1.2428	0.0158			
98	20-May	1.2334	1.2270	0.0064			
99	21-May	1.2497	1.2334	0.0163			
100	24-May	1.2360	1.2497	0.0137			
101	25-May	1.2223	1.2360	0.0137			
102	26-May	1.2309	1.2223	0.0086			
103	27-May	1.2255	1.2309	0.0054			
104	28-May	1.2384	1.2255	0.0129			
105	31-May	1.2307	1.2384	0.0077			
					0.2698	21	**0.0128**

Table 9. Mean absolute deviation for the forecasts using a moving average.

MAD For Moving Average							
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**Σ|*A-F*|**	***n***	***MAD***
85	3-May	1.3238	1.3428	0.0190			
86	4-May	1.3089	1.3388	0.0299			
87	5-May	1.2924	1.3353	0.0429			
88	6-May	1.2727	1.3291	0.0564			
89	7-May	1.2746	1.3201	0.0455			
90	10-May	1.2969	1.3098	0.0129			
91	11-May	1.2698	1.3025	0.0327			
92	12-May	1.2686	1.2913	0.0227			
93	13-May	1.2587	1.2834	0.0247			
94	14-May	1.2492	1.2762	0.0270			
95	17-May	1.2349	1.2701	0.0352			
96	18-May	1.2428	1.2647	0.0219			

97	19-May	1.2270	1.2601	0.0331			
98	20-May	1.2334	1.2501	0.0167			
99	21-May	1.2497	1.2449	0.0048			
100	24-May	1.2360	1.2422	0.0062			
101	25-May	1.2223	1.2390	0.0167			
102	26-May	1.2309	1.2352	0.0043			
103	27-May	1.2255	1.2346	0.0091			
104	28-May	1.2384	1.2321	0.0063			
105	31-May	1.2307	1.2337	0.0030			
					0.4710	21	**0.0224**

Table 10. Mean absolute deviation for the forecasts using exponential smoothing.

MAD For Exponential Smoothing							
α **= 0.9**							
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**Σ|*A-F*|**	***n***	***MAD***
85	3-May	1.3238	1.3479	0.0241			
86	4-May	1.3089	1.3262	0.0173			
87	5-May	1.2924	1.3106	0.0182			
88	6-May	1.2727	1.2942	0.0215			
89	7-May	1.2746	1.2749	0.0003			
90	10-May	1.2969	1.2746	0.0223			
91	11-May	1.2698	1.2947	0.0249			
92	12-May	1.2686	1.2723	0.0037			
93	13-May	1.2587	1.2690	0.0103			
94	14-May	1.2492	1.2597	0.0105			
95	17-May	1.2349	1.2503	0.0154			
96	18-May	1.2428	1.2364	0.0064			
97	19-May	1.2270	1.2422	0.0152			
98	20-May	1.2334	1.2285	0.0049			
99	21-May	1.2497	1.2329	0.0168			
100	24-May	1.2360	1.2480	0.0120			
101	25-May	1.2223	1.2372	0.0149			
102	26-May	1.2309	1.2238	0.0071			
103	27-May	1.2255	1.2302	0.0047			
104	28-May	1.2384	1.2260	0.0124			
105	31-May	1.2307	1.2372	0.0065			
					0.2692	21	**0.0128**

Table 11. Mean absolute deviation for the forecasts using exponential smoothing with trend adjustment.

MAD For Trend Adjustment							
α = 0.5 **β = 0.5**							
Sample	**Date**	**Actual (*A*)**	***FIT* (*F*)**	**|*A-F*|**	**Σ|*A-F*|**	***n***	***MAD***
85	3-May	1.3238	1.3492	0.0254			
86	4-May	1.3089	1.3309	0.0220			
87	5-May	1.2924	1.3087	0.0163			
88	6-May	1.2727	1.2853	0.0126			
89	7-May	1.2746	1.2606	0.0140			
90	10-May	1.2969	1.2527	0.0442			
91	11-May	1.2698	1.2710	0.0012			
92	12-May	1.2686	1.2662	0.0024			
93	13-May	1.2587	1.2639	0.0052			
94	14-May	1.2492	1.2564	0.0072			
95	17-May	1.2349	1.2462	0.0113			
96	18-May	1.2428	1.2311	0.0117			
97	19-May	1.2270	1.2304	0.0034			
98	20-May	1.2334	1.2213	0.0121			
99	21-May	1.2497	1.2230	0.0267			
100	24-May	1.2360	1.2387	0.0027			
101	25-May	1.2223	1.2390	0.0167			
102	26-May	1.2309	1.2281	0.0028			
103	27-May	1.2255	1.2277	0.0022			
104	28-May	1.2384	1.2242	0.0142			
105	31-May	1.2307	1.2325	0.0018			
					0.2559	21	**0.0122**

Table 12. Mean absolute deviation for the forecasts using linear regression.

MAD For Linear Regression							
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**Σ|*A-F*|**	***n***	***MAD***
85	3-May	1.3238	1.2947	0.0291			
86	4-May	1.3089	1.2931	0.0158			
87	5-May	1.2924	1.2914	0.0010			
88	6-May	1.2727	1.2897	0.0170			
89	7-May	1.2746	1.2881	0.0135			
90	10-May	1.2969	1.2864	0.0105			
91	11-May	1.2698	1.2848	0.0150			
92	12-May	1.2686	1.2831	0.0145			
93	13-May	1.2587	1.2814	0.0227			

94	14-May	1.2492	1.2798	0.0306			
95	17-May	1.2349	1.2781	0.0432			
96	18-May	1.2428	1.2764	0.0336			
97	19-May	1.2270	1.2748	0.0478			
98	20-May	1.2334	1.2731	0.0397			
99	21-May	1.2497	1.2714	0.0217			
100	24-May	1.2360	1.2698	0.0338			
101	25-May	1.2223	1.2681	0.0458			
102	26-May	1.2309	1.2665	0.0356			
103	27-May	1.2255	1.2648	0.0393			
104	28-May	1.2384	1.2631	0.0247			
105	31-May	1.2307	1.2615	0.0308			
					0.5656	21	**0.0269**

9.2 Mean Squared Error

The mean squared error of the forecasts using a naive approach is calculated as follows:

$$MSE = \frac{\sum_{i=85}^{n}\left(|A_i - F_i|\right)^2}{n}$$

$$MSE = \frac{\sum_{i=85}^{21}\left(|A_i - F_i|\right)^2}{21}$$

$MSE = (|A_{105} - F_{105}|^2 \ldots + |A_{85} - F_{85}|^2) / 21$

$MSE = (|A_{(\text{May 31st})} - F_{(\text{May 31st})}|^2 \ldots + |A_{(\text{May 3rd})} - F_{(\text{May 3rd})}|^2) / 21$

$MSE = (|1.2307 - 1.2384|^2 \ldots + |1.3238 - 1.3479|^2) / 21$

$MSE = (|-0.0077|^2 \ldots + |-0.0241|^2) / 21$

$MSE = [(0.0077)^2 \ldots + (0.0241)^2] / 21$

$MSE = (0.00005 \ldots + 0.00058) / 21$

$MSE = 0.0044 / 21$

$MSE = \mathbf{0.0002}$

The following tables display the mean squared errors of the forecasts for May 2010 using a naive approach, a moving average, exponential smoothing, exponential smoothing with trend adjustment, and linear regression.

Table 13. Mean squared error for the forecasts using a naive approach.

MSE For Naive Approach								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**$\|A\text{-}F\|^2$**	**$\Sigma\|A\text{-}F\|^2$**	***n***	***MSE***
85	3-May	1.3238	1.3479	0.0241	0.0006			
86	4-May	1.3089	1.3238	0.0149	0.0002			
87	5-May	1.2924	1.3089	0.0165	0.0003			
88	6-May	1.2727	1.2924	0.0197	0.0004			
89	7-May	1.2746	1.2727	0.0019	0.0000			
90	10-May	1.2969	1.2746	0.0223	0.0005			
91	11-May	1.2698	1.2969	0.0271	0.0007			
92	12-May	1.2686	1.2698	0.0012	0.0000			
93	13-May	1.2587	1.2686	0.0099	0.0001			
94	14-May	1.2492	1.2587	0.0095	0.0001			
95	17-May	1.2349	1.2492	0.0143	0.0002			
96	18-May	1.2428	1.2349	0.0079	0.0001			
97	19-May	1.2270	1.2428	0.0158	0.0002			
98	20-May	1.2334	1.2270	0.0064	0.0000			
99	21-May	1.2497	1.2334	0.0163	0.0003			
100	24-May	1.2360	1.2497	0.0137	0.0002			
101	25-May	1.2223	1.2360	0.0137	0.0002			
102	26-May	1.2309	1.2223	0.0086	0.0001			
103	27-May	1.2255	1.2309	0.0054	0.0000			
104	28-May	1.2384	1.2255	0.0129	0.0002			
105	31-May	1.2307	1.2384	0.0077	0.0001			
						0.0044	21	**0.0002**

Table 14. Mean squared error for the forecasts using a moving average.

MSE For Moving Average								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**$\|A\text{-}F\|^2$**	**$\Sigma\|A\text{-}F\|^2$**	***n***	***MSE***
85	3-May	1.3238	1.3428	0.0190	0.0004			
86	4-May	1.3089	1.3388	0.0299	0.0009			
87	5-May	1.2924	1.3353	0.0429	0.0018			
88	6-May	1.2727	1.3291	0.0564	0.0032			
89	7-May	1.2746	1.3201	0.0455	0.0021			
90	10-May	1.2969	1.3098	0.0129	0.0002			
91	11-May	1.2698	1.3025	0.0327	0.0011			
92	12-May	1.2686	1.2913	0.0227	0.0005			
93	13-May	1.2587	1.2834	0.0247	0.0006			
94	14-May	1.2492	1.2762	0.0270	0.0007			
95	17-May	1.2349	1.2701	0.0352	0.0012			
96	18-May	1.2428	1.2647	0.0219	0.0005			
97	19-May	1.2270	1.2601	0.0331	0.0011			
98	20-May	1.2334	1.2501	0.0167	0.0003			

99	21-May	1.2497	1.2449	0.0048	0.0000			
100	24-May	1.2360	1.2422	0.0062	0.0000			
101	25-May	1.2223	1.2390	0.0167	0.0003			
102	26-May	1.2309	1.2352	0.0043	0.0000			
103	27-May	1.2255	1.2346	0.0091	0.0001			
104	28-May	1.2384	1.2321	0.0063	0.0000			
105	31-May	1.2307	1.2337	0.0030	0.0000			
						0.0150	21	**0.0007**

Table 15. Mean squared error for the forecasts using exponential smoothing.

MSE For Exponential Smoothing								
α = 0.9								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|A-F|***	***|A-F|*^{2}**	**Σ*|A-F|*^{2}**	***n***	***MSE***
85	3-May	1.3238	1.3479	0.0241	0.0006			
86	4-May	1.3089	1.3262	0.0173	0.0003			
87	5-May	1.2924	1.3106	0.0182	0.0003			
88	6-May	1.2727	1.2942	0.0215	0.0005			
89	7-May	1.2746	1.2749	0.0003	0.0000			
90	10-May	1.2969	1.2746	0.0223	0.0005			
91	11-May	1.2698	1.2947	0.0249	0.0006			
92	12-May	1.2686	1.2723	0.0037	0.0000			
93	13-May	1.2587	1.2690	0.0103	0.0001			
94	14-May	1.2492	1.2597	0.0105	0.0001			
95	17-May	1.2349	1.2503	0.0154	0.0002			
96	18-May	1.2428	1.2364	0.0064	0.0000			
97	19-May	1.2270	1.2422	0.0152	0.0002			
98	20-May	1.2334	1.2285	0.0049	0.0000			
99	21-May	1.2497	1.2329	0.0168	0.0003			
100	24-May	1.2360	1.2480	0.0120	0.0001			
101	25-May	1.2223	1.2372	0.0149	0.0002			
102	26-May	1.2309	1.2238	0.0071	0.0001			
103	27-May	1.2255	1.2302	0.0047	0.0000			
104	28-May	1.2384	1.2260	0.0124	0.0002			
105	31-May	1.2307	1.2372	0.0065	0.0000			
						0.0045	21	**0.0002**

Table 16. Mean squared error for the forecasts using exponential smoothing with trend adjustment.

MSE For Trend Adjustment								
α = 0.5 β = 0.5								
Sample	**Date**	**Actual (*A*)**	***FIT* (*F*)**	**|*A-F*|**	**|*A-F*|^2**	**Σ|*A-F*|^2**	***n***	***MSE***
85	3-May	1.3238	1.3492	0.0247	0.0006			
86	4-May	1.3089	1.3309	0.0276	0.0008			
87	5-May	1.2924	1.3087	0.0275	0.0008			
88	6-May	1.2727	1.2853	0.0279	0.0008			
89	7-May	1.2746	1.2606	0.0044	0.0000			
90	10-May	1.2969	1.2527	0.0293	0.0009			
91	11-May	1.2698	1.2710	0.0050	0.0000			
92	12-May	1.2686	1.2662	0.0018	0.0000			
93	13-May	1.2587	1.2639	0.0087	0.0001			
94	14-May	1.2492	1.2564	0.0121	0.0001			
95	17-May	1.2349	1.2462	0.0179	0.0003			
96	18-May	1.2428	1.2311	0.0023	0.0000			
97	19-May	1.2270	1.2304	0.0099	0.0001			
98	20-May	1.2334	1.2213	0.0047	0.0000			
99	21-May	1.2497	1.2230	0.0223	0.0005			
100	24-May	1.2360	1.2387	0.0003	0.0000			
101	25-May	1.2223	1.2390	0.0150	0.0002			
102	26-May	1.2309	1.2281	0.0003	0.0000			
103	27-May	1.2255	1.2277	0.0040	0.0000			
104	28-May	1.2384	1.2242	0.0118	0.0001			
105	31-May	1.2307	1.2325	0.0006	0.0000			
						0.0054	21	**0.0003**

Table 17. Mean squared error for the forecasts using linear regression.

MSE For Linear Regression								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**|*A-F*|^2**	**Σ|*A-F*|^2**	***n***	***MSE***
85	3-May	1.3238	1.2947	0.0291	0.0008			
86	4-May	1.3089	1.2931	0.0158	0.0003			
87	5-May	1.2924	1.2914	0.0010	0.0000			
88	6-May	1.2727	1.2897	0.0170	0.0003			
89	7-May	1.2746	1.2881	0.0135	0.0002			
90	10-May	1.2969	1.2864	0.0105	0.0001			
91	11-May	1.2698	1.2848	0.0150	0.0002			
92	12-May	1.2686	1.2831	0.0145	0.0002			
93	13-May	1.2587	1.2814	0.0227	0.0005			
94	14-May	1.2492	1.2798	0.0306	0.0009			

95	17-May	1.2349	1.2781	0.0432	0.0019			
96	18-May	1.2428	1.2764	0.0336	0.0011			
97	19-May	1.2270	1.2748	0.0478	0.0023			
98	20-May	1.2334	1.2731	0.0397	0.0016			
99	21-May	1.2497	1.2714	0.0217	0.0005			
100	24-May	1.2360	1.2698	0.0338	0.0011			
101	25-May	1.2223	1.2681	0.0458	0.0021			
102	26-May	1.2309	1.2665	0.0356	0.0013			
103	27-May	1.2255	1.2648	0.0393	0.0015			
104	28-May	1.2384	1.2631	0.0247	0.0006			
105	31-May	1.2307	1.2615	0.0308	0.0009			
						0.0185	21	**0.0009**

9.3 Root Mean Squared Error

The root mean squared error of the forecasts using a naive approach is calculated as follows:

$$RMSE = \sqrt{\frac{\sum_{i=85}^{n}\left(\left|A_i - F_i\right|\right)^2}{n}}$$

$$RMSE = \sqrt{\frac{\sum_{i=85}^{21}\left(\left|A_i - F_i\right|\right)^2}{21}}$$

$$RMSE = \sqrt{\frac{\left|A_{105} - F_{105}\right|^2 \ldots + \left|A_{85} - F_{85}\right|^2}{21}}$$

$$RMSE = \sqrt{\frac{\left|A_{(May31st)} - F_{(May31st)}\right|^2 \ldots + \left|A_{(May3rd)} - F_{(May3rd)}\right|^2}{21}}$$

$$RMSE = \sqrt{\frac{\left|1.2307 - 1.2384\right|^2 \ldots + \left|1.3238 - 1.3479\right|^2}{21}}$$

$$RMSE = \sqrt{\frac{\left|-0.0077\right|^2 \ldots + \left|-0.0241\right|^2}{21}}$$

$$RMSE = \sqrt{\frac{(0.0077)^2 \ldots + (0.0241)^2}{21}}$$

$RMSE = \sqrt{[(0.00005 \ldots + 0.00058) / 21]}$

$RMSE = \sqrt{(0.0044 / 21)}$

$RMSE = \sqrt{0.0002}$

$RMSE = 0.0145$

The following tables display the root mean squared errors of the forecasts for May 2010 using a naive approach, a moving average, exponential smoothing, exponential smoothing with trend adjustment, and linear regression.

Table 18. Root mean squared error for the forecasts using a naive approach.

RMSE For Naive Approach									
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	**|*A-F*|**	**|*A-F*|^2**	**Σ|*A-F*|^2**	***n***	***MSE***	***RMSE***
85	3-May	1.3238	1.3479	0.0241	0.0006				
86	4-May	1.3089	1.3238	0.0149	0.0002				
87	5-May	1.2924	1.3089	0.0165	0.0003				
88	6-May	1.2727	1.2924	0.0197	0.0004				
89	7-May	1.2746	1.2727	0.0019	0.0000				
90	10-May	1.2969	1.2746	0.0223	0.0005				
91	11-May	1.2698	1.2969	0.0271	0.0007				
92	12-May	1.2686	1.2698	0.0012	0.0000				
93	13-May	1.2587	1.2686	0.0099	0.0001				
94	14-May	1.2492	1.2587	0.0095	0.0001				
95	17-May	1.2349	1.2492	0.0143	0.0002				
96	18-May	1.2428	1.2349	0.0079	0.0001				
97	19-May	1.2270	1.2428	0.0158	0.0002				
98	20-May	1.2334	1.2270	0.0064	0.0000				
99	21-May	1.2497	1.2334	0.0163	0.0003				
100	24-May	1.2360	1.2497	0.0137	0.0002				
101	25-May	1.2223	1.2360	0.0137	0.0002				
102	26-May	1.2309	1.2223	0.0086	0.0001				
103	27-May	1.2255	1.2309	0.0054	0.0000				
104	28-May	1.2384	1.2255	0.0129	0.0002				
105	31-May	1.2307	1.2384	0.0077	0.0001				
						0.0044	21	0.0002	**0.0145**

Table 19. Root mean squared error for the forecasts using a moving average.

RMSE For Moving Average									
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|A-F|***	***|A-F|*** ^{2}	Σ***|A-F|*** ^{2}	***n***	***MSE***	***RMSE***
85	3-May	1.3238	1.3428	0.0190	0.0004				
86	4-May	1.3089	1.3388	0.0299	0.0009				
87	5-May	1.2924	1.3353	0.0429	0.0018				
88	6-May	1.2727	1.3291	0.0564	0.0032				
89	7-May	1.2746	1.3201	0.0455	0.0021				
90	10-May	1.2969	1.3098	0.0129	0.0002				
91	11-May	1.2698	1.3025	0.0327	0.0011				
92	12-May	1.2686	1.2913	0.0227	0.0005				
93	13-May	1.2587	1.2834	0.0247	0.0006				
94	14-May	1.2492	1.2762	0.0270	0.0007				
95	17-May	1.2349	1.2701	0.0352	0.0012				
96	18-May	1.2428	1.2647	0.0219	0.0005				
97	19-May	1.2270	1.2601	0.0331	0.0011				
98	20-May	1.2334	1.2501	0.0167	0.0003				
99	21-May	1.2497	1.2449	0.0048	0.0000				
100	24-May	1.2360	1.2422	0.0062	0.0000				
101	25-May	1.2223	1.2390	0.0167	0.0003				
102	26-May	1.2309	1.2352	0.0043	0.0000				
103	27-May	1.2255	1.2346	0.0091	0.0001				
104	28-May	1.2384	1.2321	0.0063	0.0000				
105	31-May	1.2307	1.2337	0.0030	0.0000				
						0.0150	21	0.0007	**0.0267**

Table 20. Root mean squared error for the forecasts using exponential smoothing.

RMSE For Exponential Smoothing									
α **= 0.9**									
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|A-F|***	***|A-F|*** ^{2}	Σ***|A-F|*** ^{2}	***n***	***MSE***	***RMSE***
85	3-May	1.3238	1.3479	0.0241	0.0006				
86	4-May	1.3089	1.3262	0.0173	0.0003				
87	5-May	1.2924	1.3106	0.0182	0.0003				
88	6-May	1.2727	1.2942	0.0215	0.0005				
89	7-May	1.2746	1.2749	0.0003	0.0000				
90	10-May	1.2969	1.2746	0.0223	0.0005				
91	11-May	1.2698	1.2947	0.0249	0.0006				
92	12-May	1.2686	1.2723	0.0037	0.0000				

93	13-May	1.2587	1.2690	0.0103	0.0001				
94	14-May	1.2492	1.2597	0.0105	0.0001				
95	17-May	1.2349	1.2503	0.0154	0.0002				
96	18-May	1.2428	1.2364	0.0064	0.0000				
97	19-May	1.2270	1.2422	0.0152	0.0002				
98	20-May	1.2334	1.2285	0.0049	0.0000				
99	21-May	1.2497	1.2329	0.0168	0.0003				
100	24-May	1.2360	1.2480	0.0120	0.0001				
101	25-May	1.2223	1.2372	0.0149	0.0002				
102	26-May	1.2309	1.2238	0.0071	0.0001				
103	27-May	1.2255	1.2302	0.0047	0.0000				
104	28-May	1.2384	1.2260	0.0124	0.0002				
105	31-May	1.2307	1.2372	0.0065	0.0000				
						0.0045	21	0.0002	**0.0146**

Table 21. Root mean squared error for the forecasts using exponential smoothing with trend adjustment.

RMSE For Trend Projection									
α **= 0.5** β **= 0.5**									
Sample	**Date**	**Actual (*A*)**	***FIT* (*F*)**	***|A-F|***	***|A-F|***^{2}	Σ***|A-F|***^{2}	***n***	***MSE***	***RMSE***
85	3-May	1.3238	1.3492	0.0254	0.0006				
86	4-May	1.3089	1.3309	0.0220	0.0005				
87	5-May	1.2924	1.3087	0.0163	0.0003				
88	6-May	1.2727	1.2853	0.0126	0.0002				
89	7-May	1.2746	1.2606	0.0140	0.0002				
90	10-May	1.2969	1.2527	0.0442	0.0020				
91	11-May	1.2698	1.2710	0.0012	0.0000				
92	12-May	1.2686	1.2662	0.0024	0.0000				
93	13-May	1.2587	1.2639	0.0052	0.0000				
94	14-May	1.2492	1.2564	0.0072	0.0001				
95	17-May	1.2349	1.2462	0.0113	0.0001				
96	18-May	1.2428	1.2311	0.0117	0.0001				
97	19-May	1.2270	1.2304	0.0034	0.0000				
98	20-May	1.2334	1.2213	0.0121	0.0001				
99	21-May	1.2497	1.2230	0.0267	0.0007				
100	24-May	1.2360	1.2387	0.0027	0.0000				
101	25-May	1.2223	1.2390	0.0167	0.0003				
102	26-May	1.2309	1.2281	0.0028	0.0000				
103	27-May	1.2255	1.2277	0.0022	0.0000				
104	28-May	1.2384	1.2242	0.0142	0.0002				
105	31-May	1.2307	1.2325	0.0018	0.0000				
						0.0054	21	0.0003	**0.0161**

Table 22. Root mean squared error for the forecasts using linear regression.

RMSE For Linear Regression									
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|A-F|***	***|A-F|²***	***Σ|A-F|²***	***n***	***MSE***	***RMSE***
85	3-May	1.3238	1.2947	0.0291	0.0008				
86	4-May	1.3089	1.2931	0.0158	0.0003				
87	5-May	1.2924	1.2914	0.0010	0.0000				
88	6-May	1.2727	1.2897	0.0170	0.0003				
89	7-May	1.2746	1.2881	0.0135	0.0002				
90	10-May	1.2969	1.2864	0.0105	0.0001				
91	11-May	1.2698	1.2848	0.0150	0.0002				
92	12-May	1.2686	1.2831	0.0145	0.0002				
93	13-May	1.2587	1.2814	0.0227	0.0005				
94	14-May	1.2492	1.2798	0.0306	0.0009				
95	17-May	1.2349	1.2781	0.0432	0.0019				
96	18-May	1.2428	1.2764	0.0336	0.0011				
97	19-May	1.2270	1.2748	0.0478	0.0023				
98	20-May	1.2334	1.2731	0.0397	0.0016				
99	21-May	1.2497	1.2714	0.0217	0.0005				
100	24-May	1.2360	1.2698	0.0338	0.0011				
101	25-May	1.2223	1.2681	0.0458	0.0021				
102	26-May	1.2309	1.2665	0.0356	0.0013				
103	27-May	1.2255	1.2648	0.0393	0.0015				
104	28-May	1.2384	1.2631	0.0247	0.0006				
105	31-May	1.2307	1.2615	0.0308	0.0009				
						0.0185	21	0.0009	**0.0297**

9.4 Mean Absolute Percentage Error

The mean absolute percentage error of the forecasts using a naive approach is calculated in the following manner:

$$MAPE = \frac{100 * \left(\sum_{i=85}^{n} \frac{|A_i - F_i|}{A_i} \right)}{n}$$

$$MAPE = \frac{100 * \left(\sum_{i=85}^{21} \frac{|A_i - F_i|}{A_i} \right)}{21}$$

$$MAPE = \frac{100 * \left(\frac{|A_{105} - F_{105}|}{A_{105}} ... + \frac{|A_{85} - F_{85}|}{A_{85}} \right)}{21}$$

$$MAPE = \frac{100 * \left(\frac{|A_{(May31st)} - F_{(May31st)}|}{A_{(May31st)}} ... + \frac{|A_{(May3rd)} - F_{(May3rd)}|}{A_{(May3rd)}} \right)}{21}$$

$$MAPE = \frac{100 * \left(\frac{|1.2307 - 1.2384|}{1.2307} ... + \frac{|1.3238 - 1.3479|}{1.3238} \right)}{21}$$

$$MAPE = \frac{100 * \left(\frac{|-0.0077|}{1.2307} ... + \frac{|-0.0241|}{1.3238} \right)}{21}$$

$$MAPE = \frac{100 * \left(\frac{0.0077}{1.2307} ... + \frac{0.0241}{1.3238} \right)}{21}$$

$MAPE$ = [100 * (0.00625 … + 0.0182)] / 21

$MAPE$ = (100 * 0.213) / 21

$MAPE$ = 21.368 / 21

$MAPE$ = 1.0175

The following tables display the mean absolute percentage errors of the forecasts for May 2010 using a naive approach, a moving average, exponential smoothing, exponential smoothing with trend adjustment, and linear regression.

Table 23. Mean absolute percentage error for the forecasts using a naive approach.

MAPE For Naive Approach								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|(A-F)/A|***	***Σ|(A-F)/A|***	***100 * Σ|(A-F)/A|***	***n***	***MAPE***
85	3-May							
86	4-May	1.3238	1.3479	0.0182				
87	5-May	1.3089	1.3238	0.0114				
88	6-May	1.2924	1.3089	0.0128				
89	7-May	1.2727	1.2924	0.0155				
90	10-May	1.2746	1.2727	0.0015				

91	11-May	1.2969	1.2746	0.0172				
92	12-May	1.2698	1.2969	0.0213				
93	13-May	1.2686	1.2698	0.0009				
94	14-May	1.2587	1.2686	0.0079				
95	17-May	1.2492	1.2587	0.0076				
96	18-May	1.2349	1.2492	0.0116				
97	19-May	1.2428	1.2349	0.0064				
98	20-May	1.2270	1.2428	0.0129				
99	21-May	1.2334	1.2270	0.0052				
100	24-May	1.2497	1.2334	0.0130				
101	25-May	1.2360	1.2497	0.0111				
102	26-May	1.2223	1.2360	0.0112				
103	27-May	1.2309	1.2223	0.0070				
104	28-May	1.2255	1.2309	0.0044				
105	31-May	1.2384	1.2255	0.0104				
		1.2307	1.2384	0.0063				
					0.2137	21.3683	21	**1.0175**

Table 24. Mean absolute percentage error for the forecasts using a moving average.

MAPE For Moving Average								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|(A-F)/A|***	***Σ|(A-F)/A|***	***100 * Σ|(A-F)/A|***	***n***	***MAPE***
85	3-May	1.3238	1.3428	0.0144				
86	4-May	1.3089	1.3388	0.0229				
87	5-May	1.2924	1.3353	0.0332				
88	6-May	1.2727	1.3291	0.0443				
89	7-May	1.2746	1.3201	0.0357				
90	10-May	1.2969	1.3098	0.0099				
91	11-May	1.2698	1.3025	0.0257				
92	12-May	1.2686	1.2913	0.0179				
93	13-May	1.2587	1.2834	0.0196				
94	14-May	1.2492	1.2762	0.0216				
95	17-May	1.2349	1.2701	0.0285				
96	18-May	1.2428	1.2647	0.0176				
97	19-May	1.2270	1.2601	0.0270				
98	20-May	1.2334	1.2501	0.0136				
99	21-May	1.2497	1.2449	0.0038				
100	24-May	1.2360	1.2422	0.0051				
101	25-May	1.2223	1.2390	0.0137				
102	26-May	1.2309	1.2352	0.0035				
103	27-May	1.2255	1.2346	0.0074				
104	28-May	1.2384	1.2321	0.0051				
105	31-May	1.2307	1.2337	0.0025				
					0.3729	37.2850	21	**1.7755**

Table 25. Mean absolute percentage error for the forecasts using exponential smoothing.

MAPE For Exponential Smoothing								
α = 0.9								
Sample	**Date**	**Actual (*A*)**	**USD Forecast (*F*)**	***|(A-F)/A|***	***Σ|(A-F)/A|***	***100 * Σ|(A-F)/A|***	***n***	***MAPE***
85	3-May	1.3238	1.3479	0.0182				
86	4-May	1.3089	1.3262	0.0132				
87	5-May	1.2924	1.3106	0.0141				
88	6-May	1.2727	1.2942	0.0169				
89	7-May	1.2746	1.2749	0.0002				
90	10-May	1.2969	1.2746	0.0172				
91	11-May	1.2698	1.2947	0.0196				
92	12-May	1.2686	1.2723	0.0029				
93	13-May	1.2587	1.2690	0.0082				
94	14-May	1.2492	1.2597	0.0084				
95	17-May	1.2349	1.2503	0.0124				
96	18-May	1.2428	1.2364	0.0051				
97	19-May	1.2270	1.2422	0.0124				
98	20-May	1.2334	1.2285	0.0040				
99	21-May	1.2497	1.2329	0.0134				
100	24-May	1.2360	1.2480	0.0097				
101	25-May	1.2223	1.2372	0.0122				
102	26-May	1.2309	1.2238	0.0058				
103	27-May	1.2255	1.2302	0.0038				
104	28-May	1.2384	1.2260	0.0100				
105	31-May	1.2307	1.2372	0.0052				
					0.2130	21.3026	21	**1.0144**

Table 26. Mean absolute percentage error for the forecasts using exponential smoothing with trend adjustment.

MAPE For Trend Adjustment								
α = 0.5 **β = 0.5**								
Sample	**Date**	**Actual (*A*)**	***FIT (F)***	***|(A-F)/A|***	***Σ|(A-F)/A|***	***100 * Σ|(A-F)/A|***	***n***	***MAPE***
85	3-May	1.3238	1.3492	0.0192				
86	4-May	1.3089	1.3309	0.0168				
87	5-May	1.2924	1.3087	0.0126				
88	6-May	1.2727	1.2853	0.0099				
89	7-May	1.2746	1.2606	0.0110				

90	10-May	1.2969	1.2527	0.0341				
91	11-May	1.2698	1.2710	0.0009				
92	12-May	1.2686	1.2662	0.0019				
93	13-May	1.2587	1.2639	0.0041				
94	14-May	1.2492	1.2564	0.0058				
95	17-May	1.2349	1.2462	0.0091				
96	18-May	1.2428	1.2311	0.0094				
97	19-May	1.2270	1.2304	0.0028				
98	20-May	1.2334	1.2213	0.0098				
99	21-May	1.2497	1.2230	0.0214				
100	24-May	1.2360	1.2387	0.0022				
101	25-May	1.2223	1.2390	0.0136				
102	26-May	1.2309	1.2281	0.0023				
103	27-May	1.2255	1.2277	0.0018				
104	28-May	1.2384	1.2242	0.0115				
105	31-May	1.2307	1.2325	0.0014				
					0.2015	20.1524	21	**0.9596**

Table 27. Mean absolute percentage error for the forecasts using linear regression.

MAPE For Linear Regression								
Sample	**Date**	**Actual (A)**	**USD Forecast (F)**	**|(A-F)/A|**	**Σ|(A-F)/A|**	**100 * Σ|(A-F)/A|**	**n**	**MAPE**
85	3-May	1.3238	1.2947	0.0220				
86	4-May	1.3089	1.2931	0.0121				
87	5-May	1.2924	1.2914	0.0008				
88	6-May	1.2727	1.2897	0.0134				
89	7-May	1.2746	1.2881	0.0106				
90	10-May	1.2969	1.2864	0.0081				
91	11-May	1.2698	1.2848	0.0118				
92	12-May	1.2686	1.2831	0.0114				
93	13-May	1.2587	1.2814	0.0181				
94	14-May	1.2492	1.2798	0.0245				
95	17-May	1.2349	1.2781	0.0350				
96	18-May	1.2428	1.2764	0.0271				
97	19-May	1.2270	1.2748	0.0389				
98	20-May	1.2334	1.2731	0.0322				
99	21-May	1.2497	1.2714	0.0174				
100	24-May	1.2360	1.2698	0.0273				
101	25-May	1.2223	1.2681	0.0375				
102	26-May	1.2309	1.2665	0.0289				
103	27-May	1.2255	1.2648	0.0321				
104	28-May	1.2384	1.2631	0.0200				
105	31-May	1.2307	1.2615	0.0250				
					0.4539	45.3894	21	**2.1614**

10 RESULTS

Throughout the tables of chapter eight, it is visible that the forecasts for the month of May are often very close to their actual exchange rate values. Though the forecasts are not entirely accurate when compared to the actual values, the forecasted exchange rates contain very little error. Since a small amount of error is anticipated in forecasting, the forecasts can be considered successful. Therefore, the hypothesis that future exchange rates can be determined with the aid of a mathematical forecasting model is accepted.

According to the various measures of forecast accuracy, exponential smoothing with trend adjustment proved to be the best model out of the five chosen forecasting models, as the forecasts provided with the trend adjustment model contained the least amount of error. This is visible in the results of the measures of the mean absolute percentage errors that are displayed in the tables of chapter nine. The MAPE for exponential smoothing with trend adjustment is 0.9596%, while the naive approach is 1.0175%, the moving average is 1.7755%, exponential smoothing is 1.0144%, and linear regression is 2.1614%. With such a small measure of MAPE, it can be assumed that the forecasts provided by the trend adjustment model are the closest to the actual exchange rates for May 2010.

It is noticeable that the results of the various measures of forecast accuracy for exponential smoothing and exponential smoothing with trend adjustment are very close to each other. Moreover, the measures of the mean squared error and the root mean squared error for exponential smoothing are slightly less than for exponential smoothing with trend adjustment. This indicates that the trend adjustment model produced a few forecasts with a greater error, compared to the exponential smoothing model. Though this may be the case, the trend adjustment model is still chosen as the best forecasting model since the MAPE shows that the forecasts for exponential smoothing with trend adjustment had a greater amount of forecasts measuring closer to the actual exchange rate values.

Even though the exponential smoothing with trend adjustment model produced very pleasing results, a large element of the forecasting model's accuracy has been influenced by the smoothing constants, as the values chosen for the smoothing constants are crucial to the accuracy of the forecasts for exchange rates. Since the measures of the MSE and RMSE indicate that the exponential smoothing with trend adjustment model produced a few forecasts containing errors greater than on average, it is possible that the smoothing constants could be even further adjusted.

While the exponential smoothing with trend adjustment model provided the most accurate forecasts, the linear regression model's forecasts were the least accurate. This is visible throughout all of the measures of forecast accuracy. Since the sample size for the linear regression model included sixty-three samples, in contrast to the maximum of seven samples included in the other forecasting models, it is likely that the large amount of historical data affected the accuracy of the exchange rate forecasts using linear regression. Furthermore, since the linear regression model is the only forecasting model that forecasts by fitting the data to a straight line, it can be concluded that exchange rates rarely follow a linear trend, which supports the theory that exchange rates are independent of each other. Though this can be concluded for large sample sizes, the other four models show evidence that when the exchange rate's environment remains constant it is possible for exchange rates to be successfully forecasted with the use of smaller sample sizes.

11 CONCLUSION

The subject matter of this thesis largely relates to the theory of whether or not the daily exchange rates of the European Euro, valued in United States Dollars, could be forecasted using a variety of mathematical forecasting models. Actual values of exchange rates were collected throughout the months of January, February, and March of 2010 and were applied to a variety of forecasting models in order to forecast the daily exchange rates for the month of May 2010. The outcome proved to be successful when a significantly small measure of error was present in the forecasts, after the errors were calculated using the various measures of forecast accuracy. The hypothesis: “future exchange rates can be determined with the aid of a mathematical forecasting model” was then accepted.

The linear regression model supports the theory that exchange rates are independent of each other, however, it is possible to successfully forecast exchange rates using a limited amount of historical data over a short-term time horizon. This is proven in the results of the various measures of forecast accuracy for the other four forecasting models, as the models produced accurate forecasts using a maximum of seven samples that had been documented within a period of three months before the date of the forecast.

Unfortunately, it will be very difficult to consistently estimate exchange rates successfully using these forecasting models. There are many reasons as to why the forecasts for exchange rates will ultimately deviate from the actual exchange rate values. Firstly, foreign exchange rates show a great vulnerability to natural disasters and political/economical factors. These factors are rarely predictable and, as a result, are often unable to be forecasted in advance. Therefore, the possibility of these factors’ occurrences has not been included in the forecasting that has been conducted throughout this thesis. Additionally, the currency’s environment has been expected to remain constant. As a result, the methods that have been used in this thesis will not work properly when there is a drastic change of circumstances, thus resulting in the probability of inaccurate forecasts.

Some of the additional factors that may affect the success of exchange rate forecasts include the choice of forecasting models, and the values of the smoothing constants in exponential smoothing methods. Since the chosen values of the smoothing constants largely affect the accuracy of the exponential smoothing and exponential smoothing with trend adjustment models, choosing the correct values of the smoothing constants is crucial. As the environment of the exchange rates changes, the values of the smoothing constants will need to be re-evaluated. Additionally, though there are many forecasting methods, there is no such thing as one superior method for use in all forecasting scenarios. The forecasting models that have been applied in this thesis may be somewhat unconventional for forecasting exchange rates, however, they were chosen because they made best use of the data, as well as the author's knowledge. As circumstances change, other forecasting models may become more efficient for forecasting exchange rates.

Forecasting plays an important role in the success of many companies around the world. Since forecasts are "the only estimate of demand until actual demand becomes known"(Heizer & Render, 2004, p. 105), effective planning in the short and long run greatly depends on a forecast. Thus, it is important for forecasters to consistently make estimates of what will happen in the future. Though the projection of a good estimate is the core purpose of forecasting, inaccurate forecasts can also provide the forecaster with valuable insight. Furthermore, it is impossible to know whether a forecast will be correct or incorrect until the outcome has occurred. Therefore, to conclude, it is evident that it is always best to forecast due to there being a greater probability of benefitting from the forecast than experiencing an opportunity cost for not forecasting at all.

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APPENDICES

Appendix 1 Exponential Smoothing Forecasts with α = 0.1

May Forecasts, Exponential Smoothing				
α = 0.1				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3472	1.3479
85	3-May	1	**1.3473**	1.3238
86	4-May	1	**1.3449**	1.3089
87	5-May	1	**1.3413**	1.2924
88	6-May	1	**1.3364**	1.2727
89	7-May	1	**1.3301**	1.2746
90	10-May	1	**1.3245**	1.2969
91	11-May	1	**1.3218**	1.2698
92	12-May	1	**1.3166**	1.2686
93	13-May	1	**1.3118**	1.2587
94	14-May	1	**1.3065**	1.2492
95	17-May	1	**1.3007**	1.2349
96	18-May	1	**1.2941**	1.2428
97	19-May	1	**1.2890**	1.2270
98	20-May	1	**1.2828**	1.2334
99	21-May	1	**1.2779**	1.2497
100	24-May	1	**1.2751**	1.2360
101	25-May	1	**1.2711**	1.2223
102	26-May	1	**1.2663**	1.2309
103	27-May	1	**1.2627**	1.2255
104	28-May	1	**1.2590**	1.2384
105	31-May	1	**1.2569**	1.2307

Appendix 2 Exponential Smoothing Forecasts with α = 0.2

May Forecasts, Exponential Smoothing				
α = 0.2				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3473	1.3479
85	3-May	1	**1.3474**	1.3238
86	4-May	1	**1.3427**	1.3089
87	5-May	1	**1.3359**	1.2924

88	6-May	1	**1.3272**	1.2727
89	7-May	1	**1.3163**	1.2746
90	10-May	1	**1.3080**	1.2969
91	11-May	1	**1.3058**	1.2698
92	12-May	1	**1.2986**	1.2686
93	13-May	1	**1.2926**	1.2587
94	14-May	1	**1.2858**	1.2492
95	17-May	1	**1.2785**	1.2349
96	18-May	1	**1.2698**	1.2428
97	19-May	1	**1.2644**	1.2270
98	20-May	1	**1.2569**	1.2334
99	21-May	1	**1.2522**	1.2497
100	24-May	1	**1.2517**	1.2360
101	25-May	1	**1.2486**	1.2223
102	26-May	1	**1.2433**	1.2309
103	27-May	1	**1.2408**	1.2255
104	28-May	1	**1.2378**	1.2384
105	31-May	1	**1.2379**	1.2307

Appendix 3 Exponential Smoothing Forecasts with α = 0.3

May Forecasts, Exponential Smoothing				
α = 0.3				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3474	1.3479
85	3-May	1	**1.3476**	1.3238
86	4-May	1	**1.3404**	1.3089
87	5-May	1	**1.3310**	1.2924
88	6-May	1	**1.3194**	1.2727
89	7-May	1	**1.3054**	1.2746
90	10-May	1	**1.2962**	1.2969
91	11-May	1	**1.2964**	1.2698
92	12-May	1	**1.2884**	1.2686
93	13-May	1	**1.2825**	1.2587
94	14-May	1	**1.2753**	1.2492
95	17-May	1	**1.2675**	1.2349
96	18-May	1	**1.2577**	1.2428
97	19-May	1	**1.2532**	1.2270
98	20-May	1	**1.2454**	1.2334
99	21-May	1	**1.2418**	1.2497
100	24-May	1	**1.2442**	1.2360
101	25-May	1	**1.2417**	1.2223
102	26-May	1	**1.2359**	1.2309
103	27-May	1	**1.2344**	1.2255
104	28-May	1	**1.2317**	1.2384
105	31-May	1	**1.2337**	1.2307

Appendix 4 Exponential Smoothing Forecasts with α = 0.4

May Forecasts, Exponential Smoothing				
α = 0.4				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3475	1.3479
85	3-May	1	**1.3477**	1.3238
86	4-May	1	**1.3381**	1.3089
87	5-May	1	**1.3264**	1.2924
88	6-May	1	**1.3128**	1.2727
89	7-May	1	**1.2968**	1.2746
90	10-May	1	**1.2879**	1.2969
91	11-May	1	**1.2915**	1.2698
92	12-May	1	**1.2828**	1.2686
93	13-May	1	**1.2771**	1.2587
94	14-May	1	**1.2698**	1.2492
95	17-May	1	**1.2615**	1.2349
96	18-May	1	**1.2509**	1.2428
97	19-May	1	**1.2476**	1.2270
98	20-May	1	**1.2394**	1.2334
99	21-May	1	**1.2370**	1.2497
100	24-May	1	**1.2421**	1.2360
101	25-May	1	**1.2396**	1.2223
102	26-May	1	**1.2327**	1.2309
103	27-May	1	**1.2320**	1.2255
104	28-May	1	**1.2294**	1.2384
105	31-May	1	**1.2330**	1.2307

Appendix 5 Exponential Smoothing Forecasts with α = 0.5

May Forecasts, Exponential Smoothing				
α = 0.5				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3477	1.3479
85	3-May	1	**1.3478**	1.3238
86	4-May	1	**1.3358**	1.3089
87	5-May	1	**1.3223**	1.2924
88	6-May	1	**1.3074**	1.2727
89	7-May	1	**1.2900**	1.2746
90	10-May	1	**1.2823**	1.2969
91	11-May	1	**1.2896**	1.2698

92	12-May	1	**1.2797**	1.2686
93	13-May	1	**1.2742**	1.2587
94	14-May	1	**1.2664**	1.2492
95	17-May	1	**1.2578**	1.2349
96	18-May	1	**1.2464**	1.2428
97	19-May	1	**1.2446**	1.2270
98	20-May	1	**1.2358**	1.2334
99	21-May	1	**1.2346**	1.2497
100	24-May	1	**1.2421**	1.2360
101	25-May	1	**1.2391**	1.2223
102	26-May	1	**1.2307**	1.2309
103	27-May	1	**1.2308**	1.2255
104	28-May	1	**1.2281**	1.2384
105	31-May	1	**1.2333**	1.2307

Appendix 6 Exponential Smoothing Forecasts with α = 0.6

May Forecasts, Exponential Smoothing				
α = 0.6				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3478	1.3479
85	3-May	1	**1.3478**	1.3238
86	4-May	1	**1.3334**	1.3089
87	5-May	1	**1.3187**	1.2924
88	6-May	1	**1.3029**	1.2727
89	7-May	1	**1.2848**	1.2746
90	10-May	1	**1.2787**	1.2969
91	11-May	1	**1.2896**	1.2698
92	12-May	1	**1.2777**	1.2686
93	13-May	1	**1.2722**	1.2587
94	14-May	1	**1.2641**	1.2492
95	17-May	1	**1.2552**	1.2349
96	18-May	1	**1.2430**	1.2428
97	19-May	1	**1.2429**	1.2270
98	20-May	1	**1.2334**	1.2334
99	21-May	1	**1.2334**	1.2497
100	24-May	1	**1.2432**	1.2360
101	25-May	1	**1.2389**	1.2223
102	26-May	1	**1.2289**	1.2309
103	27-May	1	**1.2301**	1.2255
104	28-May	1	**1.2273**	1.2384
105	31-May	1	**1.2340**	1.2307

Appendix 7 Exponential Smoothing Forecasts with α = 0.7

May Forecasts, Exponential Smoothing				
α = 0.7				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3479	1.3479
85	3-May	1	**1.3479**	1.3238
86	4-May	1	**1.3310**	1.3089
87	5-May	1	**1.3155**	1.2924
88	6-May	1	**1.2993**	1.2727
89	7-May	1	**1.2807**	1.2746
90	10-May	1	**1.2764**	1.2969
91	11-May	1	**1.2908**	1.2698
92	12-May	1	**1.2761**	1.2686
93	13-May	1	**1.2708**	1.2587
94	14-May	1	**1.2623**	1.2492
95	17-May	1	**1.2531**	1.2349
96	18-May	1	**1.2404**	1.2428
97	19-May	1	**1.2421**	1.2270
98	20-May	1	**1.2315**	1.2334
99	21-May	1	**1.2328**	1.2497
100	24-May	1	**1.2446**	1.2360
101	25-May	1	**1.2386**	1.2223
102	26-May	1	**1.2272**	1.2309
103	27-May	1	**1.2298**	1.2255
104	28-May	1	**1.2268**	1.2384
105	31-May	1	**1.2349**	1.2307

Appendix 8 Exponential Smoothing Forecasts with α = 0.8

May Forecasts, Exponential Smoothing				
α = 0.8				
Sample	**Date**	**Euro**	**USD Forecast**	**Actual**
61	29-Mar	1	-	1.3471
62	30-Mar	1	1.3471	1.3482
63	31-Mar	1	1.3480	1.3479
85	3-May	1	**1.3479**	1.3238
86	4-May	1	**1.3286**	1.3089
87	5-May	1	**1.3128**	1.2924
88	6-May	1	**1.2965**	1.2727
89	7-May	1	**1.2775**	1.2746
90	10-May	1	**1.2752**	1.2969
91	11-May	1	**1.2926**	1.2698

92	12-May	1	**1.2744**	1.2686
93	13-May	1	**1.2698**	1.2587
94	14-May	1	**1.2609**	1.2492
95	17-May	1	**1.2515**	1.2349
96	18-May	1	**1.2382**	1.2428
97	19-May	1	**1.2419**	1.2270
98	20-May	1	**1.2300**	1.2334
99	21-May	1	**1.2327**	1.2497
100	24-May	1	**1.2463**	1.2360
101	25-May	1	**1.2381**	1.2223
102	26-May	1	**1.2255**	1.2309
103	27-May	1	**1.2298**	1.2255
104	28-May	1	**1.2264**	1.2384
105	31-May	1	**1.2360**	1.2307

Appendix 9 Trend Adjustment Forecasts with α = 0.1, β = 0.1

May Forecasts, Trend Adjustment						
α = 0.1 **β = 0.1**						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3481	0.0010	-	1.3479
85	3-May	1	1.3490	0.0010	**1.3500**	1.3238
86	4-May	1	1.3474	0.0007	**1.3481**	1.3089
87	5-May	1	1.3442	0.0003	**1.3445**	1.2924
88	6-May	1	1.3393	-0.0002	**1.3391**	1.2727
89	7-May	1	1.3325	-0.0008	**1.3316**	1.2746
90	10-May	1	1.3259	-0.0014	**1.3245**	1.2969
91	11-May	1	1.3217	-0.0017	**1.3200**	1.2698
92	12-May	1	1.3150	-0.0022	**1.3128**	1.2686
93	13-May	1	1.3084	-0.0026	**1.3058**	1.2587
94	14-May	1	1.3010	-0.0031	**1.2979**	1.2492
95	17-May	1	1.2931	-0.0036	**1.2895**	1.2349
96	18-May	1	1.2840	-0.0041	**1.2799**	1.2428
97	19-May	1	1.2762	-0.0045	**1.2716**	1.2270
98	20-May	1	1.2672	-0.0050	**1.2622**	1.2334
99	21-May	1	1.2593	-0.0052	**1.2541**	1.2497
100	24-May	1	1.2536	-0.0053	**1.2484**	1.2360
101	25-May	1	1.2471	-0.0054	**1.2417**	1.2223
102	26-May	1	1.2398	-0.0056	**1.2342**	1.2309
103	27-May	1	1.2338	-0.0056	**1.2282**	1.2255
104	28-May	1	1.2279	-0.0057	**1.2222**	1.2384
105	31-May	1	1.2239	-0.0055	**1.2184**	1.2307

Appendix 10 Trend Adjustment Forecasts with α = 0.2, β = 0.2

May Forecasts, Trend Adjustment						
α **= 0.2** β **= 0.2**						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3481	0.0010	-	1.3479
85	3-May	1	1.3489	0.0010	**1.3498**	1.3238
86	4-May	1	1.3446	-0.0001	**1.3445**	1.3089
87	5-May	1	1.3374	-0.0015	**1.3359**	1.2924
88	6-May	1	1.3272	-0.0033	**1.3239**	1.2727
89	7-May	1	1.3137	-0.0053	**1.3084**	1.2746
90	10-May	1	1.3016	-0.0067	**1.2950**	1.2969
91	11-May	1	1.2954	-0.0066	**1.2888**	1.2698
92	12-May	1	1.2850	-0.0073	**1.2777**	1.2686
93	13-May	1	1.2758	-0.0077	**1.2681**	1.2587
94	14-May	1	1.2663	-0.0081	**1.2582**	1.2492
95	17-May	1	1.2564	-0.0084	**1.2479**	1.2349
96	18-May	1	1.2453	-0.0090	**1.2364**	1.2428
97	19-May	1	1.2377	-0.0087	**1.2290**	1.2270
98	20-May	1	1.2286	-0.0088	**1.2198**	1.2334
99	21-May	1	1.2225	-0.0082	**1.2143**	1.2497
100	24-May	1	1.2214	-0.0068	**1.2145**	1.2360
101	25-May	1	1.2188	-0.0060	**1.2129**	1.2223
102	26-May	1	1.2148	-0.0056	**1.2092**	1.2309
103	27-May	1	1.2135	-0.0047	**1.2088**	1.2255
104	28-May	1	1.2121	-0.0040	**1.2081**	1.2384
105	31-May	1	1.2142	-0.0028	**1.2113**	1.2307

Appendix 11 Trend Adjustment Forecasts with α = 0.3, β = 0.3

May Forecasts, Trend Adjustment						
α **= 0.3** β **= 0.3**						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3481	0.0010	-	1.3479
85	3-May	1	1.3488	0.0009	**1.3497**	1.3238
86	4-May	1	1.3419	-0.0014	**1.3405**	1.3089
87	5-May	1	1.3310	-0.0043	**1.3267**	1.2924
88	6-May	1	1.3164	-0.0074	**1.3091**	1.2727
89	7-May	1	1.2982	-0.0106	**1.2875**	1.2746

90	10-May	1	1.2836	-0.0118	**1.2718**	1.2969
91	11-May	1	1.2794	-0.0095	**1.2698**	1.2698
92	12-May	1	1.2698	-0.0095	**1.2603**	1.2686
93	13-May	1	1.2628	-0.0088	**1.2540**	1.2587
94	14-May	1	1.2554	-0.0084	**1.2470**	1.2492
95	17-May	1	1.2477	-0.0082	**1.2395**	1.2349
96	18-May	1	1.2381	-0.0086	**1.2295**	1.2428
97	19-May	1	1.2335	-0.0074	**1.2261**	1.2270
98	20-May	1	1.2264	-0.0073	**1.2191**	1.2334
99	21-May	1	1.2234	-0.0060	**1.2173**	1.2497
100	24-May	1	1.2271	-0.0031	**1.2239**	1.2360
101	25-May	1	1.2276	-0.0020	**1.2255**	1.2223
102	26-May	1	1.2246	-0.0023	**1.2222**	1.2309
103	27-May	1	1.2248	-0.0015	**1.2233**	1.2255
104	28-May	1	1.2240	-0.0013	**1.2226**	1.2384
105	31-May	1	1.2274	0.0001	**1.2274**	1.2307

Appendix 12 Trend Adjustment Forecasts with α = 0.4, β = 0.4

May Forecasts, Trend Adjustment						
α = 0.4 **β = 0.4**						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3481	0.0010	-	1.3479
85	3-May	1	1.3487	0.0008	**1.3495**	1.3238
86	4-May	1	1.3392	-0.0033	**1.3359**	1.3089
87	5-May	1	1.3251	-0.0076	**1.3175**	1.2924
88	6-May	1	1.3075	-0.0116	**1.2958**	1.2727
89	7-May	1	1.2866	-0.0153	**1.2712**	1.2746
90	10-May	1	1.2726	-0.0148	**1.2578**	1.2969
91	11-May	1	1.2734	-0.0085	**1.2649**	1.2698
92	12-May	1	1.2669	-0.0078	**1.2591**	1.2686
93	13-May	1	1.2629	-0.0062	**1.2567**	1.2587
94	14-May	1	1.2575	-0.0059	**1.2516**	1.2492
95	17-May	1	1.2506	-0.0063	**1.2443**	1.2349
96	18-May	1	1.2406	-0.0078	**1.2328**	1.2428
97	19-May	1	1.2368	-0.0062	**1.2306**	1.2270
98	20-May	1	1.2292	-0.0068	**1.2224**	1.2334
99	21-May	1	1.2268	-0.0050	**1.2218**	1.2497
100	24-May	1	1.2330	-0.0005	**1.2324**	1.2360
101	25-May	1	1.2338	0.0000	**1.2339**	1.2223
102	26-May	1	1.2293	-0.0018	**1.2274**	1.2309
103	27-May	1	1.2288	-0.0013	**1.2276**	1.2255
104	28-May	1	1.2267	-0.0016	**1.2251**	1.2384
105	31-May	1	1.2304	0.0005	**1.2310**	1.2307

Appendix 13 Trend Adjustment Forecasts with α = 0.6, β = 0.6

May Forecasts, Trend Adjustment						
α = 0.6 β = 0.6						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3482	0.0010	-	1.3479
85	3-May	1	1.3484	0.0006	**1.3490**	1.3238
86	4-May	1	1.3339	-0.0085	**1.3254**	1.3089
87	5-May	1	1.3155	-0.0144	**1.3011**	1.2924
88	6-May	1	1.2959	-0.0175	**1.2783**	1.2727
89	7-May	1	1.2749	-0.0196	**1.2554**	1.2746
90	10-May	1	1.2669	-0.0126	**1.2543**	1.2969
91	11-May	1	1.2798	0.0027	**1.2825**	1.2698
92	12-May	1	1.2749	-0.0019	**1.2730**	1.2686
93	13-May	1	1.2704	-0.0035	**1.2669**	1.2587
94	14-May	1	1.2620	-0.0064	**1.2556**	1.2492
95	17-May	1	1.2517	-0.0087	**1.2430**	1.2349
96	18-May	1	1.2382	-0.0116	**1.2265**	1.2428
97	19-May	1	1.2363	-0.0058	**1.2305**	1.2270
98	20-May	1	1.2284	-0.0070	**1.2214**	1.2334
99	21-May	1	1.2286	-0.0027	**1.2259**	1.2497
100	24-May	1	1.2402	0.0059	**1.2460**	1.2360
101	25-May	1	1.2400	0.0023	**1.2423**	1.2223
102	26-May	1	1.2303	-0.0049	**1.2254**	1.2309
103	27-May	1	1.2287	-0.0029	**1.2257**	1.2255
104	28-May	1	1.2256	-0.0030	**1.2226**	1.2384
105	31-May	1	1.2321	0.0027	**1.2347**	1.2307

Appendix 14 Trend Adjustment Forecasts with α = 0.7, β = 0.7

May Forecasts, Trend Adjustment						
α = 0.7 β = 0.7						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3482	0.0010	-	1.3479
85	3-May	1	1.3483	0.0004	**1.3487**	1.3238
86	4-May	1	1.3313	-0.0118	**1.3195**	1.3089
87	5-May	1	1.3121	-0.0170	**1.2951**	1.2924
88	6-May	1	1.2932	-0.0183	**1.2749**	1.2727
89	7-May	1	1.2734	-0.0194	**1.2540**	1.2746

90	10-May	1	1.2684	-0.0093	**1.2591**	1.2969
91	11-May	1	1.2856	0.0092	**1.2948**	1.2698
92	12-May	1	1.2773	-0.0030	**1.2743**	1.2686
93	13-May	1	1.2703	-0.0058	**1.2645**	1.2587
94	14-May	1	1.2604	-0.0086	**1.2518**	1.2492
95	17-May	1	1.2500	-0.0099	**1.2401**	1.2349
96	18-May	1	1.2364	-0.0124	**1.2240**	1.2428
97	19-May	1	1.2372	-0.0032	**1.2339**	1.2270
98	20-May	1	1.2291	-0.0066	**1.2224**	1.2334
99	21-May	1	1.2301	-0.0013	**1.2289**	1.2497
100	24-May	1	1.2434	0.0090	**1.2524**	1.2360
101	25-May	1	1.2409	0.0009	**1.2418**	1.2223
102	26-May	1	1.2282	-0.0087	**1.2195**	1.2309
103	27-May	1	1.2275	-0.0031	**1.2244**	1.2255
104	28-May	1	1.2252	-0.0025	**1.2226**	1.2384
105	31-May	1	1.2337	0.0052	**1.2389**	1.2307

Appendix 15 Trend Adjustment Forecasts with α = 0.8, β = 0.8

May Forecasts, Trend Adjustment						
α = 0.8 **β = 0.8**						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3482	0.0011	-	1.3479
85	3-May	1	1.3482	0.0002	**1.3484**	1.3238
86	4-May	1	1.3287	-0.0155	**1.3132**	1.3089
87	5-May	1	1.3098	-0.0183	**1.2915**	1.2924
88	6-May	1	1.2922	-0.0177	**1.2745**	1.2727
89	7-May	1	1.2731	-0.0189	**1.2542**	1.2746
90	10-May	1	1.2705	-0.0058	**1.2647**	1.2969
91	11-May	1	1.2905	0.0148	**1.3053**	1.2698
92	12-May	1	1.2769	-0.0079	**1.2690**	1.2686
93	13-May	1	1.2687	-0.0081	**1.2605**	1.2587
94	14-May	1	1.2591	-0.0093	**1.2497**	1.2492
95	17-May	1	1.2493	-0.0097	**1.2396**	1.2349
96	18-May	1	1.2358	-0.0127	**1.2231**	1.2428
97	19-May	1	1.2389	-0.0001	**1.2387**	1.2270
98	20-May	1	1.2293	-0.0076	**1.2217**	1.2334
99	21-May	1	1.2311	-0.0002	**1.2309**	1.2497
100	24-May	1	1.2459	0.0119	**1.2578**	1.2360
101	25-May	1	1.2404	-0.0021	**1.2383**	1.2223
102	26-May	1	1.2255	-0.0123	**1.2132**	1.2309
103	27-May	1	1.2274	-0.0010	**1.2264**	1.2255
104	28-May	1	1.2257	-0.0015	**1.2241**	1.2384
105	31-May	1	1.2355	0.0076	**1.2431**	1.2307

Appendix 16 Trend Adjustment Forecasts with α = 0.9, β = 0.9

May Forecasts, Trend Adjustment						
α = 0.9 β = 0.9						
Sample	**Date**	**Euro**	**USD Forecast**	**Trend**	**FIT**	**Actual**
61	29-Mar	1	-	-	-	1.3471
62	30-Mar	1	1.3471	0.0010	-	1.3482
63	31-Mar	1	1.3482	0.0011	-	1.3479
85	3-May	1	1.3480	0.0000	**1.3480**	1.3238
86	4-May	1	1.3262	-0.0196	**1.3066**	1.3089
87	5-May	1	1.3087	-0.0178	**1.2909**	1.2924
88	6-May	1	1.2923	-0.0166	**1.2757**	1.2727
89	7-May	1	1.2730	-0.0190	**1.2540**	1.2746
90	10-May	1	1.2725	-0.0023	**1.2702**	1.2969
91	11-May	1	1.2942	0.0193	**1.3135**	1.2698
92	12-May	1	1.2742	-0.0161	**1.2580**	1.2686
93	13-May	1	1.2675	-0.0076	**1.2600**	1.2587
94	14-May	1	1.2588	-0.0086	**1.2502**	1.2492
95	17-May	1	1.2493	-0.0094	**1.2399**	1.2349
96	18-May	1	1.2354	-0.0135	**1.2219**	1.2428
97	19-May	1	1.2407	0.0034	**1.2442**	1.2270
98	20-May	1	1.2287	-0.0105	**1.2183**	1.2334
99	21-May	1	1.2319	0.0018	**1.2337**	1.2497
100	24-May	1	1.2481	0.0148	**1.2629**	1.2360
101	25-May	1	1.2387	-0.0070	**1.2317**	1.2223
102	26-May	1	1.2232	-0.0146	**1.2086**	1.2309
103	27-May	1	1.2287	0.0034	**1.2321**	1.2255
104	28-May	1	1.2262	-0.0019	**1.2242**	1.2384
105	31-May	1	1.2370	0.0095	**1.2465**	1.2307

Appendix 17 Additional Information used in Linear Regression Forecasting

Additional Information, Linear Regression								
Σx	x^2	Σx^2	$\bar{x}$	$\bar{x}^2$	Σy	$\bar{y}$	xy	Σxy
	1						1.4389	
	4						2.8884	
	9						4.3050	
	16						5.7216	
	25						7.1365	
	36						8.7168	
	49						10.1367	
	64						11.6504	
	81						13.0374	
	100						14.3740	
	121						15.8059	

	144						17.1348	
	169						18.3716	
	196						19.6896	
	225						21.2025	
	256						22.6416	
	289						23.9445	
	324						25.3296	
	361						26.5981	
	400						27.9320	
	441						29.2173	
	484						30.6614	
	529						32.1632	
	576						33.2328	
	625						34.2275	
	676						35.5550	
	729						37.1520	
	784						38.4720	
	841						39.7822	
	900						40.7160	
	961						42.1817	
	1024						43.6768	
	1089						45.2958	
	1156						46.1278	
	1225						47.3165	
	1296						49.0536	
	1369						50.2349	
	1444						51.4786	
	1521						52.6071	
	1600						54.2800	
	1681						55.4525	
	1764						56.9016	
	1849						58.6563	
	1936						60.1392	
	2025						61.1190	
	2116						62.8452	
	2209						63.7179	
	2304						65.3280	
	2401						66.9193	
	2500						68.8250	
	2601						69.8955	
	2704						71.3596	
	2809						72.9068	
	2916						73.7640	
	3025						74.5140	
	3136						75.4376	
	3249						77.0583	
	3364						77.3604	
	3481						78.8004	
	3600						80.1180	
	3721						82.1731	
	3844						83.5884	
	3969						84.9177	
2016		85344	32	1024	87.1232	1.3829		2753.2859