Modelling nonequilibrium macroscopic oscillators of interest to experimentalists.

P. De Gregorio
"RareNoise": L. Conti, M. Bonaldi, L. Rondoni

ERC-IDEAS: www.rarenoise.lnl.infn.it

Nonequilibrium Processes:
The Last 40 Years and the Future
Obergurgl, Tirol, Austria, 29 August - 2 September 2011

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In celebration of Prof. Denis J. Evans' 60th birthday

GW detectors and fluctuations

Ground-based Detectors

- Often, can ‘hear’ their own thermal fluctuations.
- Expected to approach the quantum limit in the future.

Nonequilibrium stationary states and noise

- Past studies had assumed the noise be Gaussian. However the experimentalists’ interest is in the tails of the distributions. There, they may be not.

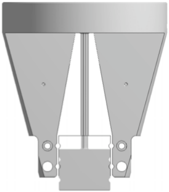
Then the question

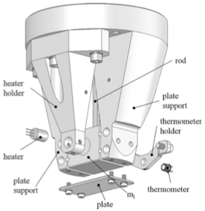
- We detect a rare burst. Is it of an external source? Or false positive due to rare nonequilibrium (and non-Gaussian) fluctuations? Knowledge correct statistics is indispensable.

The idea behind the RareNoise project

The experiment

- A heated mass is appended to a high Q rod, generating a current. Few modes of oscillation are monitored.
- Experiment performed under tunable and controllable thermodynamic conditions.

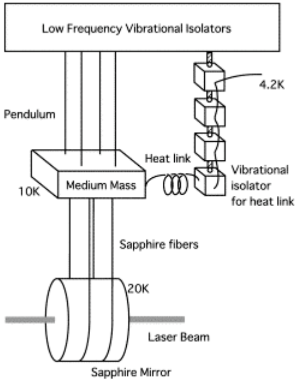




As has been described in L. Conti's presentation and M. Bonaldi's poster.

For a nice outline of the experiment see: Conti, Bonaldi, Rondoni, Class. Quantum Grav. 27, 084032 (2010)

Thermal gradients are present in actual GW detectors



LCGT Project. From: T. Tomaru et al., Phys. Lett. A 301, 215 (2002)

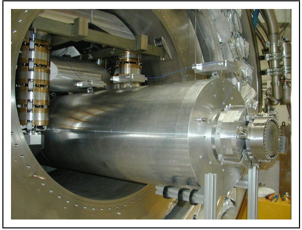
Another experimental context - AURIGA

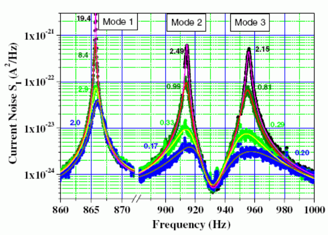
The AURIGA experiment

INFN Padova - Italy

2 tons - 3 meters long - Aluminium-alloy resonating bar cooled to an *effective* temperature of $\mathrm{O}(mK)$, via feedback currents (extra damping). The feedback employs a delay-line, generating effectively a nonequilibrium-state current.

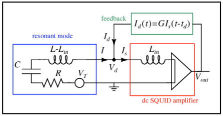
Also in M. Bonaldi's poster





A. Vinante et al., PRL 101, 033601 (2008)

Feedback cooling - the two approaches



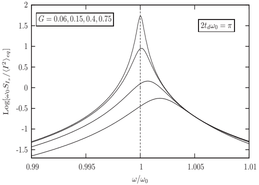
$$L\,\dot{I}(t) + R\,I(t) + \tfrac{1}{C}\,q(t) = V_T(t) \qquad \leftarrow \textbf{feedback off}$$

$$\gamma = R/L;\quad \omega_o^2 = 1/LC;\quad \langle V_T(t)V_T(t')\rangle = 2Rk_B T\,\delta(t-t')$$

$$(L-L_{in})\,\dot{I}(t) + R\,I(t) + \tfrac{1}{C}\,q(t) = V_T(t) - L_{in}\,\dot{I}_s(t)$$

$$\boxed{I_s(t) = I(t) + I_d(t)} \qquad \leftarrow \textbf{feedback on} \uparrow$$

Effective ‘cooling’



quasi harmonic appr.: $R \;\rightarrow\; R' = (R + R_d)$

$$L\,\dot{I}(t) + R'\,I(t) + \tfrac{1}{C}\,q(t) = V_T(t)$$

general case:

$$\dot{I}(t) + \int_{-\infty}^{t} f(t-t')I(t')dt' + \int_{-\infty}^{t} g(t-t')q(t')dt' = N(t)$$

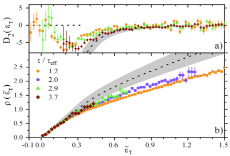
Work and heat distribution, from experiment to quasi-harmonic approximation

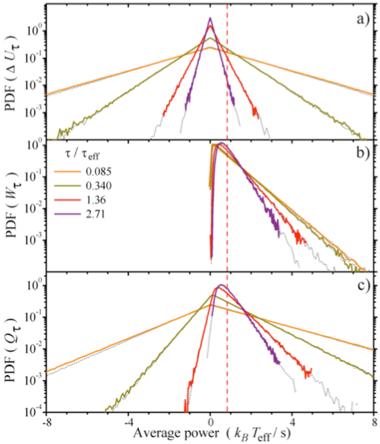
$$Q_\tau = \Delta U_\tau + W_\tau$$
$$P_\tau = Q_\tau + Q_\tau^{(\rightarrow \mathrm{bath})} \simeq Q_\tau$$

$$\rho(\epsilon_\tau) = \frac{1}{\tau} \ln \frac{\mathrm{PDF}(\epsilon_\tau)}{\mathrm{PDF}(-\epsilon_\tau)}$$

$$\epsilon_\tau = P_\tau L/(k_B T R)$$

$$D_2(\epsilon_\tau) = \frac{\partial^2}{\partial \epsilon_\tau^2}\left[\frac{\ln \mathrm{PDF}(\epsilon_\tau)}{\tau}\right]$$

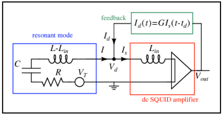




M. Bonaldi, L. Conti, PDG, L. Rondoni, G. Vedovato, A. Vinante *et al* (AURIGA team) PRL 103, 010601 (2009)

J. Farago J Stat Phys 107, 781 (2002)

Feedback cooling - the two approaches



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$\gamma = R/L;\ \ \omega_0^2 = 1/LC;\ \ \langle V_T(t)V_T(t')\rangle = 2Rk_B T\,\delta(t-t')$

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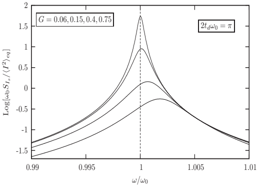
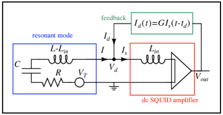
Effective ‘cooling’



Feedback cooling via the full equation



$L\,\dot{I}(t) + R\,I(t) + \frac{1}{C}\,q(t) = V_T(t)$ ← **feedback off**

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general case:

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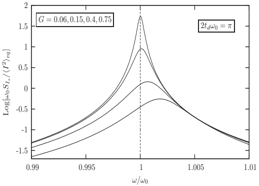
$$N(t) = \int_{-\infty}^{t} h(t-t')\eta(t')dt'$$

$\langle q(0)N(t)\rangle \neq 0;\qquad \langle I(0)N(t)\rangle \neq 0\qquad$ *Kubo's causality broken*

An interesting problem in statistical mechanics in ots own right

Effective ‘cooling’



Shift of the resonance frequency.
From the full equation

PDG, Rondoni, Bonaldi, Conti, J.Stat.Mech., P10016 (2009)

The digital protocol

- Assume $I_d(t) = GI_s(t - t_d)$; $\quad x = L_{in}/L, y = 1 - x, \; |Gy| < 1$

$$\ddot{q}_s(t) + \gamma\dot{q}_s(t) + \omega_0^2 q_s(t) - \frac{x}{y}\sum_{k=1}^{\infty}(Gy)^k[\gamma\dot{q}_s(t - kt_d) + \omega_0^2 q_s(t - kt_d)] = N(t)$$

- Generalized FDT (Kubo) would entail causality @ $t > t'$

$$\begin{cases} \langle N(t)q(t')\rangle = 0 \\ \langle N(t)\dot{q}(t')\rangle = 0 \end{cases} \Rightarrow \quad \langle N(t)N(t')\rangle = \langle \dot{q}^2\rangle f(t - t')$$

- But here

$$N(t) = \sum_{k=0}^{\infty}(Gy)^k \eta(t - kt_d) = \frac{1}{L}\sum_{k=0}^{\infty}(Gy)^k V_T(t - kt_d)$$

$$\Rightarrow \quad \exists\, k \;\; \text{s.t.} \;\; (t - kt_d) < t'$$

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Solving in Fourier space → The power spectrum

Hyp. : $$\langle \eta(t)\eta(t')\rangle = \frac{2k_B T\gamma}{L}\delta(t-t')$$

$$\Rightarrow \; S_{I_s}(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}\langle I_s(0)I_s(t)\rangle dt = \left(\frac{2k_B T\gamma}{L}\right)\frac{\omega^2}{D(\omega)}$$

$$D(\omega) = \alpha^2 + G^2\beta^2 + (1+G^2)\gamma^2\omega^2 + 2xG\gamma\omega^3 \sin(\omega t_d) - 2G(\alpha\beta + \gamma^2\omega^2)\cos(\omega t_d)$$

Def. : $\alpha = \omega^2 - \omega_0^2$, $\beta = y\,\omega^2 - \omega_0^2$

- For very high quality factors

$$\frac{\omega_0}{\gamma} \equiv Q \gg \frac{1}{xG} \gg 1 \quad \Rightarrow \quad S_{I_s}(\omega) \cong \frac{2\,z\,\omega^2}{(\omega^2-\omega_r^2)^2 + \mu^2\omega^2}$$

and if $2t_d\omega_0 = \pi$, then : $\omega_r^4 = \frac{1+G^2}{1+y^2G^2}\,\omega_0^4$; $z = \frac{k_B T\gamma}{L\,(1+y^2G^2)}$

$$\mu^2 = 2\left(\frac{\sqrt{1+G^2}}{\sqrt{1+y^2G^2}} - \frac{1+yG^2}{1+y^2G^2}\right)\omega_0^2 \xrightarrow{G\to 0} x^2G^2\omega_0^2$$

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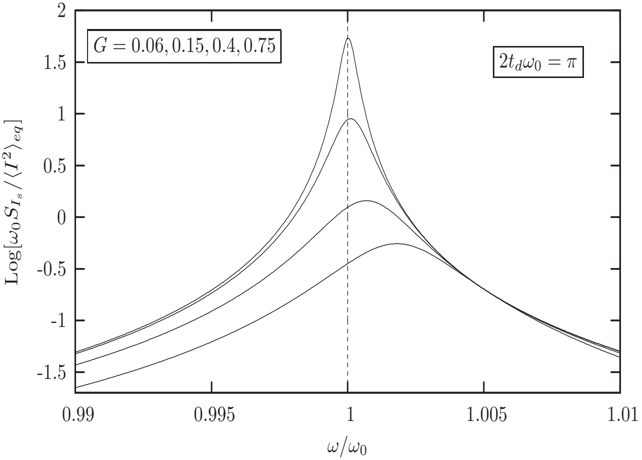
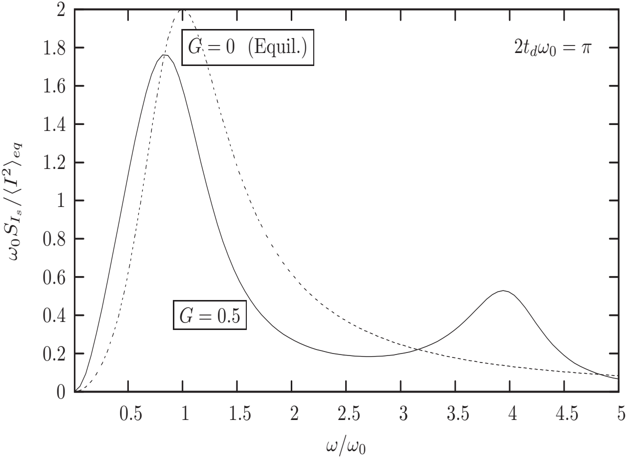
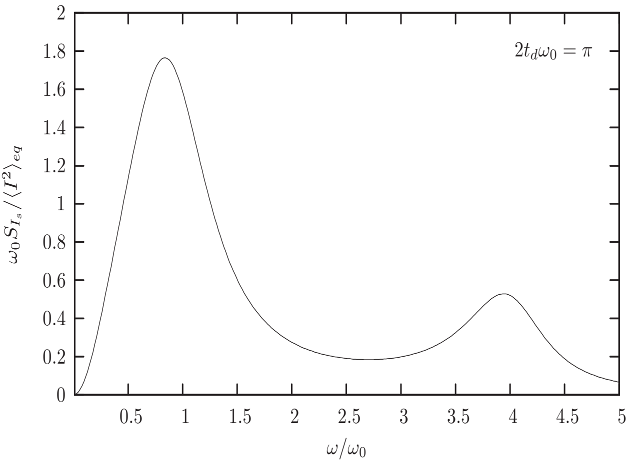
Q = 10^5 - shift of the resonance frequency



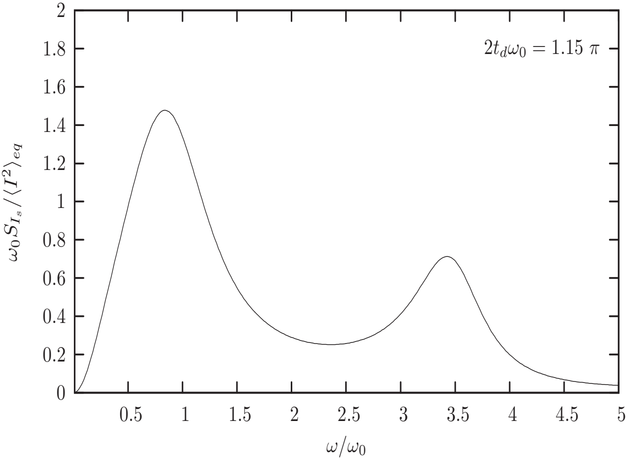
Q = 1 - "transition" of the resonance frequency



Q = 1 - changing t_d



Q = 1 - "transition" of the resonance frequency as $t_d \uparrow$



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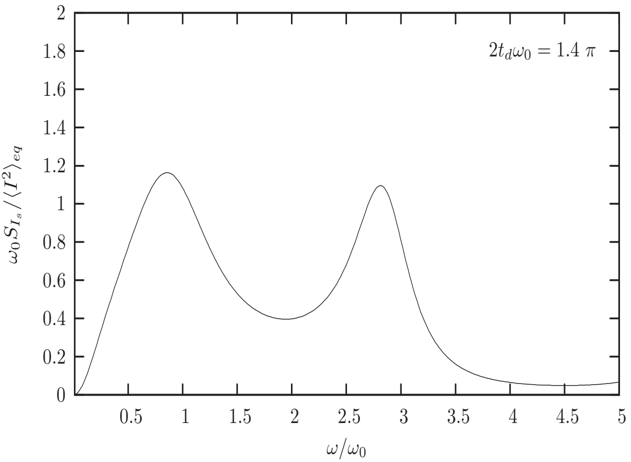

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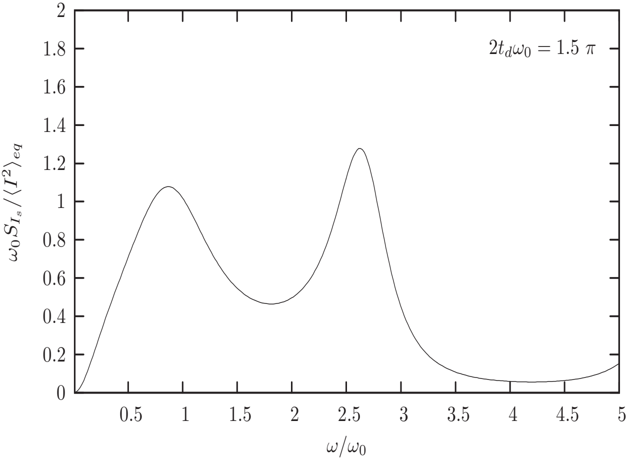

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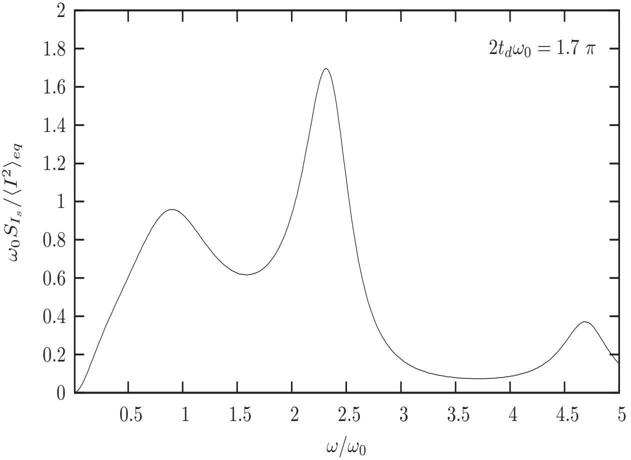


The analog protocol

- The feedback is implemented via a low-pass filter

$$\tilde{I}_d(\omega) = \frac{A\Omega}{\Omega - i\omega}\tilde{I}_s(\omega)$$

$$\Downarrow$$

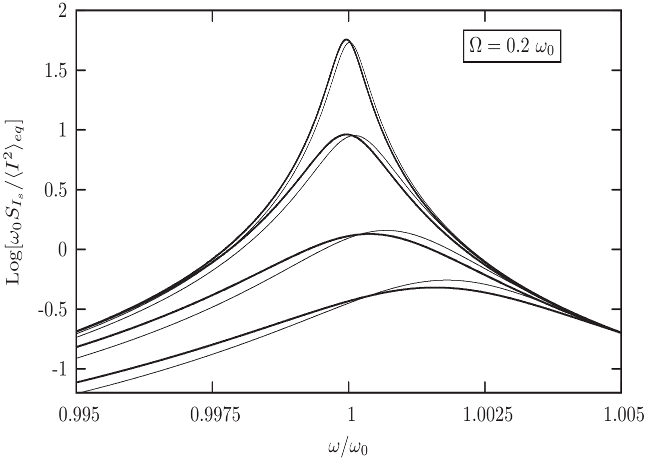
$$S_{I_s}(\omega) = \left(\frac{2k_B T\gamma}{L}\right)\frac{\omega^2\left(\omega^2+\Omega^2\right)}{B(\omega)}$$

$$B(\omega) = \omega^2(\omega^2 - F^2)^2 + (a\omega^2 - b^3)^2$$

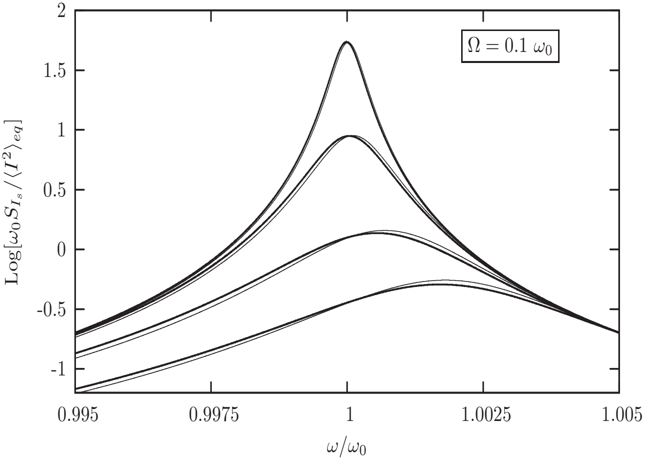
$$F^2 = \omega_0^2 + \gamma\Omega(1-A); \qquad a = (1-yA)\Omega + \gamma; \qquad b^3 = \Omega\omega_0^2(1-A)$$

- For consistency with digital protocol we set $A\Omega = G\omega_0$

Analog *vs* digital @ Q $= 10^5$



Analog *vs* digital decreasing Ω/ω_0



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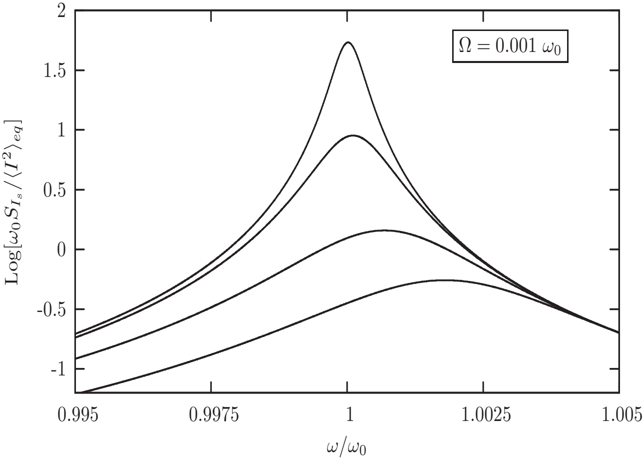
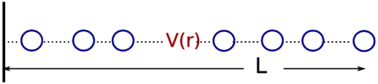


Modeling the rod with one-dimensional chains



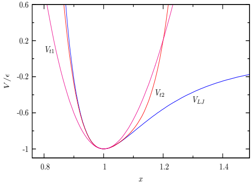
L as the observable

The idea is to define a variation of the celebrated FPU chain:

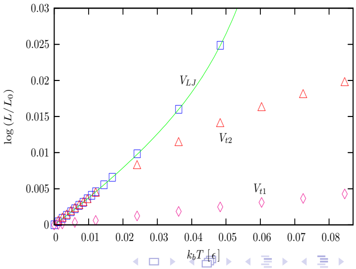
$$V_t(x) \quad = \quad h\left[(x-1)^2 - \lambda(x-1)^3 + \mu(x-1)^4\right]$$

to then compare it to a more ‘phenomelogical’ interaction potential, to overcome certain inconsistencies

$$V_{LJ}(x) = \epsilon\left(x^{-12} - 2x^{-6}\right) \qquad x = r/r_0$$



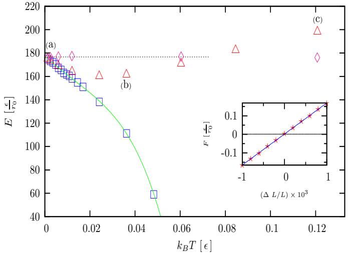
$\lambda = \mu = 1 \quad \rightarrow \quad V_{t1}$
$\lambda = 7,\ \mu = 53\,\lambda/12 \quad \rightarrow \quad V_{t2}$
(as if we expanded LJ)



Comparing thermo-elastic properties

E, the elastic modulus: $F \simeq E\,(\Delta L/L)$, if $\Delta L \ll L$.

PDG, L.Rondoni, M.Bonaldi, L.Conti, *to be published*



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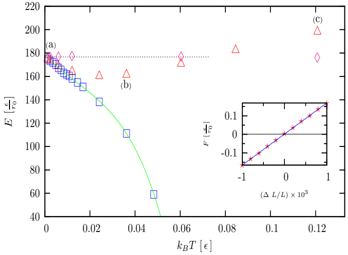

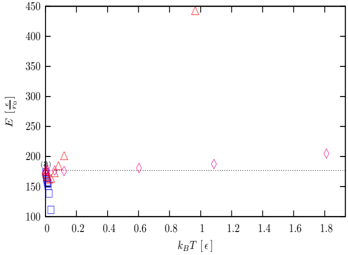

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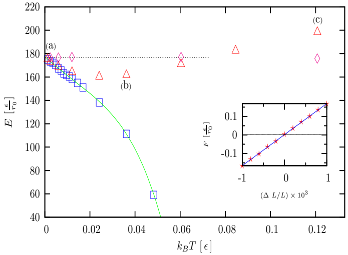
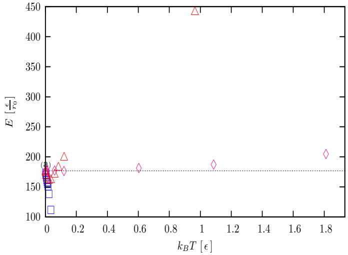
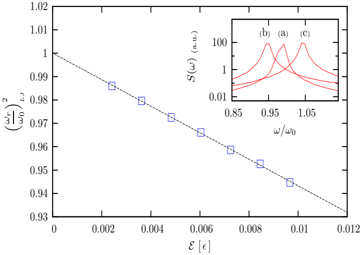
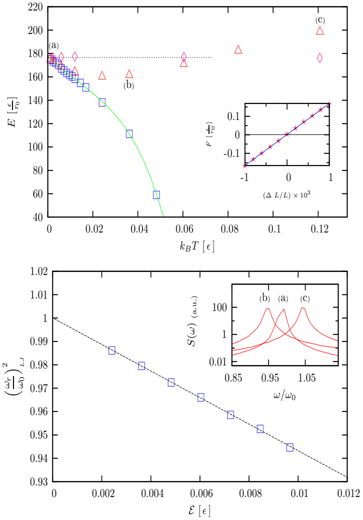




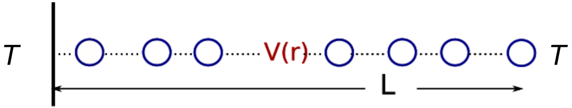


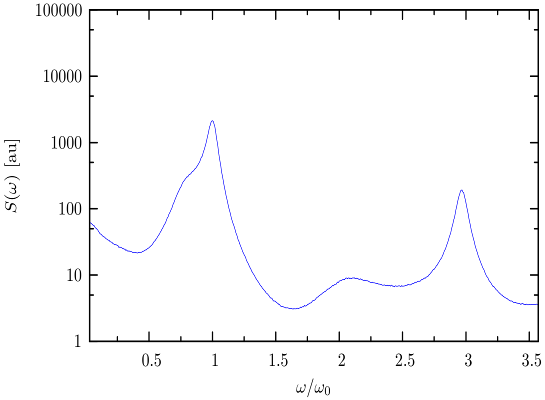
Comparing thermo-elastic properties

E, the elastic modulus: $F \simeq E\,(\Delta L/L)$, if $\Delta L \ll L$. ω_0 resonance frequency of 1st mode. $Q = \omega_0/\Delta\omega_{1/2}$

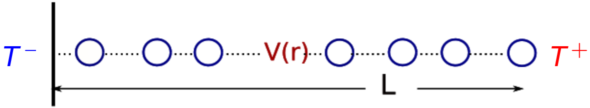


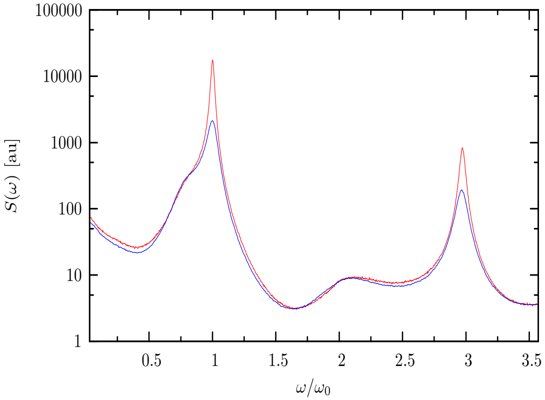
Effects of gradients (under investigation) - $\nabla T \equiv 0$



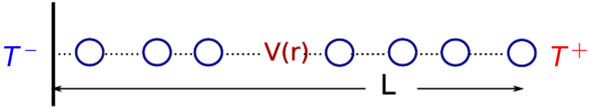


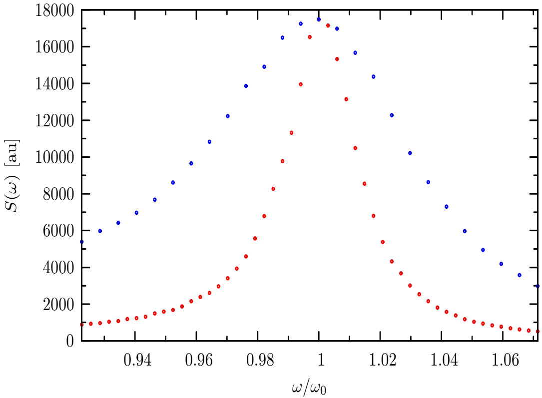
Effects of gradients on Q: $\nabla T \neq 0$ -vs- $\nabla T \equiv 0$





Effects of gradients on Q: $\nabla T \neq 0$ -vs- $\nabla T \equiv 0$





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