CURVE AND SKELETON BASED SHAPE DEFORMATIONS IN PRODUCT DESIGN

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Raluca Dumitrescu *Joris S.M. Vergeest*

Faculty of Industrial Design Engineering, Delft University of Technology, the Netherlands

{*r.dumitrescu;j.s.m.vergeest*}*@io.tudelft.nl*

Chiara E. Catalano *Franca Giannini* *Bianca Falcidieno*

Istituto di Matematica Applicata e Tecnologie Informatiche, CNR, Genoa, Italy

{*chiara;franca;bianca*}*@ge.imati.cnr.it*

In this report we present an intuitive curve and skeleton based approach for digital product modelling. Morphing–like deformations have been developed to allow for the evaluation of a larger set of alternative shapes compared to the set of shapes generated by the current modelling tools. The method helps designers to search in the product domain for alternative shapes in a straightforward way and eliminates the work–arounds. Through a slide–bar control, these alternative shapes are generated by the transformation of an initial shape into the target one by means of a suitable skeleton extraction transparent to the user and a user-defined profile curve for the target surface. The initial shape is abstracted by a skeleton and a distance function from the skeleton to the surface. For the target surface two categories have been considered, namely revolution–like and sweep–like surfaces. They are both defined through curves: an axis or a path and a profile. The user has to specify only the profile curve, as the axis or the path is represented by the skeletal curve extracted from the initial surface. The distribution of the morphing–like deformation is computed based on the skeletal curve, the distance function and the user-defined profile curve. The use of the skeleton guarantee the generated shapes belong to specific product domains and are therefore context–dependent.

Keywords: product modelling, shape deformations, curves, parameters, constraints.

1 Introduction

Nowadays, the new products are created by modifying existing digital models, possibly retrieved by virtual catalogues or reverse engineering (RE) processes. The high competition on the market demands shorter time for new product creation while still satisfying the consumers' physical and psychological needs. The chances of product success increase when new products preserve the archetype of their own class. The archetype helps consumers to distinguish between the different product classes. It refers to a typical model for a product class from which other products are instantiated. Very often it is linked with the product structure, and hence computer-aided shaping tools preserving the product archetype and allowing changes only in the shape are needed. We refer to these as semi–global shaping tools and compared to the global deformation tools where global changes might involve structural changes, the former affect only parts of the product.

Computer–aided industrial design (CAID) systems are increasingly used for product modelling and shape evaluation due to the availability of high–performance affordable personal computers. Within CAID, deformation tools are employed to perform shape refinements. There are two types of shape deformations described in the scientific literature: global and local deformations. A global deformation affects the shape of the whole object, sometimes changing also its topology. Whereas a local deformation refers to deformation of detail areas, e.g. feature–based deformations, or the manipulation of low–level shape handles (e.g. control vertices manipulations). A global deformation is introduced for example by Chen and Parent [2], where new shapes are generated based on a weighted average of two other existing shapes. An intuitive approach based on an axial deformation is proposed by Lazarus *et al.* [5], for both global and local deformations. For global deformations, the user defines an axis of the object and all vertices of the object are associated with this axis together with their local frames and distance functions. Later, when the shape of the axis is changed the object is also deformed. In the case of local deformations, the axis produces local changes in the shape based on two circular cross–section zones of influence and the approach is similar to feature insertion. Lazarus and Verroust present another global deformation in [6], where two axes inside two star–shaped objects are used to morph between these objects. A feature–based deformation is introduced by Pernot *et al.*, [10]. Here the deformation is constrained by a limiting curve on the surface and by a sketched target curve; whereas the resulting shape is controlled by the minimization of some external forces applied to a bar network coupled to the control polyhedron of the surface. Van Elsas [12] proposes the displaced feature approach, where a curve on the surface defines an area to be displaced, and a set of slide–bars control the size of the displacement and the rounding of an transit area.

All these approaches are either desired for the modification of the final shape of an object when the result is known in advance, for the insertion of features, or for the generation of one global shape as a combination of two other shapes. However, shaping tools for semi–global deformations supporting the early phases of the design process, when the stylist is still creating the product shape, not detailing it, are still lacking. This type of deformations involves changes only to one part of the product in order to shape its visual perception. The method proposed in this paper is suitable for dealing with semi–global deformations of shapes for which information about their creation are not available. However, to properly deform a digital model and fast evaluation of alternatives, we believe that context–dependent modelling tools are needed in order to reduce the many *degrees–of–freedom* of the shape by placing the necessary constraints,

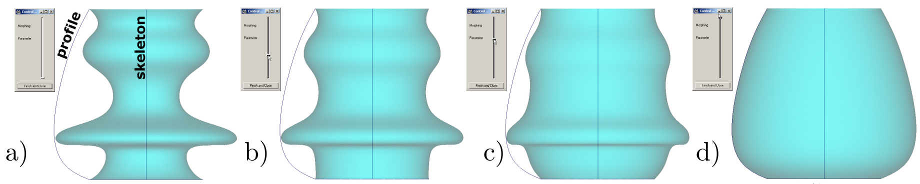

Fig. 1: a) A morphing–like deformation is applied to a vase model based on the profile curve. b)–d) Show the deformed vase with its shape towards the target shape ≈ 33%, ≈ 66%, and ≈ 100% respectively.

and allowing for the generation of a range of shapes within the specific part domain. For this, we introduce the concept of the *Shape Context* in Sec. 2. Then, we discuss how this concept can be implemented using product decomposition, shape abstraction, skeletal representation, and morphings for the deformation of parametric surfaces. In Sec. 3.1, we consider a proper decomposition of an object into its meaningful parts. Based on this decomposition, the designer selects only the part to be deformed preserving therefore the product's initial structure. This part is further abstracted by a suitable skeletal curve and a set of planar contours as described in Sec. 3.2. The skeletal curve is necessary for proper correspondence during the morphing process. Further we show how we define the target shape through higher–level shape descriptors. These descriptors provide both an intuitive definition of the target surface and reduce the shape's *degrees–of–freedom* during the deformation. Two types of surfaces have been treated in this view so far: revolution–like and sweep–like surfaces (Sec. 3.3). The user has to specify the type of the target surface and a profile curve. The target surface is generated based on the skeletal curve extracted from the selected part and the profile curve. Then, a morphing–like deformation controlled via a slide–bar is provided to the user, who can tune interactively the shape. The in–between shapes are generated through a linear interpolation between the discrete initial and target surfaces. Implementation results are shown in Sec. 3.4, further applicability of the method is discussed in Sec. 3.5, and in Sec. 4 we draw conclusions about the proposed curve and skeleton based morphing–like deformations (Fig. 1).

2 Shape Context

In product modelling, shape is of primary concern from the very beginning of idea and concept generation. It can be stated that over the entire life cycle of the product, its performance is heavily influenced by its shape, either explicitly or implicitly. Therefore, much of the designer's attention and skills need to be focused on creation of and control over shape. A central hypothesis in this research declares that effective control over shape requires having the proper shape context in place. Here the context is functioning as the cancellation (or hiding) of those *degrees–of–freedom*, or parameters, that are not relevant to the current modelling task [13]. Conversely, context can be understood as the set of parameters that are relevant to the current modelling task. A parameter should then be interpreted quite generally; it is not necessarily a numerical variable, but it could also be a profile curve, as it is considered in the next sections. It can, therefore, be anything that can be changed (or controlled) by the designer in order to perform his/her task. A task is understood here as the creation or modification of shape.

We have the following characteristics of Shape Context:

- Shape Context relates to degrees of freedom, or parameters, or shape handles. In general it is connected to those aspects that can (or need to) be changed.
- Shape Context is related to the current design intent. As far as shape is concerned, design intent can be circumscribed as *the set of shapes that would satisfy the constraints.*

A Shape Context is a *finite collection of shape variation procedures.* Each shape variation procedure is governed by a multi–component parameter.

We formally define Shape Context by its implications to the shape domain $2^{\mathbf{R}^3}$. A Shape Context C, $C = \{M_1, ..., M_n\}$, is a set of mappings, $n \geq 0$, where each mapping takes the form:

$$M_i : (2^{\mathbf{R}^3} \times P_i) \longrightarrow 2^{\mathbf{R}^3} \tag{1}$$

where P_i, called the parameter domain of M_i, can be any set. M_i transforms a shape S into a different (or possibly the same) shape S', depending on the parameter value $p \in P_i$. For a given shape $S \subset \mathbf{R}^3$, the set $I_i(S) = \{M_i(S, p) \mid p \in P_i\}$ is called the image of S through M_i. $I_i(S)$ is a family of shapes, possible including S itself. The union of all images through M_i, denoted as $R(S)$ is the range of the shape context relative to S: $R(S) = \bigcup_{i=1,n} I_i(S)$. Since we need to describe a process that takes place during a finite time interval, we account for time dependency and generalize the Shape Context equation to:

$$C(t) = \bigcup_{i=K(t)} M_i \tag{2}$$

where $t \in [t_0, t_1]$ denotes the time interval considered and $K(t) \subseteq \{1, \ldots, n\}$ is a selection function from n indices. The set $\{M_1, \ldots, M_n\}$ contains all mappings that occur in context $C(t)$ for at least one value $t \in [t_0, t_1]$.

In summary, for any t in time interval $[t_0, t_1]$, $C(t)$ specifies zero or more 'active' mappings; each mapping specifies for each shape a 'space' of shape variations, controlled by a (multi–component) parameter.

3 Curve and skeleton based shape deformations

3.1 Product Decomposition

We define a product or an object, O, as the union of a set of parts, P, a set of features, F, and the adjacency relations between them. A part represents a component of a product that always has a user–perceived functional role. On the other hand, a feature needs not necessarily have a functional role (e.g. a longitudinal depression on a vase), but it can have just an aesthetic role. Note that the set of features can be the empty set, $|F| \geq 0$; whereas the set of parts can never be empty because this lead to an inconsistent product definition. The adjacency relations between the parts are represented by: $R_P = \{(P_i, P_j) | \; P_{i,j} \in P\}$; and the adjacency relations between the features (if any) are represented by: $R_F = \{(F_i, F_j) | \; F_{i,j} \in F\}$. The adjacency relations between the parts and the features are represented by $R_{PR} = \{(P_n, F_m) | \; P_n \in$

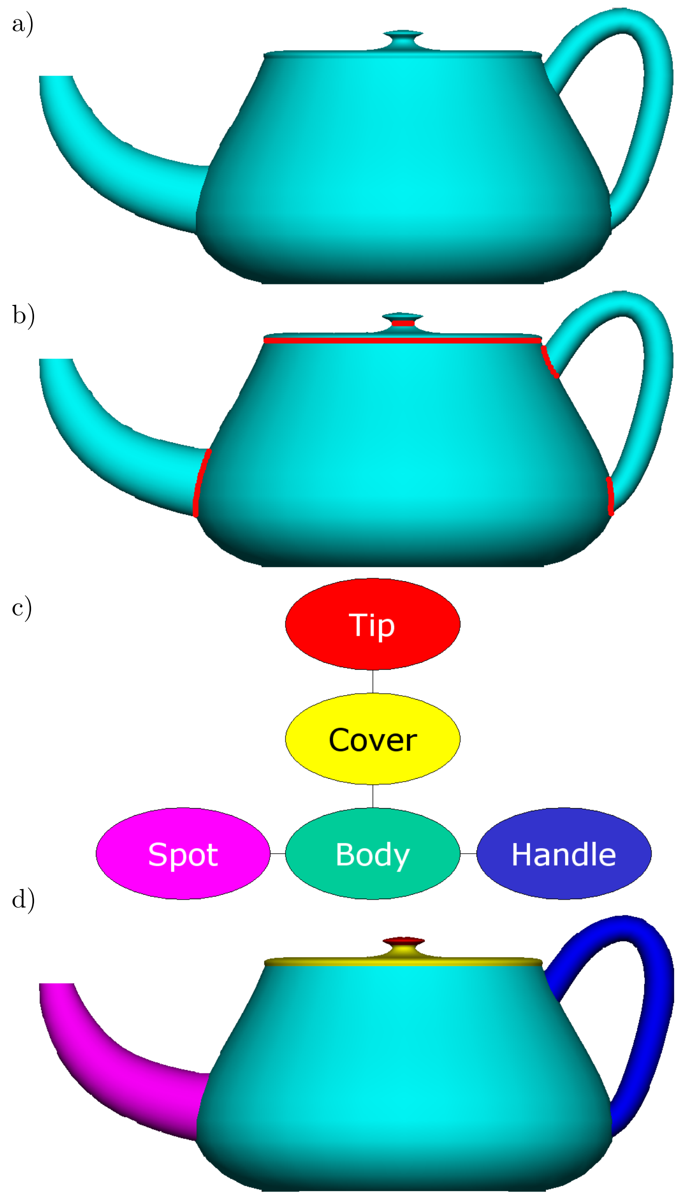

Fig. 2: a) A teapot model. b) The planar separating sections on the model. c) The shape graph of the model. d) The meaningful parts of the teapot.

$P,\ F_m \in F\}$. Therefore, we can say that each product, O, can be represented by its sets of parts and features and their adjacency relations as

$$O = \{P,\ F,\ R_P,\ R_F,\ R_{PF}\} \tag{3}$$

Human vision theory states that humans decompose the models using a minima rule that locates lines of negative curvature minima. A computer vision algorithm based on this theory has been developed by Page, [8], for the decomposition of digital models or scenes. The algorithm, called the Minima Rule Algorithm, finds the contour boundaries between the components through curvature evaluation and part salience decomposing a given object into its meaningful parts. We assume that our objects are automatically decomposed while we manually decompose them and approximate planar curves as the contour boundaries. For the domain of consumer durables, which are the focus of our interest, there commonly exist clear boundaries between the parts of the products. And, hence, the Minima Rule Algorithm would provide reliable results. An example of a consumer durable is shown in Fig. 2a. Fig. 2b shows in red the planar sections that we approximate for the separation of the parts. The model's adjacency relations can be coded into a shape graph (Fig. 2c) and the model can be decomposed into its meaningful body, spout, handle, cover and tip as shown in Fig. 2d (with different colors).

3.2 Initial Surface Abstraction

In order to provide intuitive control of the morphing of an initial shape into a target one, we need to define means to describe these two shapes by intuitive or easy to comprehend descriptors. In addition, the representation of the product archetype has been considered as an important factor in preserving the product class characteristics during the deformation. Therefore, a suitable approach to the above–mentioned issues is to describe the product model and its surfaces in terms of curves. Starting from the separating sections as discussed in Sec. 3.1, we proceed further to generate planar contours on the initial surface of the part to be deformed, and for each of these contours we extract the barycenter point. The set of all the barycenter points represent the skeletal curve of the surface. The initial surface can be abstracted by the skeletal curve and a distance function from this skeletal curve to the surface.

3.2.1 Planar Sections Extraction

The Para–Graph method proposed by Patanè, [9], to generate isocontours on parameterised mesh representations can be adapted for case of the continuous representation. For surfaces with 0–genus and 1 or 2 boundaries, we can generate planar contours on our surface. The main steps for the planar contours extraction and their corresponding barycenters computation for the case of parametric surfaces $S : [0,1] \times [0,1] \longrightarrow \mathbf{R}^3$ are presented below :

- The starting planar curve of a part represents the u_0 isocurve and an orthogonal isocurve to u_0 is assigned the v_0 isocurve. The u_1 is also computed as the intersection between a plane and the surface.
- The surface is mapped on the square unit bounded by u_0, u_1 and v_0.
- Isocontours on this square unit are generated at a desired distance one from another.
- Then these are mapped back on the surface, generating curves on the 3D surface.

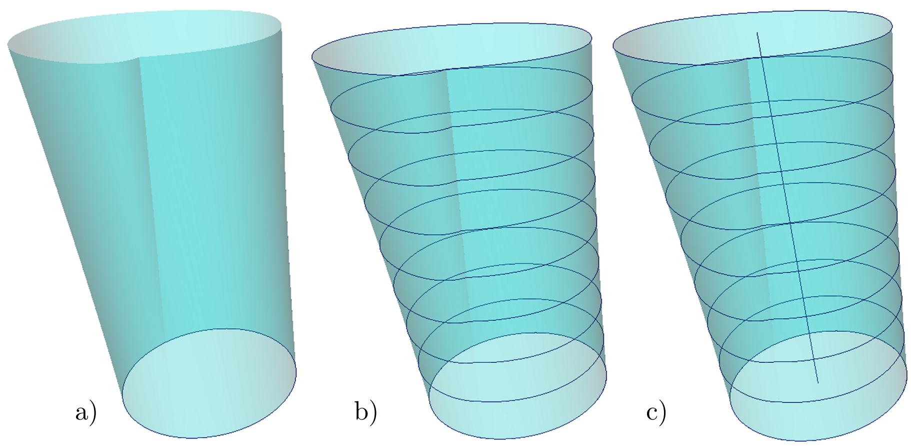

Fig. 3: a) Input surface with two boundary curves. b) Extraction of the planar contours on the surface. c) Extraction of the skeletal structure from the planar contours.

- A plane is found that approximates these isocurves, and the intersection of the plane with the surface generate the planar contours on the surface.

Fig. 3a shows a tubular surface with planar boundary isocurves, whereas in Fig. 3b the generated planar contours are shown.

3.2.2 Skeleton Extraction

The barycenter of each of these planar contours is computed as the average point of each contour that is approximated by a polygon. The set of all barycenter points represent the skeletal curve of the surface. For each barycenter we store its position, its local frame and the distance function to the contour. The contours and the skeletal curve for a tubular surface is shown in Fig. 3c.

3.3 Target Surface Types

For the case of the target surface we have considered so far two categories: the revolution–like and the sweep–like surfaces. Each of them can be defined by means of two or three curves: a profile, an axis or a path, and a scalar. However, the user specifies only the profile curve or the scaler, as the axis/path are extracted from the initial surface. Within a specific shape context, the profile curve acts as a parameter and the axis/path as a constraint during deformation. The profile curve defines the size and distribution of the deformation, while the axis/path force the surface to be defined around them, in order to preserve the same skeletal representation of the product.

3.3.1 Revolution–like Surfaces

A revolution–like surface is generated by the rotation of a two–dimensional curve about an axis. The axis is not necessary a line (Fig. 4a), but can also be a curve, as in Fig. 4b.

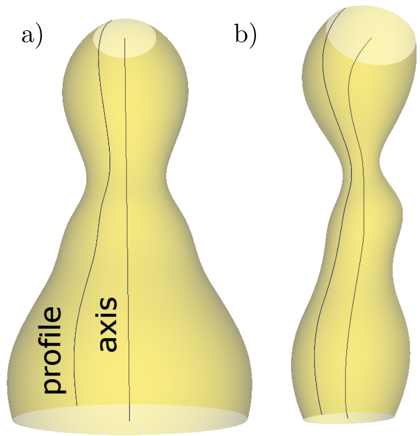

Fig. 4: Examples of revolution–like surfaces: a) surface with line axis; b) surface with curve axis.

Let $C(v) = \sum R_{j,q}(v)P_j$ be a 2D–NURBS curve on the knot vector, V, where j is the number of control points, and q is the degree of the curve.

(1) When $C(v)$ is to be revolved around a straight axis, then a surface of revolution, $S_r(u, v)$, is generated and $C(v)$ is called the *generatrix*, [11], or simple *profile curve*. Suppose that $C(v)$ lies in the xz plane, and that we revolve $C(v)$ a full 360^0 about the z axis. The required surface $S_r(u, v)$, has the characteristics such that:

- for a fixed $\bar{u}$, $S_r(\bar{u}, v)$ is the curve $C(v)$ rotated of some angle about the z axis;
- for a fixed $\bar{v}$, $S_r(u, \bar{v})$ is a full circle which lies in a plane perpendicular to the z axis, with its center on the z axis.

(2) When the profile curve $C(v)$ is to be 'revolved' around a 2D–axial curve, $C(u)$, then a tubular surface, $S_t(u, v)$, is generated. Now, $C(v)$ lies in the xz plane and its rotation considers the local frames on the axis. The required surface $S_t(u, v)$, has the characteristics such that:

- for fixed $\bar{u}$, $S_t(\bar{u}, v)$ is a deformed curve $C(v)$ depending on the shape of the axial curve;
- for fixed $\bar{v}$, $S_t(u, \bar{v})$ is a circle centered at its corresponding barycentric point, lying in the plane perpendicular to the axial curve at that barycentric point, and with the radius equal to the distance from the barycentric point to the profile curve.

3.3.2 Sweep–like Surfaces

A sweep–like surface [Fig. 5], $S(u, v)$, is created by sweeping a section curve along an arbitrary trajectory curve, [11]. If we consider the trajectory as $C_1(v)$, and the section curve as $C_2(u)$, then the general form of the sweep surface is given by the following formula:

$$S(u, v) = C_1(v) + A(v)S(v)C_2(u) \tag{4}$$

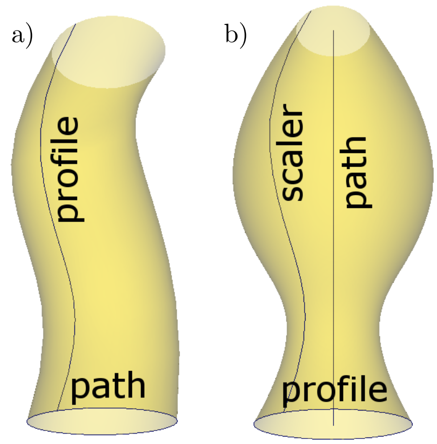

Fig. 5: Examples of sweep–like surfaces: a) profile swept along the path (or the axis); b) profile swept with scaling along the path.

(1) When $M(v) = A(v)S(v)$ is the identity matrix for all v, then $S(u, v)$ is the *generalized translational sweep*, and for each v, $C_2(u)$, is just translated by $C_1(v)$.
(2) Otherwise, the $S(v)$ in Eq. 4 is a 3×3 diagonal matrix with elements $(s_x(v), s_y(v), s_z(v))$, which incorporate the scaling of the $C_2(u)$ for each local orthogonal coordinate system. $A(v)$ is the general transformation matrix that takes the global coordinate system into the local orthogonal coordinate system. The isoparametric curves of $S(u, v)$ at fixed v values are instances of $C_2(u)$, transformed by $A(v)S(v)$ and translated by $C_1(v)$. If $C_2(u)$ passes through the global origin at $u = \bar{u}$, then $C_1(v)$ is the isoparametric curve $S(u = \bar{u}, v)$.

In Fig. 5, we show some examples of sweep surfaces, as presented by [7]. As it can be noticed in Fig. 5a, the profile and the path of the sweep surface can be any of the two curves; however, in Fig. 5b the switch between these two curves can generate two different surfaces. This is because of the scaling function defined by $S(v)$ in Eq. 4.

3.4 Semi-global Deformations and Results

A morphing (or metamorphosis) is a continuous transformation of a source (or an initial) shape into a target shape [4]. Most common, morphings are used in the animation and film industry for the creation of characters' metamorphosis. Few attempts have been recorded for their implementation to support product design [3]. An well-know problem of the morphing is the correspondence between the initial and the target shape in such a way that the transformation function form the initial to the target shape gives the same result like its inverse. Based on this reason and keeping in mind the designers understanding of surfaces in terms of its characteristic curves, we presented in the previous sections shape abstraction methods to support the morphing-like deformations from an initial surface to a target surface that also guarantee proper correspondence during the transformation. Therefore, we discussed in Sec. 3.2 how a discrete skeletal curve can be extracted from a part of a product together with the distance function to the surface. This skeletal curve is used during the deformation as a shape descriptor

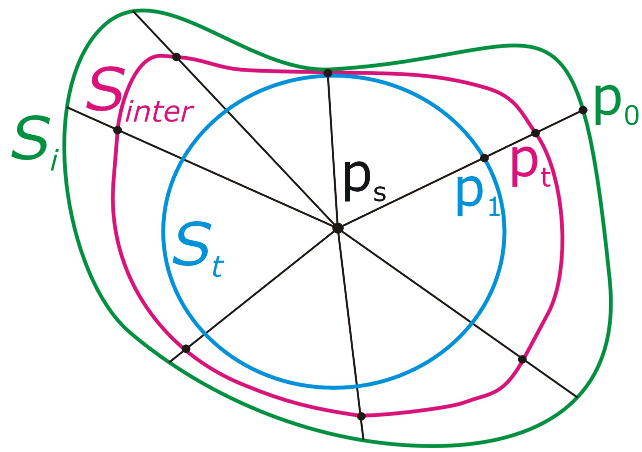

Fig. 6: Morphing from a contour on the initial shape S_i (green) to its corresponding contour on the target shape S_t (cyan). The intermediary shape S_{inter} (magenta) appears when the slider is set to $\simeq 0.5$. The correspondence between the initial and target contours is guaranteed by the barycentric point, p_s of the skeleton.

of both the initial and the target surface. Together with the profile curve specified by the user, the skeletal curve defines the target surface, as shown in Sec. 3.3.

The morphing–like deformation was implemented as a discrete morphing of the contours defining the two surfaces. While the extraction of the contours of the initial surface have been discussed previously, those of the target surface are created based on the position and orientation of the barycenter points and the surface creation rule discussed in Sec. 3.3. Therefore, for each barycenter point on the skeleton we know the two corresponding curves on the initial and target surface. We interpolate between each of these pair of curves to perform the morphing–like deformations. Given two parameters: p_0 on a curve of the initial shape and p_1 on a corresponding curve of the target shape, at times t_0 and t_1 (Fig. 6) on a line traversing the barycentric point, then a intermediary value, p_t, controlled through a slide–bar, t, is computed as a linear interpolation:

$$p_t = (1-t)p_0 + tp_1 \ , \quad t \in [0,1] \tag{5}$$

A main advantage of morphing–like deformations supporting product design is that it provide designers with modelling tools that continuously transform an initial shape into a target one, which allow them to visualise a larger set of shapes in a shorter time without having to perform work-arounds until the desired shape is reached. In addition, the user-defined profile curve necessary to define the target surface allow for the specification of an user-defined Shape Context. Therefore, a multitude of Shape Contexts for a specific part can be created by the specifications of different profile curves. The morphing–parameter is controlled through a slide–bar, as can be seen in Fig. 1.

The morphing–like deformations have been implemented in Maya$^{(R)}$ v6.1 from

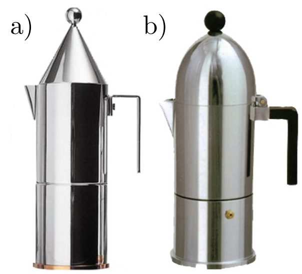

Fig. 7: Real-life coffee-maker designs by Alessi used in the examples from Fig. 8: a) "La Conica" design, b) "La Cupola" design. Source www.alessi.it.

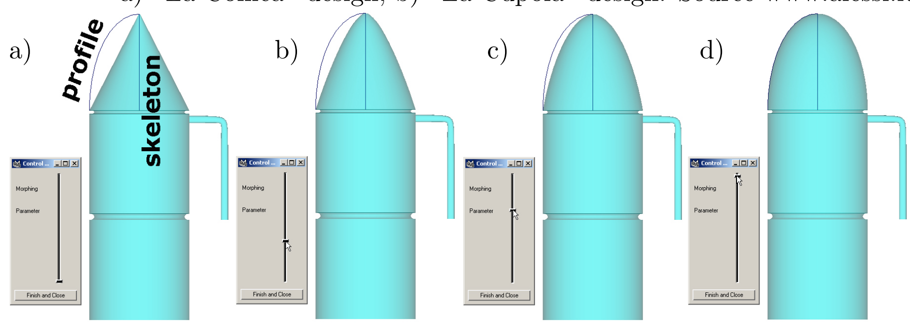

Fig. 8: a) A morphing–like deformation is applied to the lid of a coffee–maker model based on the profile curve. b)—d) Show the deformed lid with its shape towards the target shape ≈ 33%, ≈ 66% and ≈ 100% respectively.

Alias and some examples are shown in Fig. 1, Fig. 8, Fig. 9, Fig. 10, and Fig. **??**. Starting from an existing shape, the user specifies the new profile curve defining the target surface and selects the deformation icon from the menu bar. While the slide–bar window appears, the computing of the skeletal curve and the contours' correspondence is performed. Then, the user plays with the slide–bar and tune the initial shape into the target one. When a final desired shape is found and the slide–bar window is closed, the deformation completes and the geometrical information previously created is discarded.

Examples of revolution–like target surfaces with line axis are shown in Fig. 1, and Fig. 8. The latter example represents the implementation of our deformation technique on models of real-life products like those in Fig. 7. In this case, the shape of the "La Cupola" had been consider the initial surface and using the new profile curve, we were not only able to generate the shape of the "La Conica" product, but also to create other intermediary shapes by controlling the amount of the distribution of the morphing via the slide-bar. This way, increasing the set of shapes belonging to that product domain and helping designers to quickly evaluate more shapes compared to current techniques.

Deformation results for the case of revolution–like target surfaces with curve axis are shown in Fig. 9.

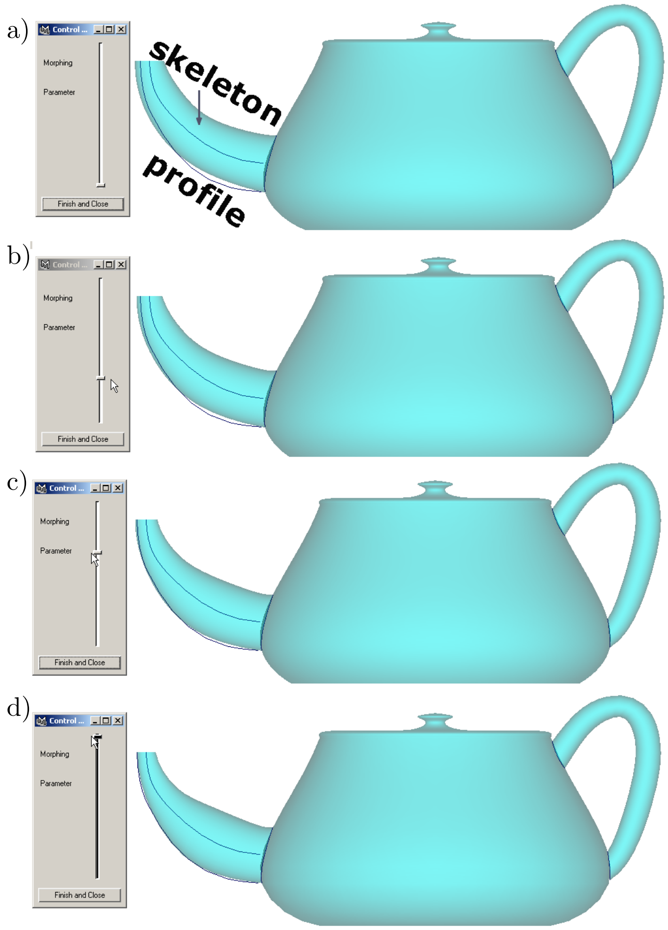

Fig. 9: a) A teapot model to which a morphing–like deformation is performed on its spout based on the profile curve. b)—d) Show the deformed spout with its shape towards the target shape ≈ 33%, ≈ 66% and ≈ 100% respectively.

The case of sweep–like target surface are still under development, but some first implementations are shown in Fig. 10 for surfaces without scaling of the profile and in Fig. 11 for surfaces with scaling. In the latter example an constraint have been applied for the position and orientation of the knob in concordance with the product shape graph.

3.5 Further possible work

The methodology presented above is implemented for semi–global deformations. However, it can also be applied for local deformations when the feature to be deformed

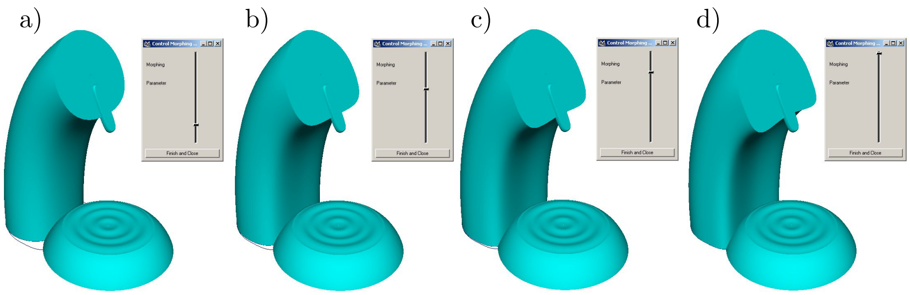

Fig. 10: a) A morphing–like deformation is applied on the body of a Senseo model based on the profile curve. b)—d) Show the deformed body with its shape towards the target shape ≈ 50% and ≈ 100% respectively.

is of same type as one of the surface types discussed in Sec. 3.3. In this respect, the approach can also be implemented for the deformation of the sweep–like features introduced by Catalano [1]. Here the profile that defines the shape of the feature can be redefined to generate the target surface; and the skeletal curve need not to be extracted from the feature, but associated with its directrix.

4 Concluding remarks

In product modelling, shape abstraction can improve user's interaction with and manipulation of the shapes. It can significantly reduce the *degrees–of–freedom* and allow for a better control over the expression of the shape. We believe that curve–based definition of surfaces provide intuitive means to specify and control their behavior during morphing–like deformations. In this report we have presented a curve and skeleton based approach to support product shape deformations. After a product is decomposed into its meaningful parts, and a part to be deformed, a type of target surface and a new profile curve are specified by the user, then a skeletal curve is extracted from the initial surface and the user-defined profile curve together with the skeletal curve are used to define the target surface. A slide–bar controls the morphing–like of the discrete initial surface into the target one.

Finally, while the other approaches preserving the product structure can generate only one shape when the deformation is applied, or a larger set of shapes is possible but producing changes also in the structure; our method provides good control and tuning over the deformation due to the intuitiveness in perceiving curves by the designers and the proper correspondence between the initial and target surface. Most important, it improves the interaction between product designers and the digital shape modelling environments by allowing the specification of the different Shape Contexts and the visualisation and evaluation of a larger set of shapes. It therefore supports creativity, but also shape reuse as the initial shapes are possibly obtained by RE processes or retrieved from previous projects.

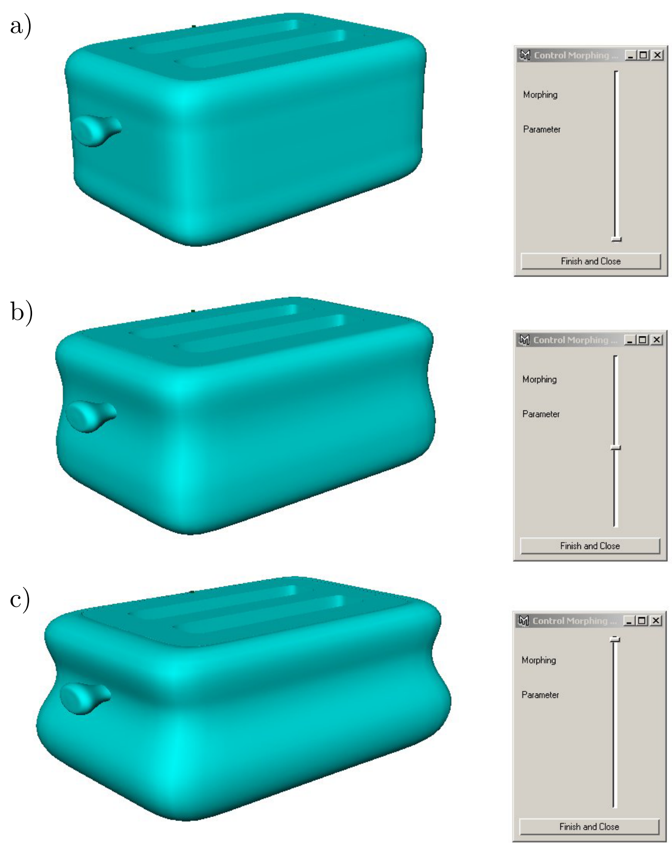

Fig. 11: a) A morphing–like deformation is applied on the body of a toaster model based on the profile (scaler) curve. b)—d) Show the deformed body with its shape towards the target shape $\approx$ 50% and $\approx$ 100% respectively.

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