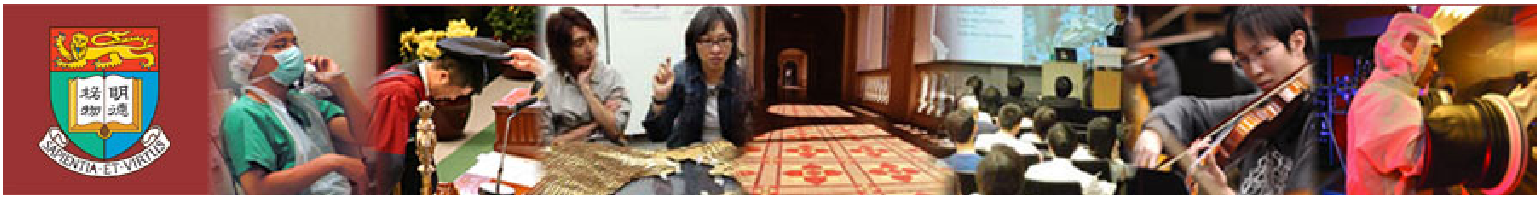

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Pricing Electricity in Pools With Wind Producers

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***Abstract*—This paper considers an electricity pool that includes a significant number of wind producers and is cleared through a network-constrained auction, one day in advance and on an hourly basis. The hourly auction is formulated as a two-stage stochastic programming problem, where the first stage represents the clearing of the market and the second stage models the system operation under a number of plausible wind production realizations. This formulation co-optimizes energy and reserve, and allows deriving both pool energy prices and balancing energy prices. These prices result in both cost recovery for producers and revenue reconciliation. A case study of realistic size is used to illustrate the functioning of the proposed pricing scheme.**

***Index Terms*—Electricity pricing, market clearing, stochastic programming, wind power.**

NOTATION

The main notation used throughout this paper is defined below for quick reference. For convenience, other symbols are defined as required in the main text.

A. Indices and Numbers:

n	Index of system buses running from 1 to N_B.
i	Index of generating units running from 1 to N_G.
ω	Index of wind power scenarios running from 1 to N_Ω.
j	Index of loads running from 1 to N_L.
q	Index of wind farms running from 1 to N_Q.

B. Variables:

$\delta^0_{n\omega}$	Voltage angle at node n at the market stage [rad].
$\delta_{n\omega}$	Voltage angle at node n under scenario ω [rad].
λ_n	Nodal price at which electricity transactions are cleared at the market stage [$/MWh].
$\lambda_{n\omega}$	Nodal price at which electricity transactions are cleared at the operation stage [$/MWh].
$L^{\rm shed}_{j\omega}$	Involuntarily load shed at bus j under scenario ω [MW].
P_i	Scheduled power output of generating unit i [MW].
$r^{\rm D}_{i\omega}$	Reserve down deployed by generator i under scenario ω [MW].
$r^{\rm U}_{i\omega}$	Reserve up deployed by generator i under scenario ω [MW].
$W^{\rm s}_q$	Wind power generation scheduled at the market stage [MW].
$W^{\rm spill}_{q\omega}$	Wind power generation spillage under scenario ω [MW].

C. Random Variables:

W_q	Random variable modeling the wind power generation [MW]. $W_{q\omega}$ represents the actual realization of the wind power production in scenario ω and therefore, it constitutes input information.

D. Constants:

B_{nr}	Absolute value of the susceptance of line (n, r) [per unit].
C_i	Production cost offer of generating unit i [$/MWh].
C_q	Offer cost of wind farm q [$/MWh].
$C^{\max}_{nr}$	Maximum capacity of line (n, r) [MW].
L_j	Power consumption by load j [MW].
$P^{\max}_i$	Capacity of unit i [MW].
π_ω	Probability of wind power scenario w.
$R^{\rm D,max}_i$	Maximum reserve down that can be provided by generator i [MW].
$R^{\rm U,max}_i$	Maximum reserve up that can be provided by generator i [MW].
$V^{\rm LOL}_j$	Value of lost load for consumer j [$/MWh].
$W^{\max}_q$	Upper bound of the power offer made by wind farm q [MW].

E. Sets:

Λ	Set of transmission lines.
$M_{\rm L}$	Mapping of the set of consumers (loads) into the set of buses.
$M_{\rm G}$	Mapping of the set of generating units into the set of buses.
$M_{\rm Q}$	Mapping of the set of wind farms into the set of buses.

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I. Introduction

A. Motivation and Approach

THE increasing participation of wind producers in electricity pools leads to the need for re-designing the corresponding clearing algorithms to recognize the variable and uncertain nature of such producers [1], [2].

The pool clearing algorithm proposed in this paper corresponds to a single-period network-constrained auction, similar to those used by ISO-New England [3] and PJM [4]. Clearing takes place the day prior to power delivery on an hourly basis.

Incorporating wind producers naturally leads to a two-stage stochastic programming formulation [5], [6]. The first stage of this stochastic programming model represents the actual market clearing one day ahead and results in the so-called "pool prices", which are obtained as dual variables of energy balance equations at the scheduling (market) level. The second stage represents all plausible realizations of the wind power production and results in the so-called "balancing prices", which correspond to the dual variables of the balance equations for all plausible wind production realizations. Since the network is explicitly represented in the formulation, both pool and balancing prices are locational marginal prices.

These prices are shown to provide cost recovery in expectation for all power producers and, also in expectation, ensure revenue reconciliation.

For the sake of simplicity and following common industry practice, non-convexities, such as start-up costs and minimum power outputs of production units, are not explicitly represented in the clearing model. These issues are considered in detail in [7]–[10].

The proposed market-clearing model is cast as a two-stage stochastic programming problem in which the first-stage represents the *anticipated dispatch* (e.g., a day-ahead schedule), while the second-stage represents the *real-time dispatch*. The term "reserve deployment" (or just "deployed reserve") is used to refer to changes in the form of energy (MWh) between first- and second-stage dispatches. Likewise, by "reserve capacity" (or just "reserve") we refer to the available capacity in MW to be eventually deployed, i.e., converted to energy (MWh), to cope with uncertain wind variations. Mathematically, the proposed clearing model materializes into a well-behaved linear programming (LP) problem that can be efficiently solved using commercially available software.

B. Literature Review and Contributions

The proposed model relates to that reported in [11], which formulates a two-stage stochastic model to represent market clearing and system operation under equipment failures. This model is extended in [6] to incorporate wind power producers. The rationale of such a type of models is analyzed in [12] using a generic mathematical framework. The model in [12], which includes a price scheme similar to the one used in this paper, allows production offers to differ between energy and reserve deployment, which may constitute a gaming incentive for some producers.

We propose a single auction that clears the market and financially settles it. Such a proposal is motivated by current practice in some European markets with high wind integration, as is the case of the electricity market in the Iberian Peninsula (with 25% of its installed capacity being wind power) or that in Western Denmark (with 32% of its installed capacity being wind power). The proposed model is essentially different than those described in [13] and [14], which do not explicitly represent the sequence market-clearing and system-operation.

As opposed to [15], we price energy through the pool prices, and deployed reserve through the balancing prices. Following sound economic principles, we do not price reserve capacity as reserve capacity does not entail specific costs to producers, and pricing reserve capacity may lead to significant price inconsistencies in case of multiple equally probable operation scenarios. This is the case in pools with significant wind power production, where pricing marginally reserve capacity may lead to disproportionate reserve capacity prices.

Within the above framework, the contributions of this paper are threefold:

1) To promote a marginal pricing scheme (including pool prices and balancing prices) that is tailored to an electricity pool that includes a significant number of wind producers.
2) To derive prices (pool and balancing) as dual variables of the energy balance constraints of an LP problem. Therefore, such prices are calculated in a simple and robust manner.
3) To report and analyze results from a detailed case study that illustrates the direct applicability of the proposed pricing scheme.

C. Paper Organization

The rest of this paper is organized as follows. Section II describes the proposed clearing model, its mathematical formulation and the consequent pricing scheme, which is subsequently illustrated using a three-node system in Section III. Section IV reports and discusses results from a realistic case study. Section V provides some relevant conclusions. Finally, the cost-recovery and revenue-adequacy properties (in expectation) of the proposed pricing scheme are proved in the Appendix.

II. Market-Clearing Model

The market-clearing model formulated below is intended for the simultaneous and anticipated dispatch of energy and reserve deployment (energy and reserve co-optimization). Out of this model, a pricing scheme is fabricated to remunerate scheduled production (at pool prices) and deployed reserve (at balancing prices). Formulation (1) below is based on the following simplifying assumptions.

1) For simplicity, only wind generation uncertainty is taken into account. Notwithstanding this, demand uncertainty would be treated analogously. Likewise, note that an N-1 criterion (protecting against generation unit and transmission line single failures) can be easily embedded within the proposed market-clearing procedure.
2) Loads are assumed to be inelastic.

3) The proposed market-clearing algorithm consists in a single-period auction. Thus, inter-temporal constraints are excluded from the market-clearing formulation. In practice, these constraints are generally enforced *ex post* by using, for example, some *ad hoc* rules as in the case of the Electricity Market of the Iberian Peninsula [16] or by setting the outcomes of each single-period market clearing as the initial conditions for the subsequent one. Ramping limits are important and may complicate the operation of the system if neither hydro resources nor CCGTs are available. In the proposed model, we disregard ramping limits for the sake of simplicity. However, note that incorporating ramping limits in the proposed model is straightforward, either if clearing a single period (myopic ramping limits) or clearing a multiple-period horizon.
4) Additionally, the minimum power output of each generating unit is assumed to be zero. This way, we sidestep the tricky issue of non-convex prices.
5) Generation-side energy production cost functions are assumed to be linear. This assumption is made to facilitate mathematical derivations.
6) Network losses are neglected, as it is common practice in market-clearing procedures.
7) The cost of deploying reserve is the cost of energy production. This way, the reserve deployment cost is calculated from the supply cost functions submitted in advance (e.g., one day ahead) by generating units.
8) Wind power uncertainty can be efficiently modeled through a finite set of scenarios. This assumption is required for the proposed market-clearing model to be computationally solvable.
9) The market administrator uses all the available information at the time of clearing the market to maximize the social welfare of the market. This information includes scenario sets modeling wind production from the different wind farms. This way, key factors such as correlation between wind sites can be accounted for in pursuit of the social welfare maximization.

In relation to assumption 9 above, note that if wind generators are allowed to submit their own estimate of their probability distributions to the market, then key information such as the correlation of the power produced by wind farms at different sites is lost. This is clearly detrimental to wind producers, especially in highly wind-penetrated markets. For instance, suppose that each wind producer submits its own scenario set independently and the proposed market is cleared accordingly. In the case that their power outputs were indeed significantly correlated, the "free capacity" scheduled day ahead could not be enough to perform the required adjustments in real time with a high probability, thus increasing social costs. Consequently, a centralized administrator collecting information from every wind farm is needed. Note that this market administrator behavior does not deviate from the market administrator behavior regarding the load, which generally involves detailed load predictions at all buses throughout the network. Additionally, this centralized task could be somehow decentralized by means of "wind aggregators" covering a sufficient number of wind farms.

The market-clearing model that results from these assumptions translates into a linear programming problem that can be efficiently solved by using commercially available software, e.g., [17].

A. Formulation

We consider a short-term electricity market that is cleared, usually one day in advance, by solving the following linear optimization problem:

$$\underset{\Xi^{\mathrm{P}}}{\text{Minimize}}$$

$$\sum_{\omega=1}^{\mathrm{N}_\Omega}\pi_\omega\left[\sum_{i=1}^{\mathrm{N}_G}C_i\left(P_i+r_{i\omega}^{\mathrm{U}}-r_{i\omega}^{\mathrm{D}}\right)+\sum_{q=1}^{\mathrm{N}_Q}C_q\left(W_{q\omega}-W_{q\omega}^{\mathrm{spill}}\right)\right.$$
$$\left.+\sum_{j=1}^{\mathrm{N}_L}V_j^{\mathrm{LOL}}L_{j\omega}^{\mathrm{shed}}\right] \tag{1a}$$

subject to

$$\sum_{i:(i,n)\in M_{\mathrm{G}}}P_i+\sum_{q:(q,n)\in M_{\mathrm{Q}}}W_q^s-\sum_{j:(j,n)\in M_{\mathrm{L}}}L_j$$
$$-\sum_{r:(n,r)\in\Lambda}B_{nr}\left(\delta_n^0-\delta_r^0\right)=0:\lambda_n,\forall n \tag{1b}$$
$$\sum_{i:(i,n)\in M_{\mathrm{G}}}\left(r_{i\omega}^{\mathrm{U}}-r_{i\omega}^{\mathrm{D}}\right)+\sum_{j:(j,n)\in M_{\mathrm{L}}}L_{j\omega}^{\mathrm{shed}}$$
$$+\sum_{q:(q,n)\in M_{\mathrm{Q}}}\left(W_{q\omega}-W_q^s-W_{q\omega}^{\mathrm{spill}}\right)$$
$$+\sum_{r:(n,r)\in\Lambda}B_{nr}\left(\delta_n^0-\delta_{n\omega}-\delta_r^0+\delta_{r\omega}\right)=0:\lambda_{n\omega},\forall n,\forall\omega \tag{1c}$$
$$-P_i\geq -P_i^{\max}:\rho_i,\forall i \tag{1d}$$
$$-W_q^s\geq -W_q^{\max}:\sigma_q,\forall q \tag{1e}$$
$$-r_{i\omega}^{\mathrm{U}}\geq -R_i^{\mathrm{U,max}}:\alpha_{i\omega}^{\mathrm{U}},\forall i,\forall\omega \tag{1f}$$
$$-r_{i\omega}^{\mathrm{D}}\geq -R_i^{\mathrm{D,max}}:\alpha_{i\omega}^{\mathrm{D}},\forall i,\forall\omega \tag{1g}$$
$$P_i+r_{i\omega}^{\mathrm{U}}-r_{i\omega}^{D}\geq 0:\varphi_{i\omega}^{\min},\forall i,\forall\omega \tag{1h}$$
$$-P_i-r_{i\omega}^{\mathrm{U}}+r_{i\omega}^{D}\geq -P_i^{\max}:\varphi_{i\omega}^{\max},\forall i,\forall\omega \tag{1i}$$
$$-B_{nr}\left(\delta_n^0-\delta_r^0\right)\geq -C_{nr}^{\max}:\varepsilon_{nr}^0,\forall(n,r)\in\Lambda \tag{1j}$$
$$-B_{nr}(\delta_{n\omega}-\delta_{r\omega})\geq -C_{nr}^{\max}:\varepsilon_{nr\omega},\forall(n,r)\in\Lambda,\forall\omega \tag{1k}$$
$$-L_{j\omega}^{\mathrm{shed}}\geq -L_j:\mu_{j\omega},\forall j,\forall\omega \tag{1l}$$
$$-W_{q\omega}^{\mathrm{spill}}\geq -W_{q\omega}:\gamma_{q\omega},\forall q,\forall\omega \tag{1m}$$
$$\delta_1^0=0:\tau_1^0 \tag{1n}$$
$$\delta_{1\omega}=0:\tau_{1\omega},\forall\omega \tag{1o}$$
$$P_i\geq 0,\forall i;r_{i\omega}^{\mathrm{U}},r_{i\omega}^{\mathrm{D}}\geq 0,\forall i,\forall\omega;$$
$$L_{j\omega}^{\mathrm{shed}}\geq 0,\forall j,\forall\omega;W_q^s\geq 0,\forall q;W_{q\omega}^{\mathrm{spill}},\forall q,\forall\omega;$$
$$\delta_n^0\text{ free},\forall n;\delta_{n\omega}\text{ free},\forall n,\forall\omega \tag{1p}$$

where $\Xi^{\mathrm{P}}=\{P_i,r_{iw}^{\mathrm{U}},r_{i\omega}^{\mathrm{D}},\forall i,\forall\omega;W_q^{\mathrm{s}},W_{q\omega}^{\mathrm{spill}},\forall q,\forall\omega;L_{j\omega}^{\mathrm{shed}},\forall j,\forall\omega;\delta_n^0,\delta_{n\omega},\forall n,\forall\omega\}$ is the set of primal variables.

The objective function (1a) to be minimized is the expected cost of the power system operation. This cost is made up of the cost of energy production, which is incurred by both the conventional power plants and wind farms, and the cost of involuntary load curtailments. Observe that the term $\sum_\omega\sum_q\pi_\omega C_q W_{q\omega}$ is constant and as such, can be removed from the objective function. Constraints (1b) and (1c) are power balance equations. Specifically, constraints (1b) enforce the power balance on *dispatched* quantities, while constraints (1c) guarantee that the unexpected deviations of the wind generation are covered by deploying reserve and/or shedding load, if necessary. Constraints (1d) limit the power dispatched for each generating unit to its capacity. Similarly, the amount of wind power production from each wind farm that is considered to settle the energy dispatch is bounded from above by constraint (1e), being the upper bound the power block offered by the wind farm under consideration. Constraints (1f) and (1g) guarantee that the reserve deployed by each generating unit at the real-time stage does not exceed its reserve capacity offer. In this sense, the reserve capacity offers $R_i^{\mathrm{U,max}}$ and $R_i^{\mathrm{D,max}}$ should be consistent with the time interval over which the real-time re-dispatch is to be carried out. In this paper, we consider hourly time steps and therefore, the reserve capacity offers should be, at most, the 1-hour ramp rate of generating units. Constraints (1h) and (1i) ensure that the power production of each generator is kept above zero and below its capacity. The set of constraints (1j) and (1k) enforces the transmission capacity limits. Constraints (1l) and (1m) are commonsense bounds according to which the amount of load that is involuntarily shed and the amount of wind power that is spilled are smaller than or equal to the actual load consumption and the actual wind power production, respectively. Constraints (1n) and (1o) set node 1 as the reference node. The series of constraints (1p) constitutes variable declarations. Note that the dual variables associated with each group of constraints are indicated after the corresponding equalities or inequalities, separated by a colon.

Problem (1) models a single-auction process that clears the market and financially settles it. We describe the proposed market organization as a *single-settlement scheme* in that the amount of energy to be traded both day ahead and in real time, and the corresponding clearing prices are simultaneously determined in a single shot, as opposed to current market practices in most countries where these electricity transactions are settled in two sequentially cleared auctions. Since the proposed market-clearing procedure considers "all possible" wind realizations (via scenarios), all possible real-time settlements are considered within the stochastic programming procedure. In other words, the outcomes of the proposed algorithm are the best clearing of the market under uncertainty and the best pre-dispatch for all possible settlements associated with all possible wind production realizations. Therefore, as long as the real-time stage of the proposed market-clearing model provides a good enough approximation of the actual real-time adjustments, the schedule resulting from the proposed model is better than or equal to that derived from a two sequentially cleared auctions in which the day-ahead and real-time dispatches are optimized independently. The resulting scheme of prices is thus consistent with the maximization of the expected social welfare in markets that are to be cleared under uncertainty. The practical interpretation of "real-time dispatch", however, may vary from market to market. In principle, it should correspond to the actual dispatch over a short period of time (typically ranging from minutes to one hour).

Additionally, it should be noted that as the proposed single market settlement encompasses, in advance, the energy transactions negotiated from day ahead to real time, it concentrates a large trading volume, thus decreasing market power. In a market organization consisting of two sequentially cleared markets, physical participants may, for instance, withhold some capacity day ahead to put it into play in real time with a view to increasing revenues. In many European markets, where real-time auctions are basically used just as markets to correct deviations, the energy volume traded in such markets is sometimes so small that real-time prices tend to be very volatile and susceptible to manipulation. Note that this put wind producers in an unfavorable and risky situation. However, in the proposed market settlement, which co-optimizes day-ahead and real-time dispatches in a single shot, exercising market power becomes more involved.

Observe that the market-clearing model (1) solely accounts for *energy* (as opposed to *capacity*), in the form of the dispatched quantities in the day-ahead schedule and the reserve deployed in the real-time stage. Notwithstanding this, the proposed market settlement is fully compatible with the so-called "reserve capacity markets" (which are currently implemented in the Danish area of the Nordpool [18], for instance) with a view to promoting and rewarding the flexibility of conventional generators, and ensuring the availability of reserve capacity in the real-time operation. Furthermore, reserve capacity costs can be easily incorporated into market-clearing model (1) in the form of reserve capacity bids, as in [6] and [11]. To this end, we just need to replace constraints (1f) and (1g) above by constraints (2) below, and add the term $\sum_{i=1}^{\mathrm{N_G}}(C_i^{\mathrm{U}}R_i^{\mathrm{U}} + C_i^{\mathrm{D}}R_i^{\mathrm{D}})$ to the objective function (1a), with R_i^{U} and R_i^{D} being the upward and downward reserve capacities scheduled for unit i, and C_i^{U} and C_i^{D} their respective declared costs:

$$0 \le R_i^{\mathrm{U}} \le R_i^{\mathrm{U,max}}, \quad \forall i \tag{2a}$$
$$0 \le R_i^{\mathrm{D}} \le R_i^{\mathrm{D,max}}, \quad \forall i \tag{2b}$$
$$r_{i\omega}^{\mathrm{U}} \le R_i^{\mathrm{U}}, \quad \forall i \tag{2c}$$
$$r_{i\omega}^{\mathrm{D}} \le R_i^{\mathrm{D}}, \quad \forall i. \tag{2d}$$

Some authors, though, are not in favor of considering reserve capacity bids, because the provision of reserve capacity does not entail intrinsic costs to producers (see, for instance, [14]). Nevertheless, as illustrated later in Section III, reserve capacity bids may be, however, useful within the context of electricity markets that are "stochastically" cleared for the following three reasons:

1) To competitively reward the availability of reserve capacity to make real-time adjustments.
2) To reduce the risk of economic losses faced by generators deploying reserve at the real-time stage. This is a direct consequence of the previous point.
3) To avoid possible multiple solutions to problem (1) by giving "priority" to the day-ahead dispatch of generators over their real-time redispatch.

In the following section, we state the pricing scheme fabricated out of the market-clearing model (1).

B. Pricing Scheme

Energy transactions settled via the market-clearing tool (1) are priced as follows:

1) Each generating unit i located at bus n is paid for its *scheduled* power production P_i at a price λ_n.
2) Each load j located at bus n is charged for its power consumption L_j at a price λ_n.
3) Each wind farm q located at bus n is paid for its *scheduled* wind power production W_q^s at a price λ_n.
4) Each generating unit i at bus n deploying upward reserve in scenario ω is paid for its overproduction $r_{i\omega}^{\mathrm{U}}$ at a price $\lambda_{n\omega}$.
5) Each generating unit i at bus n deploying downward reserve in scenario ω is charged for its power withdrawal $r_{i\omega}^{\mathrm{D}}$ at a price $\lambda_{n\omega}$. In this case, $\lambda_{n\omega}$ should be understood as a repurchase price.
6) Each wind farm q at bus n with production surplus in scenario ω is paid for its excess of generation $W_{q\omega} - W_q^s - W_{q\omega}^{\mathrm{spill}}$ at a price $\lambda_{n\omega}$.
7) Each wind farm q at bus n with generation shortage in scenario ω is charged for its production deficit $W_q^s - W_{q\omega} - W_{q\omega}^{\mathrm{spill}}$ at a price $\lambda_{n\omega}$.

This pricing scheme exhibits two important features, namely:

1) It is revenue adequate in expectation, i.e., the payments that the system/market operator must make to and receive from the participants do not cause it to incur a financial deficit. The probabilistic term *expectation* is required here due to the intrinsic dependence of the outcomes of the market-clearing tool (1) on the actual realization of wind generation. Intuitively speaking, a market settlement is said to be revenue adequate in expectation provided that it does not cause the system/market operator to run a financial deficit over time if used repeatedly over many trading periods.
2) It guarantees that the expected profit of the generating units (including the wind farms) is greater than or equal to their operating costs.

Proofs for both properties, revenue adequacy and cost recovery in expectation, are provided in the Appendix.

From an economic perspective, pool prices ("day-ahead prices", λ_n) account for the impact on the expected social welfare of a marginal *certain* increase in load. To supply this marginal certain increase in load, inflexible units can be used by means of day-ahead planning. Real-time prices $(\lambda_{n\omega})$ account for the impact on the expected social welfare of a marginal *uncertain* increase in load under a particular scenario. This marginal increase in load, for being uncertain, cannot be supplied by inflexible units, as these cannot be re-dispatched. In principle, the locational marginal prices (LMPs) derived from a deterministic day-ahead market-clearing algorithm are not related to the "day-ahead prices" obtained from the proposed clearing algorithm, as the latter also account for the *anticipated* impact of marginal load increases on the real-time dispatch costs. Likewise, the LMPs resulting from a deterministic real-time market are not necessarily connected to the real-time prices provided by the proposed market-clearing algorithm because the day-ahead dispatches in both settlements will be probably different. Nevertheless, since both sets of LMPs emanate from the same physical reality, they should not be expected to differ dramatically.

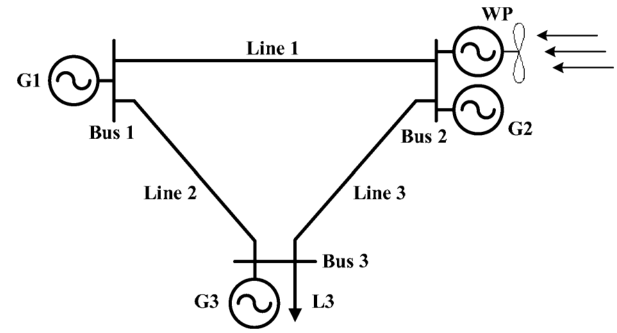

Fig. 1. Three-node system.

In practice, the real-time realization of the wind generation is not likely to be contained in the scenario set used to settle the market. In this case, according to [13, Theorem 1], balancing prices $\lambda_{n\omega}$ can always be determined by just solving the second stage of the proposed market-clearing model with the first-stage variables fixed to their optimal values, for the specific realization of the uncertain parameters. Note that this does not mean that a separate "real-time market" needs to be organized. It just means that balancing prices need to be recomputed for the specific uncertainty outcome. Moreover, if the uncertainty modeling is accurate enough, by definition, these real-time prices will approximately preserve the theoretical properties stated above, i.e., revenue reconciliation and cost recovery for producers in expectation.

Finally, we would like to point out that financial transmission rights (FTRs) can be implemented within the proposed market framework and used to refund congestion revenues collected by the transmission system operator to agents and to better allow agents to hedge congestion costs resulting from their energy purchases and sales in the market. However, FTRs are outside the scope of this paper.

III. Illustrative Example

The proposed pricing scheme is illustrated next using the three-node system sketched in Fig. 1. Line reactances and capacities are all equal to 0.13 p.u. and 100 MW, respectively. The system includes three conventional generators (G1, G2, and G3) and one wind power plant (WP). Data for the conventional units are provided in Table I. Note that, comparatively speaking, unit G1 is cheap, but inflexible; unit G2 is relatively cheap, but flexible; and unit G3 is expensive, but flexible. The wind plant is located at node 2. Its uncertain power output is modeled by means of three scenarios, which are referred to as *medium* (35 MW), *high* (50 MW), and *low* (10 MW), with probabilities of occurrence equal to 0.5, 0.2, and 0.3, in that order. The power block offered by the wind producer is assumed to be equal to its forecasted power production (i.e., 30.5 MW). The three-bus system also includes an inelastic load (L3) of 200 MW located at node 3, with a value of lost load equal to \$1000/MWh.

The market is cleared based on this information. Market outcomes related to dispatched quantities and deployed reserve are collated in Table II. The scheduled wind power production

TABLE I
GENERATOR DATA—THREE-BUS SYSTEM

Generator i	1	2	3
$P_i^{\max}$ (MW)	100	50	100
C_i ($/MWh)	20	25	30
$R_i^{\mathrm{U,max}}$ (MW)	0	20	30
$R_i^{\mathrm{D,max}}$ (MW)	0	20	30

TABLE II
MARKET OUTCOMES—THREE-BUS SYSTEM. POWERS IN MW

Unit	P_i	$r_{i\omega}^{\mathrm{U}}$			$r_{i\omega}^{\mathrm{D}}$		
		Medium	High	Low	Medium	High	Low
G1	100	0	0	0	0	0	0
G2	50	0	0	0	0	0	0
G3	30	0	0	10	15	30	0

TABLE III
ENERGY AND BALANCING PRICES—THREE-BUS SYSTEM. PRICES IN $/MWH

$\lambda_n, \forall n$	$\frac{\lambda_{n\omega}}{\pi_\omega}, \forall n$		
	Medium	High	Low
29	30	25	30

(W_q^s) is equal to 20 MW. The resulting energy and balancing prices are shown in Table III. Note that electricity prices are the same at all nodes in the system, because the network does not become congested in any of the three considered wind power scenarios. Given the energy and balancing prices in Table III and the dispatched quantities in Table II, the payments to market participants per scenario can be computed. For instance, the payment to generator G3 in scenario *low* is given by $30 \times 29 + 10 \times 30 = \$1170$. Furthermore, considering that the energy production cost of unit G3 is equal to $30/MWh, the profit that it makes in scenario *low* is $1170 - 30 \times 40 = -30\$/\mathrm{MWh}$. Table IV provides the benefit obtained by market participants both per scenario and in expectation. Observe that the profit made by generator G3 is indeed a random variable whose expected value $(-30 \times 0.5 + 120 \times 0.2 - 30 \times 0.3)$ is equal to zero. Generator G3 can be seen then as the marginal unit in a stochastic sense. The randomness of its profit is inherited from the uncertain character of the reserve deployment service, which in turn depends on the actual wind power realization. The proposed market settlement guarantees cost recovery for generating units in expectation, but this does not prevent generator G3 from incurring economic losses in scenarios *medium* and *low* (see Table IV).

To reduce the risk of negative profits faced by market participants that are willing to make real-time adjustments, reserve capacity bids can be introduced in the proposed market settlement as stated in Section II-A. Consider that generator G2 offers both downward and upward reserve capacity at a cost of $1/MW, while generator G3 does it at a cost of $2/MW. Table V shows the day-ahead schedule and the real-time redispatch in this case. The wind power production scheduled at the day-ahead stage is 20 MW again. Likewise, Tables VI and VII list, respectively, the clearing prices and the profit made by market participants

TABLE IV
PROFIT OF MARKET PARTICIPANTS—THREE-BUS SYSTEM. PROFIT IN $

	Expected	Per scenario		
		Medium	High	Low
G1	900	900	900	900
G2	200	200	200	200
G3	0	−30	120	−30
WP	865	1030	1330	280
L3	−5800	−5800	−5800	−5800

TABLE V
MARKET OUTCOMES WITH RESERVE CAPACITY BIDS—THREE-BUS SYSTEM. POWERS IN MW

Unit	P_i	$r_{i\omega}^{\mathrm{U}}$			$r_{i\omega}^{\mathrm{D}}$		
		Medium	High	Low	Medium	High	Low
G1	100	0	0	0	0	0	0
G2	40	10	0	10	0	0	0
G3	40	0	0	0	25	30	0

TABLE VI
ENERGY AND BALANCING PRICES WITH RESERVE CAPACITY BIDS—THREE-BUS SYSTEM. PRICES IN $/MWH

$\lambda_n, \forall n$	$\frac{\lambda_{n\omega}}{\pi_\omega}, \forall n$		
	Medium	High	Low
30	30	20	36.67

TABLE VII
PROFIT OF MARKET PARTICIPANTS WITH RESERVE CAPACITY BIDS—THREE-BUS SYSTEM. PROFIT IN $

	Expected	Per scenario		
		Medium	High	Low
G1	1000	1000	1000	1000
G2	260	250	200	316.7
G3	60	0	300	0
WP	835	1050	1200	233.3
L3	−6000	−6000	−6000	−6000

per scenario and in expectation when the aforementioned reserve capacity bids are taken into account to clear the market. As an example, observe that the benefit of generator G3 in scenario *low* is now given by $40 \times 30 - 40 \times 30 = \$0$, where we ignore the "costs" related to the reserve capacity bids inasmuch as the provision of reserve capacity does not entail specific costs to generators. Note, indeed, that generator G3 does not incurr economic losses in any of the three considered scenarios. Furthermore, its expected profit is equal to 60, i.e., greater than 0. Therefore, the possibility of bidding reserve capacity serves to competitively reward the capability of and the willingness to make real-time adjustments, thus promoting the flexibility of market participants in an efficient manner.

Lastly, we point out that accounting for reserve capacity bids avoids the existence of possible multiple solutions to the market-clearing model (1). For instance, Table VIII provides an alternative market scheduling to that shown in Table II.

TABLE VIII
ALTERNATIVE MARKET SCHEDULING TO THAT IN TABLE II. POWERS IN MW

Unit	P_i	$r_{i\omega}^{\mathrm{U}}$			$r_{i\omega}^{\mathrm{D}}$		
		Medium	High	Low	Medium	High	Low
G1	100	0	0	0	0	0	0
G2	39.5	10.5	10.5	10.5	0	0	0
G3	30	0	0	10	15	30	0

TABLE IX
RESERVE CAPACITIES OFFERED BY GENERATING UNITS ($* = \{\mathrm{U}, \mathrm{D}\}$)

Unit type	$R_i^{*,\max}$ (MW)
U76	20
U100	70
U155	30
U197	30
U12	12
U350	40

Observe that, in terms of social welfare, the following two actions are equivalent:

1) To dispatch generator G2 to 50 MW and the wind farm to 20 MW at the day ahead-stage (see Table II).
2) To dispatch generator G2 to 39.5 MW and the wind farm to 30.5 MW at the day-ahead stage and then redispatch generator G2 to 50 MW at the real-time stage by deploying 10.5 MW of upward reserve in any of the three possible wind power realizations considered (see Table VIII).

In contrast, if the market settlement allows for reserve capacity bids (different from zero), such a type of dispatch decisions are not equivalent anymore.

IV. CASE STUDY

The pricing scheme described in Section II-B is further illustrated using a 24-bus system based on the single-area version of the IEEE Reliability Test System—1996 [19]. For simplicity, the generating units of this well-known system are grouped by node and type. The only purpose behind this grouping is to facilitate the presentation and analysis of the simulation results. Thus, the simplified system consists of 34 lines, 12 generating units, and 17 loads. On the assumption of a perfectly competitive electricity market, the energy offers submitted by generating units represent their marginal costs of energy production, which are indicated in [19, Table VI]. We assume that nuclear and hydro power producers offer their energy production at zero price. The amount of reserve capacity that each generating unit is willing to provide, either downward or upward, is listed in Table IX. We assume that the nuclear and hydro generators are not technically able to provide reserve. No reserve capacity costs are considered.

Two wind farms comprising 2.5-MW wind turbines Nordex N80/2500 with a hub height of 105 m are located at nodes 7 and 8. The power curve of this turbine model is publicly available in [20]. Wind speeds at both wind sites are described by means of the same Weibull distribution with scale and shape parameters equal to 9.7 and 1.6, respectively. This probability distribution for wind speeds, in combination with the considered wind turbine model, results in a capacity factor for both wind farms of approximately 40%. This capacity factor has been estimated using the *Wind Turbine Power Calculator* provided in [20]. Besides, wind speeds at both wind sites are assumed to be correlated with a correlation coefficient of 0.5. Correlated samples are then obtained by using the sampling procedure described in [21]. An original set of 10 000 samples is first generated and subsequently reduced to 100 by applying the scenario reduction technique proposed in [22] and [23]. Selecting the right number of scenarios constitutes a tradeoff between model accuracy and tractability. We believe that current computational machinery allows considering a large enough number of scenarios. Note that the number of scenarios should be large enough so that adding any additional scenario does not change the market outcomes (preferably) or minimally changes them. We assume that wind power producers offer their forecast production at zero price. Note that nowadays this offering strategy constitutes a common practice for wind producers participating in electricity markets [24], [25].

The results reported below corresponds to one single period characterized by a total system demand of 2850 MW. This demand is geographically distributed among nodes as specified in [19, Table V]. Loads are assumed to be inelastic with a value of lost load equal to \$2000/MWh. Results for two different wind penetrations levels, 12.3% and 26.3%, are presented. The wind penetration level is given as the ratio of the installed wind capacity to the total system demand. The number of wind turbines in the farms at nodes 7 and 8 that are required to achieve these penetration levels are 40 and 100, and 100 and 200, respectively.

The market-clearing problem has been solved using CPLEX 9.0.2 under GAMS [17] on a Windows-based personal computer Intel(R) Core(TM) i5 with four processors clocking at 2.4 GHz and 6 GB of RAM. The required computational time is around five seconds.

For low wind penetration levels, such as 12.3%, there is enough *room* in the transmission network to accommodate the energy transactions settled at the market stage plus the subsequent energy redispatches in the form of deployed reserve without the occurrence of congestion events. Therefore, the wind energy injected at nodes 7 and 8 is able to reach every node in the system and as a result, no differences in prices exist among nodes. In particular, for a wind penetration level of 12.3%, the energy price (λ_n) is equal to \$19.9/MWh at every node. Likewise, the probability-removed balancing prices under the highest and lowest wind production scenarios, which will be denoted, respectively, by $\lambda_{n\overline{\omega}}/\pi_{\overline{\omega}}$ and $\lambda_{n\underline{\omega}}/\pi_{\underline{\omega}}$ hereafter, are \$16.0/MWh and \$21.7/MWh in that order, irrespective of the node under consideration. These probability-removed balancing prices are obtained by dividing each dual variable $\lambda_{n\omega}$ by its associated probability π_ω. This way, the energy prices and the probability-removed balancing prices are of the same order of magnitude. Further, according to [13], the balancing prices so transformed are dual optimal for the real-time market model that results from problem (1) once the wind power uncertainty is disclosed and first-stage variables (scheduled quantities) are fixed to their optimal values.

On the contrary, for high enough wind penetration levels, e.g., 26.3%, network bottlenecks become probable and consequently, the nodal prices differ. As an example, Table X shows, for this wind penetration level, the values of the energy price and the probability-removed balancing prices under the highest and

TABLE X
ENERGY AND BALANCING PRICES ($/MWH) AT SELECTED NODES. WIND PENETRATION LEVEL OF 26.3%

Bus #	1	3	5	7	8	9	10	15	24
λ_n	19.0	18.8	19.1	16.8	16.9	18.6	19.2	18.9	18.8
$\frac{\lambda_{n\overline{\omega}}}{\pi_{\overline{\omega}}}$	13.9	12.7	14.7	0	0	11.5	15.5	13.2	13.0
$\frac{\lambda_{n\underline{\omega}}}{\pi_{\underline{\omega}}}$	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7

TABLE XI
PAYMENTS TO MARKET PARTICIPANTS

Wind penetration (%)	Payments ($) to			
	Conventional producers	Wind producers	Loads	Total
12.3	54055.6	2657.8	-56713.4	0
26.3	49104.6	3449.7	-53282.4	-728.1

lowest wind production realizations at some selected nodes. In light of these prices, the following three observations are in order:

1) In the scenario of highest wind power production, the balancing prices are zero at the nodes where the wind farms are connected, namely, nodes 7 and 8. The reason for this is that any marginal increment of load at these nodes is satisfied, in this scenario, by the wind energy production that would be otherwise spilled due to the network congestion.
2) The balancing price in the scenario of lowest wind power production is node-independent. This is so because, under this scenario, the network does not become congested.
3) Even though the *scheduled* productions do not cause network congestion at the market stage, the energy price differs among nodes. This highlights the coupling between energy and balancing prices induced by the two-stage stochastic programming approach. Intuitively speaking, the energy price *anticipates* probable network bottlenecks during the real-time operation of the power system.

The payments to market participants under the proposed pricing scheme are indicated in Table XI for the two considered wind penetration levels. While the system operator makes payments to producers, it receives payments from consumers. This is why the *payments to loads* in this table are expressed in negative numbers. Observe that if the wind penetration level grows, the payments to conventional producers diminish, whereas the payments to wind producers increase. Logically, there is a transfer of revenues from conventional generators to wind producers at the same time that the payments from loads are reduced due to the *free* character of the wind energy. In either case, revenue adequacy in expectation is guaranteed. In fact, for a wind power penetration level of 26.3% (condition such that network congestion events are probable) the system operator is expected to incur a financial surplus of $728.1.

Lastly, Tables XII and XIII provide, respectively, the expected profits achieved by conventional and wind producers under the proposed pricing scheme. Observe that all the participants recover their production costs in expectation, thus making an expected profit greater than or equal to zero. In general, the

TABLE XII
EXPECTED PROFIT OF CONVENTIONAL GENERATING UNITS ($)

Unit #	Bus #	Type	Wind penetration	
			12.3%	26.3%
1	1	U76	1012.1	886.4
2	2	U76	1012.1	885.8
3	7	U100	138.1	203.5
4	13	U197	190.8	221.9
5	15	U12	0	0
6	15	U155	1468.9	1309.3
7	16	U155	1468.9	1310.6
8	18	U400	7959.8	7549.4
9	21	U400	7959.8	7548.9
10	22	U50	5969.8	5662.0
11	23	U155	2937.9	2626.1
12	23	U350	3218.2	2866.2
Total	–	–	33336.5	31070.1

TABLE XIII
EXPECTED PROFIT OF WIND PRODUCERS ($)

Bus #	Wind penetration	
	12.3%	26.3%
7	768.0	1215.2
8	1889.9	2234.5
Total	2657.8	3449.7

expected profits of conventional producers decrease as they are displaced from the energy supply by an increasing wind power penetration. Only generating units 3 and 4 see their expected profit increased due to the fact that they get more involved in the deployment of reserve with the increment in wind power penetration.

V. CONCLUSIONS AND FUTURE WORK

This paper describes a pricing scheme for a pool that incorporates a significant number of wind producers. The analysis carried out leads to the following conclusions:

1) The proposed pricing scheme is adapted to the specificities of wind producers, characterized by their variability and unpredictability. This constitutes no harm to conventional producers.
2) This pricing scheme is marginal and results in both cost recovery for producers and revenue reconciliation, both in expectation.
3) Two sets of marginal prices are derived: pool prices that reflect energy scheduling and balancing prices that reflect system operation.
4) The proposed prices are derived from the solution of an LP problem. Thus, they are obtained in an easy and robust manner.
5) The pricing scheme described in this paper does not embody non-convexities (e.g., start-up costs or minimum power output constraints). Future work is needed to incorporate such non-convexities.

APPENDIX

A. Revenue Adequacy in Expectation

Mathematically, the proposed market settlement is revenue adequate in expectation if, at the optimum, it holds

$$\sum_{n=1}^{N_B}\lambda_n^*\left(\sum_{i:(i,n)\in M_G} P_i^* + \sum_{q:(q,n)\in M_Q} W_q^{s*} - \sum_{j:(j,n)\in M_L} L_j\right) + \sum_{\omega=1}^{N_\Omega}\sum_{n=1}^{N_B}\pi_\omega\left(\frac{\lambda_{n\omega}^*}{\pi_\omega}\right) \times\left[\sum_{i:(i,n)\in M_G}\left(r_{i\omega}^{U*}-r_{i\omega}^{D*}\right) - \sum_{q:(q,n)\in M_\Omega}\left(W_q^{s*}+W_{q\omega}^{\text{spill}*}-W_{q\omega}\right)\right] \leq 0, \tag{3}$$

where $\lambda_{n\omega}^*/\pi_\omega$ is the probability-removed real-time prices and superscript "$*$" denotes optimal values.

Using the power balance (1b) and (1c), expression (3) can be equivalently rewritten as follows:

$$\sum_{n=1}^{N_B}\lambda_n^*\sum_{r:(n,r)\in\Lambda} B_{nr}\left(\delta_n^{0*}-\delta_r^{0*}\right) - \sum_{\omega=1}^{N_\Omega}\sum_{n=1}^{N_B}\pi_\omega\left(\frac{\lambda_{n\omega}^*}{\pi_\omega}\right) \times\left[\sum_{j:(j,n)\in M_L} L_j^{\text{shed}*} + \sum_{r:(n,r)\in\Lambda} B_{nr}\left(\delta_n^{0*}-\delta_{n\omega}^*-\delta_r^{0*}+\delta_{r\omega}^*\right)\right] \leq 0. \tag{4}$$

Let us consider the following partial Lagrangian function of problem (1):

$$\begin{aligned}\mathcal{L} = &\sum_{\omega=1}^{N_\Omega}\pi_\omega\left[\sum_{i=1}^{N_G} C_i\left(P_i+r_{i\omega}^U-r_{i\omega}^D\right)+\sum_{q=1}^{N_Q} C_q\left(W_{q\omega}-W_{q\omega}^{\text{spill}}\right) + \sum_{j=1}^{N_L} V_j^{\text{LOL}} L_{j\omega}^{\text{shed}}\right] \\ &+\sum_{n=1}^{N_B}\lambda_n\left[\sum_{j:(j,n)\in M} L_j - \sum_{i:(i,n)\in M_G} P_i - \sum_{q:(q,n)\in M_Q} W_q^s + \sum_{r:(n,r)\in\Lambda} B_{nr}\left(\delta_n^0-\delta_r^0\right)\right] \\ &-\sum_{\omega=1}^{N_\Omega}\sum_{n=1}^{N_B}\lambda_{n\omega}\left[\sum_{i:(i,n)\in M_G}\left(r_{i\omega}^U-r_{i\omega}^D\right) + \sum_{q:(q,n)\in M_Q}\left(W_{q\omega}-W_q^s-W_{q\omega}^{\text{spill}}\right) + \sum_{j:(j,n)\in M_L}^{N_L} L_{j\omega}^{\text{shed}} + \sum_{r:(n,r)\in\Lambda} B_{nr}\left(\delta_n^0-\delta_{n\omega}-\delta_r^0+\delta_{r\omega}\right)\right].\end{aligned} \tag{5}$$

Since problem (1) is linear and thus convex, $\mathcal{L}$ is minimized subject to the rest of constraints, i.e., constraints (1d)–(1p), at the optimum. Note that by moving the power balance (1b) and (1c) to the objective function (1a) to form the partial Lagrangian function $\mathcal{L}$, the resulting optimization problem [minimize (5), subject to (1d)–(1p)] can be decomposed into appropriate minimization subproblems for any given set of prices $\{\lambda_n, \forall n; \lambda_{n\omega}, \forall n, \forall\omega\}$. In particular, the summation of the following terms, extracted from (5)

$$\sum_{\omega=1}^{N_\Omega}\sum_{j=1}^{N_L}\pi_\omega V_j^{\text{LOL}} L_{j\omega}^{\text{shed}} + \sum_{n=1}^{N_B}\lambda_n\left[\sum_{r:(n,r)\in\Lambda} B_{nr}\left(\delta_n^0-\delta_r^0\right)\right] - \sum_{\omega=1}^{N_\Omega}\sum_{n=1}^{N_B}\lambda_{n\omega}\left[\sum_{j:(j,n)\in M_L} L_{j\omega}^{\text{shed}} + \sum_{r:(n,r)\in\Lambda} B_{nr}\left(\delta_n^0-\delta_{n\omega}-\delta_r^0+\delta_{r\omega}\right)\right] \tag{6}$$

is minimized subject to constraints (1j)–(1l) and (1n)–(1p) at the optimum.

A solution such that $\delta_n^0 = 0, \forall n; \delta_{n\omega} = 0, \forall n, \forall\omega; L_{j\omega}^{\text{shed}} = 0, \forall j, \forall\omega$, is feasible for the minimization subproblem [minimize (6), subject to (1j)–(1l) and (1n)–(1p)]. This solution allows us to set the upper bound of expression (6) to zero. Given that $\sum_{\omega=1}^{N_\Omega}\sum_{j=1}^{N_L}\pi_\omega V_j^{\text{LOL}} L_{j\omega}^{\text{shed}} \geq 0$, inequality (4) holds and therefore, the proposed market settlement is revenue adequate in expectation.

B. Cost Recovery in Expectation

The proposed market settlement ensures that the generating units (including wind farms) recover their energy production costs in expectation. Mathematically, this is expressed as follows:

$$C_i P_i^* + \sum_{\omega=1}^{N_\Omega}\pi_\omega C_i\left(r_{i\omega}^{U*}-r_{i\omega}^{D*}\right) - \sum_{n:(i,n)\in M_G}\left[\lambda_n^* P_i^* + \sum_{\omega=1}^{N_\Omega}\pi_\omega\left(\frac{\lambda_{n\omega}^*}{\pi_\omega}\right)\left(r_{i\omega}^{U*}-r_{i\omega}^{D*}\right)\right] \leq 0, \quad \forall i; \tag{7}$$

and

$$\sum_{\omega=1}^{N_\Omega}\pi_\omega C_q\left(W_{q\omega}-W_{q\omega}^{\text{spill}*}\right) - \sum_{n:(q,n)\in M_Q}\left[\lambda_n^* W_q^{s*} + \sum_{\omega=1}^{N_\Omega}\pi_\omega\left(\frac{\lambda_{n\omega}^*}{\pi_\omega}\right)\left(W_{q\omega}-W_q^{s*}-W_{q\omega}^{\text{spill}*}\right)\right] \leq 0, \quad \forall q, \tag{8}$$

where superscript "$*$" denotes optimal values.

Let us consider again the partial Lagrangian function (5). At the optimum, this function is minimized subject to constraints (1d)–(1p). As stated in the proof for revenue adequacy in expectation, the optimization problem [minimize (5), subject to (1d)–(1p)] can be decomposed into appropriate minimization subproblems for any given set of prices $\{\lambda_n, \forall n; \lambda_{n\omega}, \forall n, \forall \omega\}$. Specifically, the series of terms extracted from (5)

$$\sum_{\omega=1}^{N_\Omega} \pi_\omega C_i\left(P_i + r_{i\omega}^U - r_{i\omega}^D\right) - \sum_{n:(i,n)\in M_G} \left[\lambda_n P_i + \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(\frac{\lambda_{n\omega}}{\pi_\omega}\right)\left(r_{i\omega}^{\mathrm{U}} - r_{i\omega}^{\mathrm{D}}\right)\right], \text{ and} \quad (9)$$

$$\sum_{\omega=1}^{N_\Omega} \pi_\omega C_q\left(W_{q\omega} - W_{q\omega}^{\mathrm{spill}}\right) - \sum_{n:(q,n)\in M_Q} \left[\lambda_n W_q^s + \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(\frac{\lambda_{n\omega}}{\pi_\omega}\right)\left(W_{q\omega} - W_q^s - W_{q\omega}^{\mathrm{spill}}\right)\right] \quad (10)$$

are minimized, for all i and for all q, subject to the set of constraints [(1d), (1f)–(1i), (1p)] and [(1e), (1m), (1p)], respectively.

The collection of primal variables such that $P_i = 0, \forall i$ (here, we require that generators can be dispatched to zero), and $r_{i\omega}^U = r_{i\omega}^D = 0, \forall i, \forall \omega$, constitutes a feasible solution to the minimization subproblem made up of the objective function (9) and the group of constraints (1d), (1f)–(1i), and (1p). Likewise, the set of primal variables such that $W_q^s = 0, \forall q$, and $W_{q\omega}^{\mathrm{spill}} = W_{q\omega}, \forall q, \forall \omega$, is a feasible solution to the minimization subproblem composed of the objective function (10) and constraints (1e), (1m), and (1p). This pair of solutions sets the upper bound of expressions (9) and (10) to zero and consequently, inequalities (7) and (8) hold. Therefore, the proposed pricing scheme guarantees cost recovery in expectation for both conventional and wind power producers.

It is important to note that the decomposition-based reasoning above cannot be used to prove cost recovery per scenario due to the market-stage variables P_i and W_q^s, which link all the scenarios together. In fact, numerical results show that the suggested pricing scheme allows power producers to incur economic losses in some scenarios as long as they recover their production costs in expectation, i.e., in the long run and under similar conditions.

The reasoning above is also valid if reserve capacity bids are considered within the proposed market-clearing model (1). In this case, the cost recovery in expectation leads to inequality (11) below, which states that the expected profit made by generator i from selling energy is greater than or equal to its reserve capacity costs. Therefore, insofar as the provision of reserve capacity does not entail specific costs to generating units, reserve capacity bids allow producers to reduce the risk of incurring economic losses:

$$-\left[C_i P_i^* + \sum_{\omega=1}^{N_\Omega} \pi_\omega C_i \left(r_{i\omega}^{U*} - r_{i\omega}^{D*}\right)\right] + \sum_{n:(i,n)\in M_G} \left[\lambda_n^* P_i^* + \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(\frac{\lambda_{n\omega}^*}{\pi_\omega}\right)\left(r_{i\omega}^{\mathrm{U}*} - r_{i\omega}^{\mathrm{D}*}\right)\right] \geq C_i^{\mathrm{U}} R_i^{\mathrm{U}} + C_i^{\mathrm{D}} R_i^{\mathrm{D}}, \quad \forall i. \quad (11)$$

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