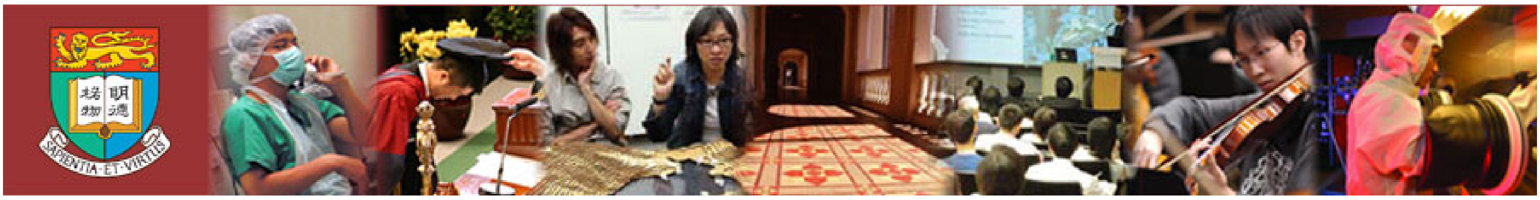

Ductility Design of High-Strength Concrete Beams and Columns

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Abstract: High-strength concrete (HSC) is increasingly used for the construction of tall buildings and long span bridges. However, most engineers just focus on how to better utilize the strength potential of HSC and little attention is paid to ensure that the HSC structures would have sufficient ductility. In this regard, it should be noted that the current design codes, which do not provide any guidelines for ductility design, are not applicable to HSC structures. In recent years, the authors have been conducting research on how the use of HSC would affect the flexural ductility of concrete members. Herein, an overall summary of their research is presented. It will be shown that depending on the reinforcement detailing and loading conditions, the ductility of HSC structures is not necessarily lower than that of normal concrete structures. Finally, guidelines for the ductility design of HSC beams and columns, augmented with design formulas and charts, are given.

Key words: beams, columns, ductility, high-strength concrete.

1. INTRODUCTION

Compared to normal-strength concrete (NSC), high-strength concrete (HSC) has the advantage of providing a higher strength for carrying more loads but also the disadvantage of rendering a lower ductility for resisting accidental overloading, impact and earthquake. For instance, it has been found that HSC beams, when heavily reinforced such that compression failure occurs, would fail a more brittle manner than NSC beams (Pecce and Fabbrocino 1999; Lin and Lee 2001; Pam *et al.* 2001a, b; Debernardi and Taliano 2002). Likewise, HSC columns, when subjected to the same axial load level (the same axial load to axial load capacity ratio), are generally more brittle than NSC columns (Li *et al.* 1991; Bayrak and Sheikh 1998; Ho and Pam 2003a; Pam and Ho 2009). Hence, the ductility of HSC members has been a major concern.

From the safety point of view, ductility should be regarded as crucial as strength (Park and Paulay 1975; Watson and Park 1994; Marefat *et al.* 2006; Xiao and Zhang 2006; Oehlers *et al.* 2008; Wilson 2009). Therefore, seismic resistant buildings or bridges are usually designed using performance-based approach (Li and Wu 2006; Liang 2007; Xuan *et al.* 2008; Xue *et al.* 2008). To improve the ductility of concrete beams and columns, one of the general methods is to provide significant amount of confinement in the form of reinforcing steel (Ho and Pam 2003a; Wu *et al.* 2004; Su and Wong 2007; Yang *et al.* 2009; Yalim *et al.* 2009; Zheng and Xie 2009), or in the form of steel tube (Han *et al.* 2005, 2008; Choi *et al.* 2006; Cai and Long 2007; Lu and Sun 2007; Shan *et al.* 2007; Tao *et al.* 2007; Zhang and Guo 2007; Choi *et al.* 2008; Park *et al.* 2008) or in the form of FRP (Al-Emrani and Kliger 2006; Haritos *et al.* 2006; Jiang and Teng 2007; Wong *et al.* 2008; Lam and Teng 2009; Ilki *et al.* 2009, Wu and Wei 2010). The ductility of reinforced concrete beams can also be improved significantly by compressive yielding

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of perforated P-blocks (Wu 2006, 2008). However, compared to strength analysis, ductility analysis is more difficult. To evaluate the flexural ductility, it is necessary to conduct nonlinear moment-curvature analysis extended into the post-peak range. Moreover, since strain reversal (decrease of strain despite monotonic increase of curvature) could occur, the stress-path dependence of the steel reinforcement needs to be taken into account (Pam *et al.* 2001a). Because of such complexities, it is not usually practical to demand ductility analysis and, therefore, in the current design codes (Standard Australia 2001; Ministry of Construction 2002; Buildings Department 2004; European Committee for Standardization 2004; ACI Committee 318 2008), only empirical deemed-to-satisfy rules, which limit the maximum tension steel or neutral axis depth in beams and impose minimum confinement in columns, are stipulated to ensure the provision of nominal ductility. However, these existing rules were developed in the past for NSC members and are not really applicable to HSC members (Li *et al.* 1991; Bayrak and Sheikh 1998; Ho and Pam 2003a) because the maximum tension steel/neutral axis depth and minimum confinement should vary with the concrete strength.

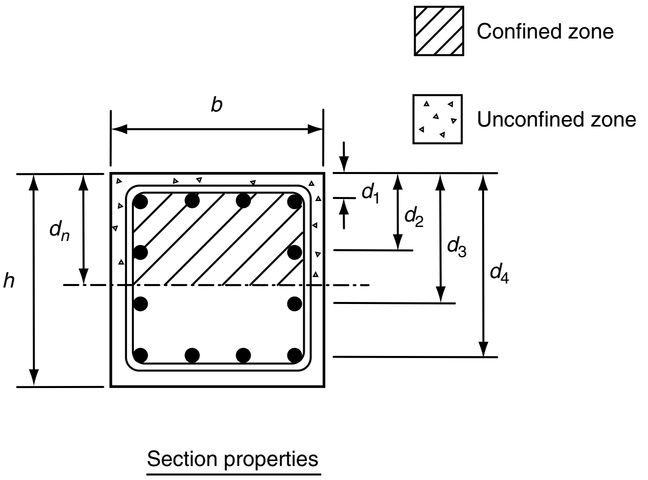
As HSC is becoming more and more commonly used and many engineers are just following the existing rules with little attention paid to the provision of sufficient ductility, it is now a matter of urgency to develop appropriate rules for the ductility design of HSC members. To resolve this problem, the authors have been studying in recent years the effects of various structural parameters, including the concrete strength, steel yield strength, compression steel ratio, tension steel ratio, confinement and axial load, on the flexural ductility of NSC and HSC members (Pam *et al.* 2001a, b; Kwan *et al.* 2002, 2004a, 2004b, 2006; Ho *et al.* 2003, 2004, 2009; Lam *et al.* 2009a, b). Based on these studies, design formulas for direct evaluation of the flexural ductility of beams and columns have been derived. Furthermore, to ensure the provision of a consistent minimum level of ductility to HSC members at not lower than the minimum level being provided to NSC members, a new ductility design method that is applicable at all concrete strength levels has been developed. Lastly, guidelines supplementing the existing deemed-to-satisfy detailing rules for incorporation into the design codes are proposed.

2. NONLINEAR MOMENT-CURVATURE ANALYSIS

To study the flexural behaviour and ductility of reinforced concrete members, nonlinear moment-curvature analysis was employed. For the concrete, the stress-strain curve was based on the model developed by Attard and Setunge (1996), which is applicable to both confined and unconfined concrete up to a concrete strength of 130 MPa. For the steel reinforcement, the stress-strain curve was assumed to be linearly elastic-perfectly plastic and, to allow for strain reversal, the descending branch was assumed to have the same gradient as the initial elastic branch. In the analysis, it was assumed that: (1) Plane sections remain plane after bending; (2) The tensile strength of concrete is negligible; (3) There is no slip between concrete and steel reinforcement; (4) The concrete core is confined while the concrete cover is unconfined. (5) The confining pressure provided to the concrete core by confinement is assumed to be constant throughout the concrete compression zone. Assumptions (1) to (4) are commonly accepted and have been adopted by various researchers (Pam *et al*. 2001a; Au and Kwan 2004; Wu *et al*. 2004). Assumption (5) is not exact because the confining pressure varies in the concrete compression zone with strain gradient. However, as this happens within a narrow range of concrete strain, the differences in the confined concrete compressive force and moment capacity are not significant (Ho *et al*. 2010). The moment-curvature behaviour of the section was analysed by applying prescribed curvatures incrementally starting from zero. At a prescribed curvature, the stresses developed in the concrete and steel were determined from the strain profile and their respective stress-strain curves. Then, the neutral axis depth and resisting moment were evaluated from the equilibrium conditions. Such procedure was repeated until the section had entered well into the post-peak stage.

Using the above moment-curvature analysis, a series of parametric studies have been carried out. The sections analysed are shown in Figure 1. In order to study the effects of the various structural parameters, the in-situ concrete strength f_{co} was varied from 40 to 100 MPa, the steel yield strength f_y was varied from 250 to 600 MPa, the confining pressure f_r was varied from 0 to 4 MPa (the confining pressure may be evaluated using the method developed by Mander *et al*. 1988), the compression steel ratio ρ_c of beam section was varied from 0 to 2%, the tension steel ratio ρ_t of beam section was varied from 0.4 to 2 times the balanced steel ratio ρ_b, the longitudinal steel ratio ρ of column section was varied from 1 to 6%, and the axial load level $P/A_g f_{co}$ of column section was varied from 0.1 to 0.6.

Three failure modes have been observed: (1) Tension failure, in which the tension steel yields during failure; (2) Balanced failure, in which the most highly stressed tension steel just yields during failure; and (3) Compression failure, in which none of the tension steel yields during failure. In beams, the three failure modes occur when the tension steel ratio ρ_t is smaller than,



Beams

- b = 300 mm
- h = 600 mm
- d_1 = 50 mm
- d_4 = 550 mm
- d_2 and d_3 not exist
- $\rho_t = A_{st}/bd$ = 0.4 to 2.0ρ_b
- $\rho_c = A_{sc}/bd$ = 0 to 2%

Columns

- $b = h$ = 1000 mm
- $A_g = bh$ = 1.0 m^2
- d_1 = 80 mm
- d_2 = 360 mm
- d_3 = 640 mm
- d_4 = 920 mm
- $P/A_g f_{co}$ = 0.1 to 0.6
- $\rho = A_s/A_g$ = 1 to 6%

Figure 1. Beam and column sections analysed

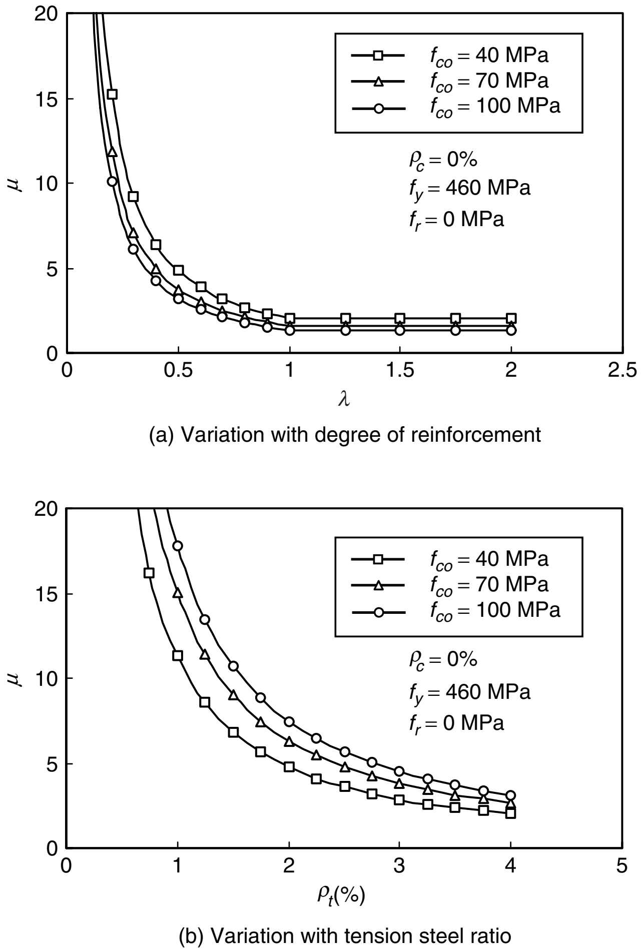


Figure 2. Flexural ductility of beams at different concrete strengths

equal to and larger than the balanced steel ratio ρ_b, respectively. It has been found that the balanced steel ratio ρ_b of a beam section is related to the balanced steel ratio ρ_{bo} of the same beam section with no compression reinforcement by $\rho_b = \rho_{bo} + \rho_c$. Using regression analysis, a formula for direct evaluation of ρ_{bo} when high-yield steel (f_y = 460 MPa) is used has been derived as:

$$\rho_{bo} = 0.005\ \left(f_{co}\right)^{0.58}\left(1+1.2 f_r\right)^{0.3} \tag{1}$$

In columns, the three failure modes occur when the axial load level $P/A_g f_{co}$ is lower than, equal to and higher than the balanced axial load level $(P/A_g f_{co})_b$, respectively. Using regression analysis, a formula for direct evaluation of $(P/A_g f_{co})_b$ when high-yield steel (f_y = 460 MPa) is used has been derived as:

$$\left(P/A_g f_{co}\right)_b = 3.1\ \left(f_{co}\right)^{-0.5}\left(1+2 f_r\right)^{0.3} \tag{2}$$

Both the above two formulas are accurate to within 10% error.

From the moment-curvature curve, the flexural ductility of each section may be evaluated in terms of the curvature ductility factor μ defined by Park and Paulay (1975) as $\mu = \phi_u/\phi_y$ where ϕ_u and ϕ_y are the ultimate and yield curvatures, respectively. The ultimate curvature ϕ_u is taken as the curvature when the resisting moment has, after reaching the peak moment M_p, dropped to 0.8 M_p. The yield curvature ϕ_y is taken as the curvature at which the peak moment M_p would be reached if the stiffness of the section is equal to the secant stiffness at 0.75 M_p (Watson and Park 1994). Based on the curvature ductility factors so evaluated, the effects of various structural parameters on the flexural ductility of beams and columns have been studied, as presented in the following sections.

3. FLEXURAL DUCTILITY OF BEAMS

In the case of beams, the effect of the concrete strength f_{co} has been found to be dependent on the amounts of tension and compression reinforcement, which determine the degree of reinforcement of the section. Herein, the degree of reinforcement is denoted by λ and explicitly defined as $\lambda = (\rho_t - \rho_c)/\rho_{bo}$. When $\lambda < 1$, λ = 1 and $\lambda > 1$, the section is under-reinforced, balanced and over-reinforced, respectively. To study the effect of the concrete strength f_{co}, the curvature ductility factor μ is plotted for different concrete strengths of f_{co} = 40, 70 and 100 MPa against the degree of reinforcement λ in Figure 2(a) and against the tension steel ratio ρ_t in

Figure 2(b). From these figures, it can be seen that as λ or ρ_t increases, μ decreases until it reaches a relatively low and constant value when the section becomes over-reinforced. It can also be seen that at the same degree of reinforcement λ, the ductility factor μ is lower at a higher concrete strength f_{co}. This is because of the gradual reduction in material ductility as the concrete strength f_{co} increases. However, at the same tension steel ratio ρ_t, the ductility factor μ is higher at a higher concrete strength f_{co}. This is because as the concrete strength f_{co} increases, the balanced steel ratio ρ_{bo} also increases, leading to decrease in the degree of reinforcement λ and increase in the ductility factor μ. Hence, the flexural ductility of a HSC beam is not necessarily lower, albeit the HSC is more brittle *per se*.

To study the effect of the confining pressure f_r, the curvature ductility factor μ is plotted for different confining pressures of f_r = 0, 1 and 2 MPa against the degree of reinforcement λ in Figure 3(a) and against the tension steel ratio ρ_t in Figure 3(b). It is observed that at a fixed degree of reinforcement λ, the ductility factor μ increases significantly with the confining pressure f_r. This is because of the gradual increase in material ductility of the confined concrete as the confining pressure f_r increases. It is also observed that at a fixed tension steel ratio ρ_t, the ductility factor μ increases substantially with the confining pressure f_r. This is because apart from the material ductility of the confined concrete, the balanced steel ratio ρ_{bo} also increases with the confining pressure f_r, leading to decrease in the degree of reinforcement λ and further increase in the ductility factor μ. Hence, in general, the provision of confinement is an effective means of improving the flexural ductility of beams.

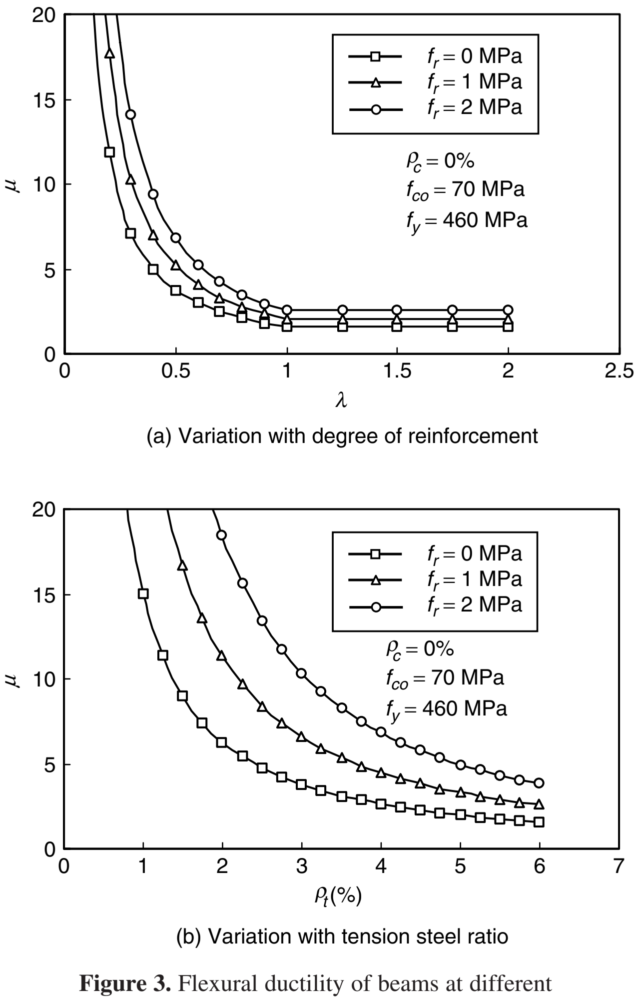


Figure 3. Flexural ductility of beams at different confining pressures

To enable direct evaluation of the flexural ductility of beams without conducting any nonlinear moment-curvature analysis, the values of μ obtained from the parametric studies are correlated to the various structural parameters using regression analysis to produce the following formulas:

$$\mu = 10.7m(\lambda)^{-1.25n}(f_{co})^{-0.45}(f_y/460)^{-0.25} \quad \text{(3a)}$$

$$m = 1 + 2.5(f_{co})^{0.5}(f_r/f_{co}) \quad \text{(3b)}$$

$$n = 1 + 5.0(f_r/f_{co}) \quad \text{(3c)}$$

where all strength values are in MPa and λ is to be taken as 1.0 when it is larger than 1.0. Within the range of parameters studied, the above formulas for direct evaluation of μ are accurate to within 10% error.

As the use of HSC in place of NSC could increase the flexural strength but at the same time decrease the flexural ductility, some engineers may query how much net benefit could be derived from HSC. To appraise the benefit of using HSC, it is necessary to consider the concurrent flexural strength and ductility that could be achieved. In Figure 4, the concurrent flexural strength and ductility that could be achieved at different concrete strengths with or without compression reinforcement provided are plotted in the form of μ vs. M_p/bd^2 curves. From the figure, it is apparent that the μ vs. M_p/bd^2 curve is generally higher at a higher concrete strength. This implies that the use of HSC could increase the flexural strength at a given flexural ductility, increase the flexural ductility at a given flexural strength or increase both the flexural strength and flexural ductility. Likewise, the μ vs. M_p/bd^2 curve is also generally higher when compression reinforcement is provided. Hence, the provision of compression reinforcement could also increase the flexural strength and/or flexural

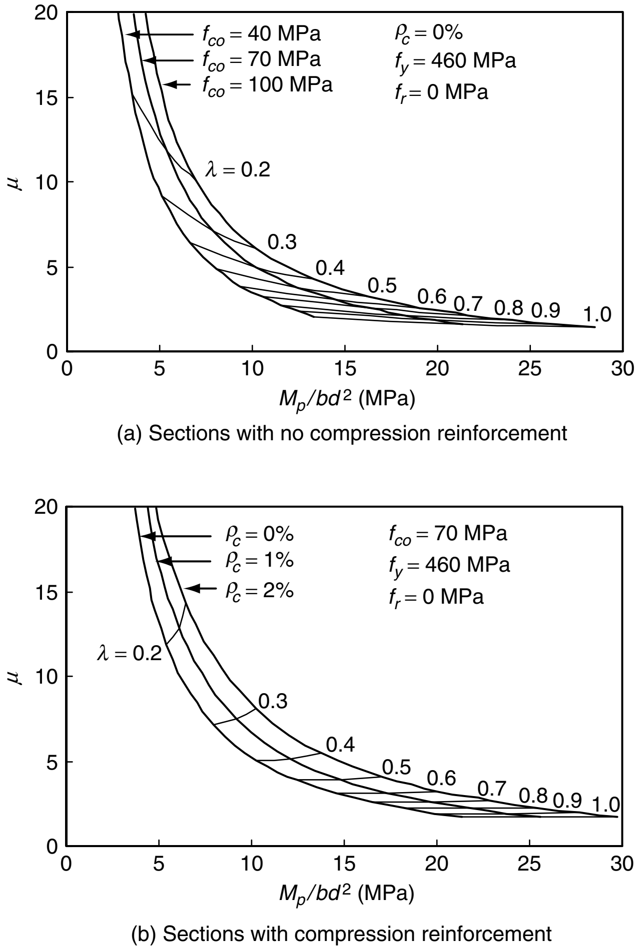


Figure 4. Concurrent flexural strength and ductility performance of beams

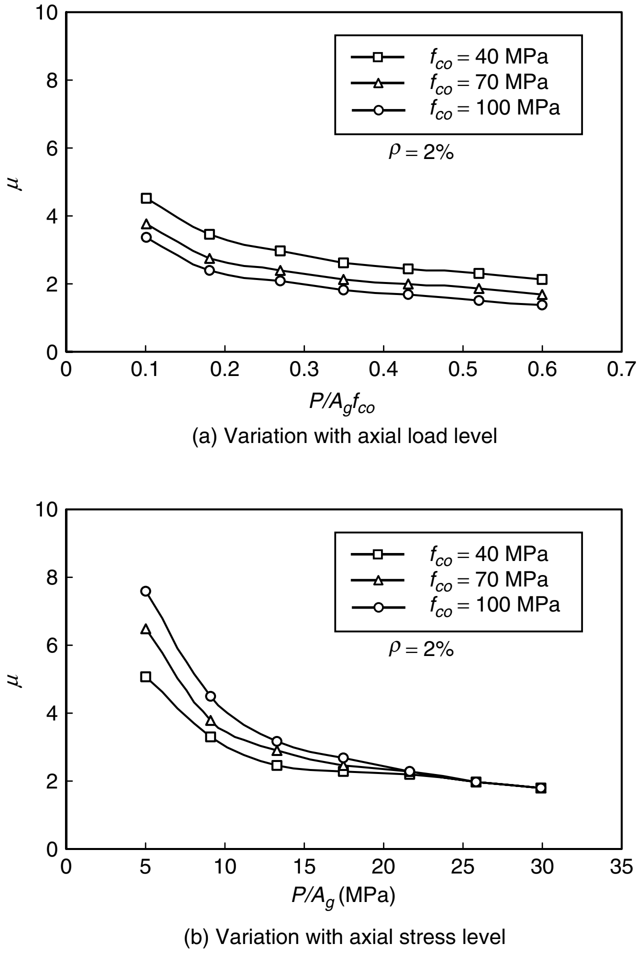


Figure 5. Flexural ductility of columns at different concrete strengths

ductility. For designing beams to meet with concurrent flexural strength and ductility requirements, Figure 4 may be used as a design chart.

4. FLEXURAL DUCTILITY OF COLUMNS

In the case of columns, the effect of the concrete strength f_{co} has been found to be dependent on the axial load. To study the effect of the concrete strength f_{co}, the curvature ductility factor μ is plotted for different concrete strengths of f_{co} = 40, 70 and 100 MPa against the axial load level $P/A_g f_{co}$ in Figure 5(a) and against the axial stress level P/A_g in Figure 5(b). From these figures, it can be seen that as $P/A_g f_{co}$ or P/A_g increases, μ decreases at a gradually decreasing rate. It can also be seen that at the same axial load level $P/A_g f_{co}$, the ductility factor μ is lower at a higher concrete strength f_{co}. This is because of the gradual reduction in material ductility as the concrete strength f_{co} increases. However, at the same axial stress level P/A_g, the ductility factor μ is higher at a higher concrete strength f_{co}. This is because as the concrete strength f_{co} increases, the axial load capacity $A_g f_{co}$ also increases, leading to decrease in the axial load level $P/A_g f_{co}$ and increase in the ductility factor μ. Hence, the flexural ductility of a HSC column is not necessarily lower, albeit the HSC is more brittle *per se*.

To study the effect of the longitudinal steel ratio ρ, the curvature ductility factor μ is plotted against the longitudinal steel ratio ρ at constant axial load level $P/A_g f_{co}$ in Figure 6(a) and at constant axial stress level P/A_g in Figure 6(b). From both figures, it can be seen that when $P/A_g f_{co} < 0.3$ or $P/A_g < 20$ MPa, the ductility factor μ is relatively high and gradually decreases as the longitudinal steel ratio ρ increases but when $P/A_g f_{co} \geq 0.3$ or $P/A_g \geq 20$ MPa, the ductility factor μ is relatively low and almost independent of the longitudinal steel ratio ρ. Hence, the longitudinal steel ratio ρ does have some effect when the flexural ductility is relatively high but has little effect when the flexural ductility is relatively low and causing concern (Wu *et al*. 2004).

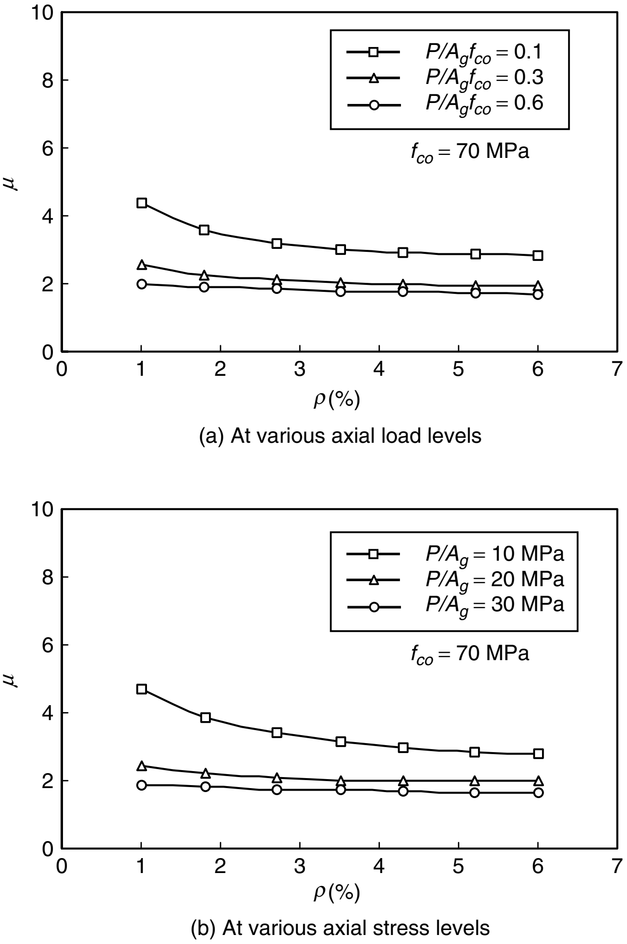


Figure 6. Variation of ductility of columns with longitudinal steel ratio

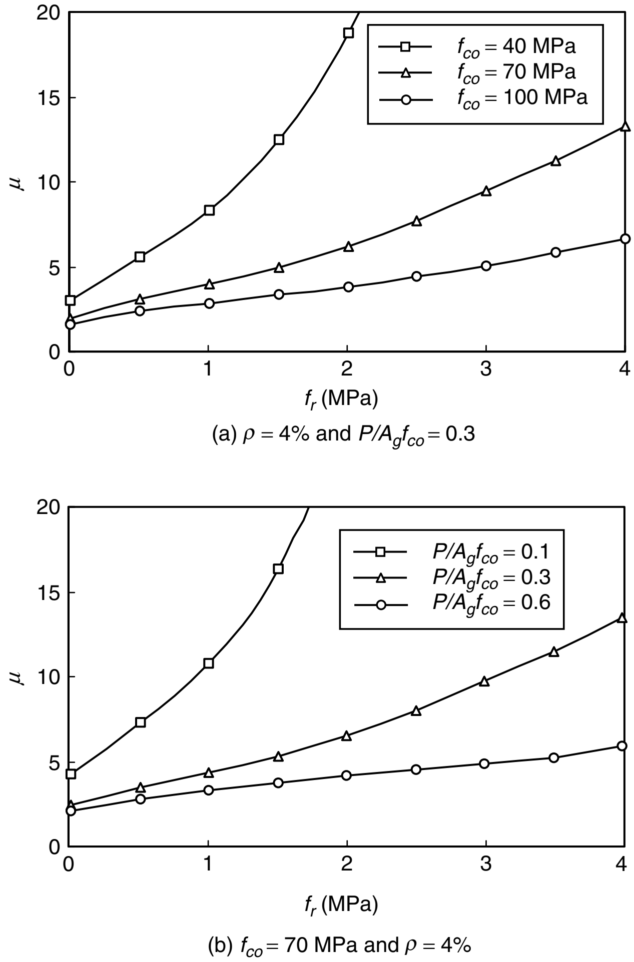


Figure 7. Variation of ductility of columns with confining pressure

To study the effect of the confining pressure f_r, the curvature ductility factor μ is plotted against the confining pressure f_r at constant concrete strength f_{co} in Figure 7(a) and at constant axial load level $P/A_g f_{co}$ in Figure 7(b). From both figures, it is obvious that in all cases, the provision of confinement is an effective means of improving the flexural ductility of columns. However, the effectiveness of providing confinement is generally higher at lower concrete strength or lower axial load level. When the concrete strength and/or axial load level is relatively high, in which case the ductility tends to be low, the effectiveness of providing confinement is lower and thus a disproportionately larger amount of confinement may be needed for ductility improvement.

From the above, it may be concluded that the major factors affecting the flexural ductility of columns are the concrete strength, axial load level, confining pressure and longitudinal steel ratio. However, their effects are dependent on the failure mode and thus when evaluating the flexural ductility of columns, columns failing in tension and columns failing in compression have to be dealt with separately. Using regression analysis of the data obtained from the parametric studies, the following formulas for direct evaluation of the flexural ductility of columns have been developed:

When tension failure or balanced failure occurs:

$$\mu = 10.7m\left(\frac{\rho_t - \rho_c}{\rho_{bo}} + \frac{P}{A_{sb} f_y}\right)^{-1.25n} \left(f_{co}\right)^{-0.45}\left(f_y/460\right)^{-0.25} \tag{4}$$

When compression failure occurs:

$$\mu = 14.0\left(\frac{P/A_g f_{co}}{\left(P/A_g f_{co}\right)_b}\right)^{-0.45} \left(f_{co}\right)^{-0.45}\left(1 + 30\left(f_r/f_{co}\right)\right) \tag{5}$$

where m and n are the same as those given by Eqns 3b and 3c, and A_{sb} is the balanced steel area (the area of tension steel causing balanced failure). Both the above formulas are accurate to within 15%.

5. COMPARISON WITH EXPERIMENTAL RESULTS

To verify the validity of the above formula for concrete beams and columns, the flexural ductility predicted by Eqn 3 for concrete beams, and Eqns 4 and 5 for concrete columns, have been compared with experimental results obtained by other researchers. For the sake of comparison, it should be noted that the value of f_{co} for each beam and column is taken as $0.85\eta f_c'$, where η is the ratio of the average concrete stress of the equivalent rectangular stress block to the cylinder strength f_c' as stipulated in EC2 (European Committee for Standardization 2004). However, if only the concrete cube strength f_{cu} is provided, f_{co} is then taken as $0.72f_{cu}$ (Ho *et al.* 2002). On the other hand, the value of confining pressure is evaluated according to the formula provided by Mander *et al.* (1988). For symmetrically reinforced concrete columns, the following equation can be used:

$$f_r = 0.5k_e\rho_s f_{ys} \tag{6}$$

where k_e is the confinement effectiveness factor, ρ_s is the volumetric ratio of confinement and f_{ys} is the yield strength of confinement.

For concrete beams, the curvature ductility factors evaluated by Eqn 3 are compared with the curvature ductility factors obtained by Pecce and Fabbrocino (1999), Lin and Lee (2001) and Debernardi and Taliano (2002). The comparison is summarised in Table 1. From the table, it is evident that the predicted curvature ductility factors using Eqn 3 are all very close to the respective curvature ductility factors obtained by other researchers from experiment. It thus verifies the accuracy and applicability of Eqn 3 in predicting the ductility of normal- and high-strength concrete beams.

For unconfined concrete columns, where f_r is negligible, the curvature ductility factors evaluated by Eqn 4 or 5 are compared with the curvature ductility factors obtained by Sheikh and Yeh (1990) and Ho and Pam (2003a, b). The comparison is summarised in Table 2. From the table, it is evident that the predicted curvature ductility factors using Eqn 4 for columns failing in tension, and Eqn 5 for columns failing in compression, are all very close to the respective curvature ductility factors obtained by other researchers from experiment. It thus verifies the accuracy and applicability of Eqns 4 and 5 in predicting the ductility of unconfined normal- and high-strength concrete columns.

Table 1. Comparison of predicted ductility factors of concrete beams

					Curvature ductility factor μ		
Specimen code	**f_c' (MPa)**	**f_{co} (MPa)**	**f_y (MPa)**	**$\rho_t[\rho_c]$ (%)**	**By Eqn 3 [1]**	**Test Result [2]**	**$\frac{[1]}{[2]}$**
Debernardi and Taliano (2002)							
T1A1	27.7	23.5	587	0.67[0.30]	34.9	34.1	1.02
T2A3	27.7	23.5	587	1.33[0.59]	14.7	15.2	0.97
T3A1	27.7	23.5	587	2.00[0.59]	6.6	7.1	0.93
T5A3	27.7	23.5	587	0.63[0.22]	30.7	30.0	1.02
T6A1	27.7	23.5	587	1.28[0.22]	9.4	7.5	1.25
T7A3	27.7	23.5	587	1.98[0.23]	5.0	4.0	1.25
T10A3	27.7	23.5	587	0.60[0.13]	26.0	22.2	1.17
T11A1	27.7	23.5	587	1.21[0.13]	9.2	9.6	0.96
Lin and Lee (2001)							
N2	41.0	34.9	490	1.29[0.65]	20.3	21.3	0.95
N3	41.0	34.9	490	1.96[0.63]	8.2	10.0	0.82
T2	41.0	34.9	490	1.29[0.65]	20.3	18.7	1.09
T3	41.0	34.9	490	1.96[0.63]	8.2	12.0	0.68
T4	41.0	34.9	490	2.94[0.71]	4.3	6.1	0.70
Pecce and Fabbrocino (1999)							
A	41.3	35.1	471	2.59[0.10]	3.8	3.8	1.00
B	41.3	35.1	454	1.08[0.10]	12.3	12.7	0.97
C	42.3	36.0	534	2.20[0.04]	4.4	4.5	0.98
AH	93.8	63.8	471	2.59[0.10]	4.5	6.3	0.71
BH	93.8	63.8	454	1.08[0.10]	14.5	12.6	1.15
CH	95.4	64.9	534	2.20[0.04]	5.2	5.2	1.00

Table 2. Comparison of predicted ductility factors of unconfined concrete columns

Specimen code	f_{cu} or [f_c'] (MPa)	f_{co} (MPa)	$(P/A_g f_{co})$	$(P/A_g f_{co})_b$	Curvature ductility factor μ #By Eqn [1]	Test Result [2]	$\frac{[1]}{[2]}$
Ho and Pam (2003a)							
BS-80-01-09-R6	89.6	64.5	0.150	0.472	6.5	7.3	0.89
BS-80-01-09-R8	85.4	61.5	0.149	0.484	6.8	7.2	0.94
BS-80-01-09-R10	83.2	59.9	0.154	0.506	6.6	7.7	0.86
Ho and Pam (2003b)							
BS-60-06-61-S	56.5	40.7	0.753	0.596	2.2	2.3	0.96
BS-60-06-61-C	60.4	43.5	0.738	0.595	2.1	2.4	0.88
BS-100-03-24-S	95.1	68.5	0.393	0.505	2.1	2.9	0.72
BS-100-03-24-C	109.5	78.8	0.425	0.470	1.8	1.7	1.06
Sheikh and Yeh (1990)							
E-8	[25.9]	22.0	0.913	0.989	2.7	3.2	0.84
A-11	[27.9]	23.7	0.867	0.929	2.6	4.3	0.60
F-12	[33.4]	28.4	0.707	0.858	2.9	3.7	0.78
D-14	[26.9]	22.9	0.880	0.954	2.7	4.3	0.63
A-16	[33.9]	28.8	0.706	0.876	3.0	5.3	0.57

If $(P/A_g f_{co}) \le (P/A_g f_{co})_b$, then Eqn 4 is used.
If $(P/A_g f_{co}) > (P/A_g f_{co})_b$, then Eqn 5 is used.

Table 3. Comparison of predicted ductility factors of confined concrete columns

Specimen code	f_{cu} or [f_c'] (MPa)	f_{co} (MPa)	$(P/A_g f_{co})$	$(P/A_g f_{co})_b$	f_r (MPa)	Curvature ductility factor μ #By Eqn [1]	Test Result [2]	$\frac{[1]}{[2]}$
Ho (2003)								
NEW-100-03-61-C	108.8	78.4	0.417	0.681	4.1	9.8	11.3	0.87
NEW-80-03-24-S	90.4	65.1	0.389	0.712	3.4	9.3	9.8	0.95
Ho and Pam (2003a)								
NEW-80-01-09-R12	85.9	61.8	0.151	0.589	1.2	12.5	12.8	0.98
Ho and Pam (2003b)								
NEW-100-03-24-C	108.4	78.0	0.419	0.687	4.2	7.6	9.0	0.84
NEW-60-06-61-C	62.4	44.9	0.735	0.870	3.6	8.4	10.2	0.82
NEW-60-06-61-S	57.1	41.1	0.738	0.889	3.3	9.0	8.3	1.08
Sheikh *et al.* (1994)								
AS-3H	[54.1]	46.0	0.728	0.818	3.0	2.1	2.8	0.75
AS-18H	[54.7]	46.5	0.752	0.932	5.0	2.4	2.3	1.04
AS-20H	[53.6]	45.6	0.756	1.034	7.0	2.7	3.0	0.90
Sheikh and Khoury (1993)								
FS-9	[32.4]	27.5	0.894	1.057	3.0	3.2	3.5	0.91
ES-13	[32.5]	27.7	0.894	1.033	2.7	3.1	3.4	0.91
AS-3	[33.2]	28.2	0.706	1.045	3.0	5.5	6.9	1.08
AS-17	[31.3]	26.6	0.906	1.076	3.0	3.3	3.1	1.06
AS-18	[32.8]	27.8	0.906	1.204	5.0	3.6	3.3	1.09
AS-19	[32.3]	27.4	0.553	0.993	2.3	9.4	12.0	0.78
Sheikh and Yeh (1990)								
E-2	[31.4]	26.7	0.719	1.063	2.9	5.8	5.4	1.07
F-4	[32.2]	27.4	0.700	1.051	2.9	5.9	6.8	0.87
D-5	[31.2]	26.5	0.541	1.067	2.9	8.2	10.6	0.77
F-6	[27.2]	23.1	0.879	1.139	2.8	4.6	4.8	0.96
E-10	[26.3]	22.3	0.901	1.164	2.9	4.7	3.9	1.21
E-13	[27.2]	23.1	0.868	1.139	2.8	4.8	5.5	0.87
D-15	[26.9]	22.9	0.880	1.152	2.9	4.8	5.5	0.87

If $(P/A_g f_{co}) \le (P/A_g f_{co})_b$, then Eqn 4 is used.
If $(P/A_g f_{co}) > (P/A_g f_{co})_b$, then Eqn 5 is used.

For confined concrete columns, where f_r is significant, the curvature ductility factors evaluated by Eqn 4 or 5 are compared with the curvature ductility factors obtained by Sheikh and Yeh (1990), Sheikh and Khoury (1993), Sheikh *et al.* (1994) and Ho and Pam (2003a, b). The comparison is summarised in Table 3. From the table, it is evident that the predicted curvature ductility factors using Eqn 4 for columns failing in tension, and Eqn 5 for columns failing in compression, are all very close to the respective curvature ductility factors obtained by other researchers from experiment. It thus verifies the accuracy and applicability of Eqns 4 and 5 in predicting the ductility of confined normal- and high-strength concrete columns.

6. MINIMUM DUCTILITY DESIGN

In the current design codes, there are no specific guidelines for the ductility design of any concrete members. Only nominal ductility is provided by stipulating some empirical deemed-to-satisfy detailing rules. The curvature ductility factors achieved by following these rules vary significantly and are generally lower at a higher concrete strength. With a view to maintain the flexural ductility when using HSC in place of NSC, it is advocated that instead of following the existing empirical rules, the members should be designed to achieve a consistent minimum curvature ductility factor, denoted herein by $\mu_{\min}$, which should not be lower than that being provided to NSC members. Based on previous studies by the authors (Au and Kwan 2004; Ho *et al.* 2004; Kwan *et al.* 2006; Lam *et al.* 2009b), it is proposed to set $\mu_{\min} = 3.32$ for non-earthquake resistant structures. For earthquake resistant structures, a higher value of $\mu_{\min}$ should be adopted, depending on the actual ductility demand of the member.

For beams, the ductility design may be carried out by imposing the condition that the value of μ evaluated by Eqn 3 must be higher than $\mu_{\min}$. This condition implies that for any given values of f_{co}, f_y and f_r, there is a maximum allowable value of λ and a maximum allowable value of $(\rho_t - \rho_c)$, which are denoted by $\lambda_{\max}$ and $(\rho_t - \rho_c)_{\max}$, respectively. For illustration, the values of $(\rho_t - \rho_c)_{\max}$ for achieving $\mu_{\min} = 3.32$ when no confinement is provided are listed in Table 4. These values indicate that the value of $(\rho_t - \rho_c)_{\max}$ increases as the concrete strength f_{co} increases but decreases as the steel yield strength f_y increases. An alternative ductility design method is to limit the neutral axis depth d_n to not more than a certain fraction of the effective depth d, as in most of the current design codes (Standard Australia 2001; Ministry of Construction 2002; Buildings Department 2004; European Committee for Standardization 2004). The maximum allowable values of d_n/d for achieving $\mu_{\min} = 3.32$ when no confinement is provided are also listed in the table. It is clear from these values that the maximum allowable value of d_n/d decreases significantly as the concrete strength f_{co} and/or steel yield strength f_y increases. For comparison, the respective flexural strengths that can be achieved with no compression reinforcement provided are also presented in the table. From these strength values, it is evident that the use of a higher strength concrete would allow a higher flexural strength to be attained while maintaining the same $\mu_{\min}$, whereas the use of a higher strength steel would reduce the flexural strength that can be attained at the same $\mu_{\min}$.

For columns, the ductility design may be carried out by imposing the condition that the value of μ evaluated by Eqn 4 or 5 must be higher than $\mu_{\min}$. It follows from this condition that for any given values of f_{co}, f_y and f_r, there is a maximum allowable value of $P/A_g f_{co}$ and for any given values of f_{co}, f_y and $P/A_g f_{co}$, there is a minimum allowable value of f_r. The maximum allowable value of $P/A_g f_{co}$ and the minimum allowable value of f_r are denoted by $(P/A_g f_{co})_{\max}$ and $(f_r)_{\min}$,

Table 4. Maximum steel and neutral axis depth for minimum ductility design of beams

	Maximum allowable value of $(\rho_t - \rho_c)$ (%)		Maximum allowable value of (d_n/d)		Maximum M_p/bd^2 at $\rho_c = 0\%$ (MPa)	
f_{co} (MPa)	f_y = 460 MPa	f_y = 600 MPa	f_y = 460 MPa	f_y = 600 MPa	f_y = 460 MPa	f_y = 600 MPa
40	2.67	1.76	0.428	0.371	10.24	9.05
50	2.93	1.92	0.391	0.338	11.49	10.07
60	3.15	2.05	0.364	0.314	12.55	10.90
70	3.35	2.18	0.343	0.295	13.51	11.71
80	3.53	2.29	0.327	0.281	14.37	12.40
90	3.69	2.38	0.313	0.268	15.14	12.98
100	3.89	2.47	0.302	0.258	15.82	13.55

Table 5. Maximum axial load level for minimum ductility design of columns

	Maximum axial load level $(P/A_g f_{co})$max					
f_{co} (MPa)	f_r = 0 MPa	f_r = 0.5 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	0.26	0.56	0.75	0.97	> 1.0	> 1.0
50	0.20	0.35	0.62	0.82	0.97	> 1.0
60	0.16	0.32	0.53	0.71	0.86	0.94
70	0.12	0.27	0.39	0.63	0.76	0.85
80	0.10	0.27	0.32	0.57	0.68	0.77
90	0.09	0.23	0.29	0.50	0.61	0.70
100	0.08	0.22	0.26	0.42	0.56	0.63

Table 6. Minimum confining pressure for minimum ductility design of columns

	Minimum confining pressure $(f_r)_{min}$ (MPa)					
f_{co} (MPa)	$P/A_g f_{co}$ = 0.1	$P/A_g f_{co}$ = 0.2	$P/A_g f_{co}$ = 0.3	$P/A_g f_{co}$ = 0.4	$P/A_g f_{co}$ = 0.5	$P/A_g f_{co}$ = 0.6
40	0.00	0.00	0.06	0.30	0.39	0.50
50	0.00	0.00	0.23	0.52	0.68	0.98
60	0.00	0.02	0.39	0.73	1.00	1.43
70	0.00	0.09	0.57	0.95	1.34	1.88
80	0.00	0.20	0.78	1.20	1.68	2.27
90	0.02	0.34	1.07	1.51	2.02	2.89
100	0.07	0.46	1.35	1.85	2.47	3.47

respectively. The values of $(P/A_g f_{co})_{max}$ and $(f_r)_{min}$ for achieving $\mu_{min} = 3.32$ at different concrete strengths and a fixed steel yield strength of 460 MPa are presented in Tables 5 and 6, respectively. From Table 5, it can be seen that the value of $(P/A_g f_{co})_{max}$ decreases as the concrete strength f_{co} increases but increases as the confining pressure f_r increases. Hence, when HSC is used in place of NSC with no corresponding increase in the confining pressure, the maximum axial load level has to be reduced, thus limiting the beneficial use of HSC. From Table 6, it can also be seen that the value of $(f_r)_{min}$ increases as the concrete strength f_{co} or the axial load level $P/A_g f_{co}$ increases. Hence, for heavily load HSC columns, it is particularly important to provide a sufficiently high confining pressure to maintain a minimum level of flexural ductility.

7. SUPPLEMENTARY GUIDELINES

To remedy the situation that the existing deemed-to-satisfy detailing rules would yield a lower flexural ductility at a higher concrete strength, supplementary design guidelines for incorporation into the design codes are formulated herein.

For the design of beams, the current practices (ACI Committee 318 2008; Standard Australia 2001; Buildings Department 2004, European Committee for Standardization 2004) are either to limit the tension steel ratio ρ_t at not larger than 0.75 times the balanced steel ratio ρ_{bo} or to limit the neutral axis depth d_n at not larger than 0.50 times the effective depth d. To ensure that HSC beams so designed would achieve a minimum flexural ductility of $\mu_{min} = 3.32$, it is proposed to add compression and/or confining reinforcement to make up the reduction in flexural ductility due to the use of HSC. The compression steel ratio ρ_c and confining pressure f_r required have been evaluated as:

When ρ_t is limited to $0.75\rho_{bo}$:

$$3350\rho_c + 54 f_r = f_{co} - 30 \qquad (7)$$

When d_n is limited to $0.50d$:

$$2570\rho_c + 39 f_r = f_{co} - 30 \qquad (8)$$

where f_{co} and f_r are in MPa and $40 \le f_{co} \le 100$ MPa.

For the design of columns, the current practice (Standard Australia 2001; Ministry of Construction 2002; Buildings Department 2004; European Committee for Standardization 2004; ACI Committee 318 2008) is to set limits on the minimum size and maximum spacing of the confining reinforcement so as to provide certain nominal confinement. It has been found that the nominal confinement would provide a confining pressure of only about 0.20 MPa (Lam *et al.* 2009b). With such low confining pressure provided, the flexural ductility could become dangerously low when HSC is used. To resolve this problem and ensure that the HSC columns would achieve a minimum flexural ductility of μ_{min} = 3.32, it is proposed to set a limit on either the maximum axial load level $(P/A_g f_{co})_{max}$ or the minimum confining pressure $(f_r)_{min}$. The required limits have been tabulated in Tables 5 and 6. For easy application, these tabulated limits have been transformed into the following inequality by regression analysis:

$$\frac{\left(P/A_g f_{co}\right)}{\left(1+3.5 f_r\right)^{0.65}} \le 24.5\left(f_{co}\right)^{-1.20} \tag{9}$$

where f_{co} and f_r are in MPa and $40 \le f_{co} \le 100$ MPa.

8. CONCLUSIONS

The flexural ductility of high-strength concrete beams and columns has been studied by extensive parametric studies using nonlinear moment-curvature analysis. It was found that for beams, the major factors affecting the flexural ductility are the concrete strength, steel yield strength, degree of reinforcement and confining pressure. Generally, at the same degree of reinforcement, the flexural ductility decreases as the concrete strength increases while with the same reinforcement details, the flexural ductility increases as the concrete strength increases. More importantly, the reduction in flexural ductility due to the use of higher strength concrete and/or steel can always be compensated by adding more compression and/or confining reinforcement.

For columns, the major factors affecting the flexural ductility are the concrete strength, steel yield strength, axial load/stress level and confining pressure. Generally, at the same axial load level, the flexural ductility decreases as the concrete strength increases while at the same axial stress level, the flexural ductility increases as the concrete strength increases. As the axial load level is seldom adjusted downwards when high-strength concrete is used, the flexural ductility could become dangerously low as the concrete strength increases. Nevertheless, the reduction in flexural ductility due to the use of higher strength concrete and/or steel can always be compensated by adding more confining reinforcement.

For practical applications, formulas for direct evaluation of the balanced steel ratio and balanced axial load level, which are needed to determine the failure mode, have been developed. To avoid cumbersome nonlinear moment-curvature analysis, formulas for direct evaluation of the flexural ductility of beams and columns have also been developed. These formulas have been compared with the curvature ductility factors of normal- and high-strength concrete beams and columns obtained by other researchers from experiment. From the comparison, it is evident that the proposed formulas can predict the ductility of concrete beams and columns fairly accurately.

Based on these formulas, a minimum ductility design method for ensuring the achievement of a minimum ductility of μ_{min} = 3.32 has been proposed. Lastly, in order to remedy the situation that the existing deemed-to-satisfy detailing rules could yield an unacceptably low ductility at high concrete strength, supplementary guidelines on the addition of compression and confining reinforcement to beams and on the maximum axial load level and minimum confinement to be imposed to columns have been formulated for incorporation into the design codes.

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REFERENCES

ACI Committee 318 (2008). *Building Code Requirements for Reinforced Concrete and Commentary ACI 318M-08*, Manual of Concrete Practice, American Concrete Institute, Michigan, USA.

Al-Emrani, M. and Kliger, R. (2006). "Experimental and numerical investigation of the behaviour and strength of composite steel-CFRP members", *Advances in Structural Engineering*, Vol. 9, No. 6, pp. 819–831.

Attard, M.M. and Setunge, S. (1996). "The stress strain relationship of confined and unconfined concrete", *ACI Materials Journal*, Vol. 93, No. 5, pp. 432–442.

Au, F.T.K. and Kwan, A.K.H. (2004). "A minimum ductility design method for non-rectangular high-strength concrete beams", *Computers and Concrete*, Vol. 1, No. 2, pp. 115–130.

Bayrak, O. and Sheikh, S.A. (1998). "Confinement reinforcement design consideration for ductile HSC columns", *Journal of Structural Engineering*, ASCE, Vol. 124, No. 9, pp. 999–1010.

Buildings Department (2004). *Code of Practice for Structural Use of Concrete 2004*, The Government of Hong Kong Special Administrative Region.

Cai, J. and Long, Y.L. (2007). "Axial load behavior of rectangular CFT stub columns with binding bars", *Advances in Structural Engineering*, Vol. 10, No. 5, pp. 551–565.

Choi, S., Lee, S., Hong, S. and Kim, J. (2008). "Structural capacities of tension side for CFT square column-to-bean connections with combined-cross diaphragm", *Advances in Structural Engineering*, Vol. 11, No. 2, pp. 209–227.

Choi, S.M., Hong, S.D. and Kim, Y.S. (2006). "Modeling analytical moment-rotation curves of semi-rigid connections for CFT square columns and steel beams", *Advances in Structural Engineering*, Vol. 9, No. 5, pp. 697–706.

Debernardi, P.G. and Taliano, M. (2002). "On evaluation of rotation capacity for reinforced concrete beams", *ACI Structural Journal*, Vol. 99, No. 3, pp. 360–368.

European Committee for Standardization (2004). *Eurocode 2: Design of Concrete Structures: Part 1-1: General Rules and Rules for Buildings*, UK.

GB 50010 (2002). *Code for Design of Concrete Structures*, Ministry of Construction, Beijing, China.

Han, L.H., You, J.T. and Lin, X.K. (2005). "Experimental behaviour of self-consolidating concrete (SCC) filled hollow structural steel (HSS) columns subjected to cyclic loadings", *Advances in Structural Engineering*, Vol. 8, No. 5, pp. 497–512.

Han, L.H., Lin, X.K. and Yang, Y.F. (2008). "Cyclic performance of concrete filled steel tubular columns after exposure to fire: analysis and simplified model", *Advances in Structural Engineering*, Vol. 11, No. 4, pp. 455–473.

Haritos, N., Hira, A., Mendis, P. and Perera, U. (2006). "Repair and strengthening of reinforced concrete structures using CFRPs", *Advances in Structural Engineering*, Vol. 9, No. 1, pp. 1–9.

Ho, J.C.M., Kwan, A.K.H. and Pam, H.J. (2002). "Ultimate concrete strain and equivalent rectangular stress block for design of high-strength concrete beams", *The Structural Engineer*, Vol. 80, No. 16, pp. 26–32.

Ho, J.C.M., Kwan, A.K.H. and Pam, H.J. (2003). "Theoretical analysis of post-peak flexural behaviour of normal- and high-strength concrete beams", *The Structural Design of Tall and Special Buildings*, Vol. 12, No. 2, pp. 109–125.

Ho, J.C.M., Kwan, A.K.H. and Pam, H.J. (2004). "Minimum flexural ductility design of high-strength concrete beams", *Magazine of Concrete Research*, Vol. 56, No. 1, pp. 13–22.

Ho, J.C.M., Lam, J.Y.K. and Kwan, A.K.H. (2010). "Effectiveness of adding confinement for ductility improvement of high-strength concrete columns", *Engineering Structures*, Vol. 32, No. 3, pp. 714–725.

Ho, J.C.M. and Pam, H.J. (2003a). "Inelastic design of low-axially loaded high-strength reinforced concrete columns", *Engineering Structures*, Vol. 25, No. 8, pp. 1083–1096.

Ho, J.C.M. and Pam, H.J. (2003b). "Influence of transverse steel configuration on post-elastic behaviour of high-strength reinforced concrete columns", *Transactions of the Hong Kong Institution of Engineers*, Vol. 10, No. 2, pp. 1–9.

Ilki, A., Demir, C., Bedirhanoglu, I. and Kumbasar, N. (2009). "Seismic retrofit of brittle and low strength RC columns using fibre reinforced polymer and cementitious composites", *Advances in Structural Engineering*, Vol. 12, No. 3, pp. 325–347.

Jiang, T. and Teng, J.G. (2007). "Analysis-oriented stress-strain models for FRP-confined concrete", *Engineering Structures*, Vol. 29, No. 11, pp. 2968–2986.

Kwan, A.K.H., Au, F.T.K. and Chau, S.L. (2004a). "Effects of confinement on flexural strength and ductility design of HS concrete beams", *The Structural Engineers*, Vol. 84, No. 23–24, pp. 38–44.

Kwan, A.K.H., Chau, S.L. and Au, F.T.K. (2006). "Improving flexural ductility of high-strength concrete beams", *Proceedings, Institution of Civil Engineers*, *Structures and Buildings*, Vol. 159, No. SB6, pp. 339–347.

Kwan, A.K.H., Ho, J.C.M. and Pam, H.J. (2002). "Flexural strength and ductility of reinforced concrete beams", *Proceedings, Institution of Civil Engineers*, *Structures and Buildings*, Vol. 152, No. 4, pp. 339–347.

Kwan, A.K.H., Ho, J.C.M. and Pam, H.J. (2004b). "Effects of concrete grade and steel yield strength on flexural ductility of reinforced concrete beams", *Australian Journal of Structural Engineering*, Vol. 5, No. 2, pp. 119–138.

Lam, J.Y.K., Ho, J.C.M. and Kwan, A.K.H. (2009a). "Flexural ductility of high-strength concrete columns with minimal confinement", *Materials and Structures*, Vol. 42, No. 7, pp. 909–921.

Lam, J.Y.K., Ho, J.C.M. and Kwan, A.K.H. (2009b). "Maximum axial load level and minimum confinement for ductility design of concrete columns", *Computer and Concrete*, Vol. 6, No. 5, pp. 357–376.

Lam, L. and Teng, J.G. (2009). "Stress-strain model for FRP-confined concrete under cyclic axial compression", *Engineering Structures*, Vol. 31, No. 2, pp. 308–321.

Li, B., Park, R. and Tanaka, H. (1991). "Effect of confinement on the behaviour of high strength concrete columns under seismic loading", *Proceedings, Pacific Conference on Earthquake Engineering*, Auckland, pp. 67–78.

Li, Y.H. and Wu, B. (2006). "Determination of equivalent damping relationships for direct displacement-based seismic design method", *Advances in Structural Engineering*, Vol. 9, No. 2, pp. 279–291.

Liang, Q.Q. (2007). "Performance-based optimization: a review", *Advances in Structural Engineering*, Vol. 10, No. 6, pp. 739–753.

Lin, C.H. and Lee, F.S. (2001). "Ductility of high-performance concrete beams with high-strength lateral reinforcement", *ACI Structural Journal*, Vol. 98, No. 4, pp. 600–608.

Lu, F.W. and Sun, G. (2007). "Nonlinear equivalent simulation of mechanical properties of expansive concrete-filled steel tube columns", *Advances in Structural Engineering*, Vol. 10, No. 3, pp. 273–281.

Mander, J.B., Priestley, M.J.N. and Park, R. (1988). "Theoretical stress-strain model for confined concrete", *Journal of Structural Engineering*, ASCE, Vol. 114, No. 8, pp. 1804–1825.

Marefat, M.S., Khanmohammadi, M., Bahrani, M.K. and Goli, A. (2006). "Experimental assessment of reinforced concrete columns with deficient seismic details under cyclic load", *Advances in Structural Engineering*, Vol. 9, No. 3, pp. 337–347.

Oehlers, D.J., Mohamed Ali, M.S. and Griffith, M.C. (2008). "Concrete Component of the rotational ductility of reinforced concrete flexural members", *Advances in Structural Engineering*, Vol. 11, No. 3, pp. 281–291.

Pam, H.J. and Ho, J.C.M. (2009). "Length of critical region for confinement steel in limited ductility high-strength reinforced concrete columns", *Engineering Structures*, Vol. 31, No. 12, pp. 2896–2908.

Pam, H.J., Kwan, A.K.H. and Ho, J.C.M. (2001a). "Post-peak behavior and flexural ductility of doubly reinforced normal- and high-strength concrete beams", *Structural Engineering and Mechanics*, Vol. 12, No. 5, pp. 459–474.

Pam, H.J., Kwan, A.K.H. and Islam, M.S. (2001b). "Flexural strength and ductility of reinforced normal- and high-strength concrete beams", *Proceedings*, *Institution of Civil Engineers*, *Structures and Buildings*, Vol. 146, No. 4, pp. 381–389.

Park, S., Choi, S., Park, Y., Kim, Y. and Kim, J. (2008). "Ductility characteristics of partially restrained beam-to-column composite connections in concrete filled square tubes", *Advances in Structural Engineering*, Vol. 11, No. 5, pp. 565–575.

Park, R. and Paulay, T. (1975). *Reinforced Concrete Structures*, Wiley, New York, U.S.A.

Pecce, M. and Fabbrocino, G. (1999). "Plastic rotation capacity of beams in normal and high-performance concrete", *ACI Structural Journal*, Vol. 96, No. 2, pp. 290–296.

Shan, J.H., Chen, R., Zhang, W.X., Xiao, Y., Yi, W.J. and Lu, F.Y. (2007). "Behavior of concrete filled tubes and confined concrete filled tubes under high speed impact", *Advances in Structural Engineering*, Vol. 10, No. 2, pp. 209–218.

Sheikh, S.A. and Khoury, S.S. (1993). "Confined concrete columns with stubs", *ACI Structural Journal*, Vol. 90, No. 4, pp. 414–431.

Sheikh, S.A., Shah, D.V. and Khoury, S.S. (1994). "Confinement of high-strength concrete columns", *ACI Structural Journal*, Vol. 91, No. 1, pp. 100–111.

Sheikh, S.A. and Yeh, C.C. (1990). "Tied concrete columns under axial load and flexure", *Journal of Structural Division*, ASCE, Vol. 116, No. 10, pp. 2780–2801.

Standard Australia (2001). *Australian Standard for Concrete Structures AS 3600-2001*, Australia.

Su, R.K.L. and Wong, S.M. (2007). "Seismic behaviour of slender reinforced concrete shear walls under high axial load ratio", *Engineering Structures*, Vol. 29, No. 8, pp. 1957–1965.

Tao, Z., Han, L.H. and Zhuang, J.P. (2007). "Axial loading behavior of CFRP strengthened concrete-filled steel tubular stub columns", *Advances in Structural Engineering*, Vol. 10, No. 1, pp. 37–46.

Watson, S. and Park, R. (1994). "Simulated seismic load tests on reinforced concrete columns", *Journal of Structural Engineering*, ASCE, Vol. 120, No. 6, pp. 1825–1849.

Wilson, J.L. (2009). "The cyclic behaviour of reinforced concrete chimney sections with and without openings", *Advances in Structural Engineering*, Vol. 12, No. 3, pp. 411–420.

Wong, Y.L., Yu, T., Teng, J.G. and Dong, S.L. (2008). "Behavior of FRP-confined concrete in annular section columns", *Composites Part B: Engineering*, Vol. 39, No. 3, pp. 451–466.

Wu, Y.F. (2006). "New avenue of achieving ductility for reinforced concrete members", *Journal of Structural Engineering*, ASCE, Vol. 132, No. 9, pp. 1502–1506.

Wu, Y.F. (2008). "Ductility demand of compression yielding fiber-reinforced polymer-reinforced concrete beams", *ACI Structural Journal,* Vol. 105, No. 1, pp. 104–110.

Wu, Y.F., Oehlers, D.J. and Griffith M.C. (2004). "Rational definition of the flexural deformation capacity of RC column sections", *Engineering Structures*, Vol. 26, No. 5, pp. 641–650.

Wu, Y.F. and Wei, Y.Y. (2010). "Effects of cross-sectional aspect ratio on the strength of CFRP-confined rectangular concrete columns", *Engineering Structures*, Vol. 32, pp. 32–45.

Xiao, J. and Zhang, C. (2006). "Experimental investigation on the limitation of axial load level of RC columns in seismic regions", *Advances in Structural Engineering*, Vol. 9, No. 3, pp. 349–359.

Xuan, G., Shahrooz, B.M., Harries, K.A. and Rassati, G.A. (2008). "A performance-based design approach for coupled core wall systems with diagonally reinforced concrete coupling beams", *Advances in Structural Engineering*, Vol. 11, No. 3, pp. 253–268.

Xue, Q., Wu, C.W., Chen, C.C. and Chen, K.C. (2008). "The draft code for performance-based seismic design of buildings in Taiwan", *Engineering Structures*, Vol. 30, No. 6, pp. 1535–1547.

Yalim, B., Kalayci, A.S. and Mirmiran, A. (2009). "Performance of FRP-strengthened RC beams with surface out-of-flatness", *Advances in Structural Engineering*, Vol. 12, No. 2, pp. 241–255.

Yang, Y.F., Han, L.H. and Zhu, L.T. (2009). "Experimental performance of recycled aggregate concrete-filled circular steel tubular columns subjected to cyclic flexural loadings", *Advances in Structural Engineering*, Vol. 12, No. 2, pp. 183–194.

Zhang, S. and Guo, L. (2007). "Behaviour of high strength concrete-filled slender RHS steel tubes", *Advances in Structural Engineering*, Vol. 10, No. 4, pp. 337–351.

Zheng, W.Z. and Xie, H.Y. (2009). "Experimental research and analysis on encased steel box concrete beams", *Advances in Structural Engineering*, Vol. 12, No. 2, pp. 211–230.

NOTATION

A_g	area of beam or column section ($= bh$)
A_s	total area of longitudinal steel reinforcement
A_{sb}	balanced steel area
A_{sc}	area of compression reinforcement
A_{st}	area of tension reinforcement
b	breadth of beam or column section
d	effective depth of beam or column section
d_i	depth to centroid of steel at i^{th} layer from extreme compressive fibre

d_n depth to neutral axis
f_c' concrete cylinder strength
f_{co} peak stress on stress-strain curve of unconfined concrete
f_{cu} concrete cube strength
f_r confining pressure produced by confining reinforcement
$(f_r)_{\min}$ minimum allowable confining pressure
f_y yield strength of steel reinforcement
f_{ys} yield strength of confinement
h total depth of the beam or column section
k_e confinement effectiveness factor as per Mander *et al.* (1988)
M_p peak moment
P axial load applied at centroid
$(P/A_g f_{co})_b$ balanced axial load level
$(P/A_g f_{co})_{\max}$ maximum allowable axial load level
η ratio of equivalent concrete stress to cylinder strength as per EC2
λ degree of reinforcement
$\lambda_{\max}$ maximum allowable degree of reinforcement
μ curvature ductility factor
$\mu_{\min}$ minimum curvature ductility factor to be achieved
ϕ_u ultimate curvature
ϕ_y yield curvature
ρ longitudinal steel ratio ($= A_s/A_g$)
ρ_b balanced steel ratio ($= A_{sb}/bd$)
ρ_{bo} balanced steel ratio for beam section with no compression reinforcement
ρ_c compression steel ratio ($= A_{sc}/bd$)
ρ_c volumetric ratio of confinement
ρ_t tension steel ratio ($= A_{st}/bd$)