

Consiglio Nazionale delle Ricerche

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Generalized Random Waypoint Mobility Model
for Wireless Ad Hoc Networks**

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IIT TR-10/2002

Technical report

Maggio 2002



Istituto di Informatica e Telematica

The Node Spatial Distribution of the Generalized Random Waypoint Mobility Model for Wireless Ad Hoc Networks

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May 2002

Abstract

In this paper we investigate the node spatial distribution generated by nodes moving according to the random waypoint model, which is widely used in the simulation of mobile ad hoc networks. More specifically, we consider a generalization of this model, called generalized random waypoint model, in which nodes can remain stationary for the entire simulation time with a given probability, and where the pause time between consecutive movements is chosen according to an arbitrary probability distribution. Furthermore, we allow the nodes to be initially distributed according to an arbitrary density. We show that the structure of the asymptotic spatial density resulting from this mobility model is composed by three distinct components: the initial, the uniform and the non-uniform component. The relative values of these components depends on the mobility parameters. We also derive an explicit formula of the two-dimensional node spatial density, which is proved to fit very well data obtained through experimentation.

1 Introduction

Performance analysis in presence of mobility is of primary importance in the design of wireless networks. For example, in cellular networks user's movement patterns are used to optimize the cell and location area layout, the dynamic channel allocation scheme, and so on. Since real movement patterns are often difficult to obtain, a common approach is to use a synthetic mobility model, which resembles to some extent the behavior of real "mobile entities". Based on this model, useful conclusions regarding critical network parameters can be provided. However, the accuracy of the results heavily depends on how close the chosen model is to the real scenario.

Mobility modeling has been extensively studied in the field of cellular networks. For example, Zonoozi and Dassanayake [20] have shown (by means of extensive simulations and "goodness-of-fit" tests) that the cell residence time of a mobile user moving according to a given mobility pattern follows a generalized gamma distribution. In the Brownian mobility model, it has been shown that, given the user's location at time t_0 , the probability distribution of the physical location at time $t > t_0$ can be calculated [12]. Very accurate mobility models for cellular networks are presented in [13, 17]. For a survey of mobility modeling of cellular networks see [2].

Although interesting, these results are not applicable to ad hoc networks. In fact, most of them concern specific properties of cellular networks (e.g., cell residence time, number of handoffs during a call, and so on). Furthermore, the emphasis is usually on modeling the properties of a single mobile user, rather than on the global mobile user's distribution. Finally, the mobility patterns considered resemble the typical motions that occur in cellular systems, i.e. human and vehicular motion. Hence, an accurate study of the probabilistic properties of a mobile ad hoc network is, to the best of our knowledge, still lacking.

A very common mobility model used in the wireless ad hoc networks community is the random waypoint model [11]. In this model, every node chooses uniformly at random a destination in the (bounded) deployment region R , and moves toward it with a certain velocity. When it reaches the destination, it remains stationary for a predefined pause time, and then it starts moving again with the same rule. As it has been observed in [2, 3, 5], nodes moving according to the random waypoint model tend to concentrate in the middle of the deployment region R , originating the so called *border effect*. The intensity of this

effect for different values of the mobility parameters has been carefully investigated in [5], where it is shown that for large values of the pause time the border effect is limited, and the spatial distribution can be well approximated with the uniform distribution. However, for other combinations of these parameters the border effect becomes very intense, and the resemblance with the uniform distribution is no longer valid. This indicates that the node spatial distribution generated by the random waypoint model *is not* uniform.

The fact that the spatial distribution generated by the random waypoint model is not uniform has two important consequences. First, it reduces the applicability of existing theoretical results concerning mobile ad hoc networks [1, 8, 14], which are based on the so called uniformity assumption: i.e., they assume that in any snapshot of the mobile network the nodes are distributed uniformly and independently at random in the deployment area. Second and more important, it implies that the representativeness of the huge amount of experimental data obtained using the random waypoint model could be impaired. In fact, typical settings for an experimentation based on the random waypoint model are the following: a few tenths or a hundred of nodes are distributed uniformly at random in a square region; then, they start moving according to the random waypoint model with a certain velocity. The behavior of the mobile network is observed for a number of steps (where one step often corresponds to one second) in the order of, at most, one thousand. Similar settings have been used, for instance, in the evaluation of routing [6, 7, 9, 10, 11, 18], multicast [15] and energy-conserving [19] protocols. Since typical values of the pause time are in the order of hundreds of steps, it follows that nodes in the above described scenario performs in general only a very limited number of movements during the observation period. Considering that the initial node distribution (which is uniform) is *different* from the asymptotic distribution generated by the random waypoint model (which *is not* uniform), this implies that few node movements could not be enough to reach the “steady state” of the network. In other words, observing the network for relatively few steps after the initial node deployment could be not representative of the actual long term behavior of the system.

In order to circumvent this problem, the node spatial distribution generated by the random waypoint model must be investigated. A first step in this direction was done in [4], where the node spatial distribution of one-dimensional networks (i.e., nodes moving along a line) when the pause time is 0 is derived. In this paper, we derive an explicit formula of the long run two-dimensional node spatial distribution of the random waypoint model for arbitrary values of the pause time. Our analysis will also show that a similar expression holds for a generalized and more realistic version of the random waypoint model. In this generalized random waypoint model, nodes can remain stationary for the entire simulation time with a given probability, and the pause time between consecutive movements is chosen according to an arbitrary probability distribution. Furthermore, we allow the nodes to be initially distributed according to an arbitrary density. We show that the structure of the asymptotic spatial density resulting from this mobility model is composed by three distinct components: the initial, the uniform and the non-uniform component. The relative values of these components depends on the mobility parameters. The quality of our formula is validated through extensive experimentation, which shows that the theoretical curve fits very well experimental data for many combinations of the mobility parameters.

We remark that the formula of the node spatial distribution obtained in this paper has a great practical relevance. By initially distributing the nodes according to our formula, we put the network in its asymptotic “steady state”, thus avoiding the large number of mobility steps (in the order of thousands) needed to make the system converge to this “steady state”. Thus, the computational resources can be used to investigate the behavior of the network *after* the “steady state” has been reached, rather than wasted in “calculating” it.

2 The node spatial distribution of the random waypoint model

In the random waypoint model [11], every node chooses uniformly at random a destination in the deployment region R . In this paper, we assume that R is the unit square $[0, 1]^2$. The node moves toward the destination with a velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. When it reaches the destination, it remains stationary for a predefined pause time t_{pause} , and then it starts moving again according to the same rule. We have also included a further parameter in the model, namely the probability p_{stat} that a node remains stationary during the entire simulation time. Hence, only $(1 - p_{stat}) \cdot n$ nodes

(on the average) will move. Introducing p_{stat} in the model accounts for those situations in which some nodes are not able to move. For example, this could be the case when sensors are spread from a moving vehicle, and some of them remain entangled, say, in a bush or tree. This can also model a situation where two types of nodes are used, one type that is stationary and another type that is mobile.

We are interested in characterizing the node spatial distribution that results from a network whose nodes move according to the random waypoint model for a very large number of mobility steps. First, we observe that in this mobility model each node moves independently of each other, hence we can concentrate our attention on the spatial distribution of a single node. More formally, we want to determine the probability density function $PDF_{X(t)}$ as t grows to infinity, where $X(t)$ denotes the position of the node in the deployment region at time t .

As it can be easily seen, this density function is composed by three distinct components: the *initial*, the *pause* and the *mobility* component. The initial component accounts for the fact that a node can remain stationary for the entire network operational time. The pause component accounts for the fact that a node “rest” before starting a new movement. Finally, the mobility component accounts for the time that a node is actually moving.

Let us consider each component separately. The initial component can be easily determined by observing that a node remains stationary during the entire network operational time with probability p_{stat} , and that nodes are initially distributed uniformly at random in the deployment region. Since a stationary node will never move, the initial component f_I of the density function is:

$$f_I(x, y) = \begin{cases} p_{stat} & \text{if } (x, y) \in [0, 1]^2 \\ 0 & \text{otherwise} \end{cases},$$

independently of the time t at which we observe the node position. Hence, the initial component is the uniform distribution except for the fact the $\int_R f_I(x, y) dx dy$ equals p_{stat} and not 1 as in the uniform case. Thus, the initial component can be seen as a uniform distribution “weighted” by p_{stat} .

Since the starting and ending point of a node movement in the random waypoint model are chosen uniformly at random in R , the pause component has a formula similar to that of the initial component. More precisely, we have:

$$f_{P(t)}(x, y) = \begin{cases} (1 - p_{stat})p(t) & \text{if } (x, y) \in [0, 1]^2 \\ 0 & \text{otherwise} \end{cases},$$

where $f_{P(t)}$ denotes the pause component of the density function at time t , and $p(t)$ is the probability that a node is “resting” at time t . The term $(1 - p_{stat})$ accounts for the fact that a resting node is not stationary. Since we are interested in characterizing the asymptotic density (i.e., the density as $t \rightarrow \infty$), we must determine the value of $p(t)$ as $t \rightarrow \infty$. This way we obtain the asymptotic expression of the pause component, denoted f_P . For the sake of simplicity, assume that $v_{min} = v_{max} = v$, i.e. that the node always moves with the same velocity v . In the random waypoint model, a node alternates rest periods (lasting t_{pause} steps each) and movement periods. Under our simplifying hypothesis $v_{min} = v_{max} = v$, the duration of a movement period depends only on the distance between the starting and destination point of the movement. Let $i(t)$ denotes the number of movements completed before time t , and let t_{move_j} denotes the duration of the j -th movement (see Figure 1). We have:

$$\lim_{t \rightarrow \infty} p(t) = \lim_{t \rightarrow \infty} \frac{\sum_{j=1}^{i(t)} \frac{t_{pause}}{t_{pause} + t_{move_j}}}{i(t)} = \lim_{t \rightarrow \infty} \frac{t_{pause} \sum_{j=1}^{i(t)} \frac{1}{t_{pause} + t_{move_j}}}{i(t)}$$

By the law of large numbers, we obtain:

$$\lim_{t \rightarrow \infty} \frac{t_{pause} \sum_{j=1}^{i(t)} \frac{1}{t_{pause} + t_{move_j}}}{i(t)} = \frac{t_{pause}}{t_{pause} + E[t_{move}]},$$

where $E[t_{move}]$ is the expected duration of the movement period. Denoting with $E[d]$ the expected distance between a pair of points chosen uniformly at random in R , we can conclude that $\lim_{t \rightarrow \infty} p(t) = \frac{t_{pause}}{t_{pause} + \frac{E[d]}{v}} = p$.

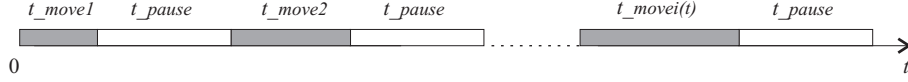


Figure 1: A node alternates between rest periods (t_{pause}) and movement periods (t_{move_j}).

Contrary to the case of the initial and pause component, the mobility component *is not* uniform. In fact, this is the component that generates the border effect observed in [2, 3, 5]. Denoting with $f_M(x, y)$ the limit of the mobility component as $t \rightarrow \infty$, we can summarize our discussion in the following theorem:

Theorem 1. *The asymptotic spatial density function of a node moving in R according to the random waypoint model of parameters p_{stat} , t_{pause} and $v_{min} = v_{max} = v$, is*

$$f(x, y) = \begin{cases} p_{stat} + (1 - p_{stat})p + (1 - p_{stat})(1 - p)f_M(x, y) & \text{if } (x, y) \in [0, 1]^2 \\ 0 & \text{otherwise} \end{cases},$$

where $p = \frac{t_{pause}}{t_{pause} + \frac{E[d]}{v}}$.

The formula for $f(x, y)$ of Theorem 1 is fully coherent with the statistical analysis presented in [5]: the density is the sum of a uniform ($f_I(x, y) + f_P(x, y)$) and a non-uniform ($f_M(x, y)$) component. As p_{stat} and/or t_{pause} increase, the uniform component of the density becomes predominant, and $f(x, y)$ can be well approximated with the uniform distribution. Conversely, for small values of p_{stat} and/or t_{pause} , the non-uniform component dominates and generates a significant border effect. The influence of the velocity on $f(x, y)$ is less evident: in general, higher velocities cause a decreased value of t_{move} and, consequently, a more uniform distribution. However, for extreme values of t_{pause} the effect of the velocity is negligible. For example, if $t_{pause} = 0$ then the pause component of the density is 0, regardless of the value of v . Similarly, if t_{pause} is very large, then $p \approx 1$ regardless of the value of v .

In order to give a precise formula for $f(x, y)$, we need to evaluate $E[d]$ and $f_M(x, y)$. While it is known that $E[d] = 0.521405$ [16], deriving f_M is not straightforward. The expression of f_M for one-dimensional motion (i.e., the node moves along a line of length 1) has been derived in [4], using a technique based on the construction of a temporal histogram that counts how often the node is observed in any point $x \in [0, 1]$. Since the expected distance between a pair of points chosen uniformly at random in $[0, 1]$ is $1/3$ [16], we can state the exact formula of the asymptotic spatial distribution of a node moving according to the one-dimensional random waypoint model:

Theorem 2. *The asymptotic spatial density function of a node moving in $[0, 1]$ according to the random waypoint model of parameters p_{stat} , t_{pause} and $v_{min} = v_{max} = v$, is*

$$f(x) = \begin{cases} p_{stat} + (1 - p_{stat})p + (1 - p_{stat})(1 - p)6x(1 - x) & \text{if } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases},$$

where $p = \frac{t_{pause}}{t_{pause} + \frac{1}{3v}}$.

Function $f(x)$ for different combinations of the mobility parameters is shown in figures 2 and 3. The correspondent cumulative density function $CDF(x)$ is also reported. For the sake of comparison, the figures show the curves obtained with the uniform distribution. As it is seen, for increasing values of p_{stat} and/or t_{pause} the plots of $f(x)$ and $CDF(x)$ approach those of the uniform distribution. It is also seen that the non-uniform component of the density (obtained when $p_{stat} = t_{pause} = 0$) is parabolic.

In the next section, we derive the expression of the mobility component f_M of the density for two-dimensional networks.

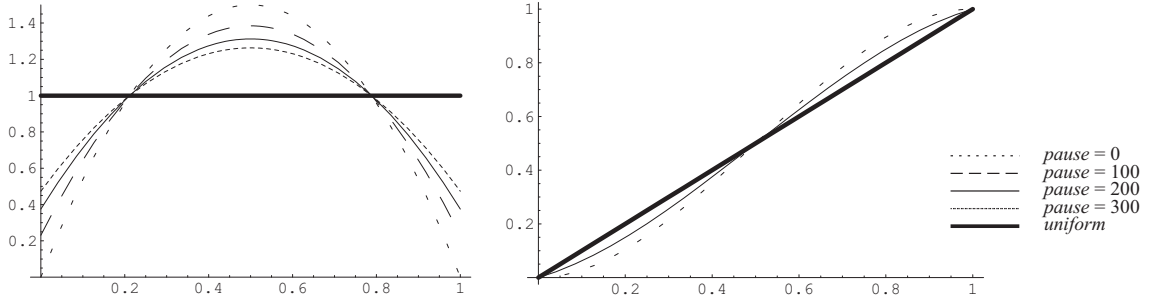


Figure 2: Density and cumulative density function of the one-dimensional node spatial distribution for different values of the pause time. p_{stat} and v are set to 0 and 0.001, respectively.

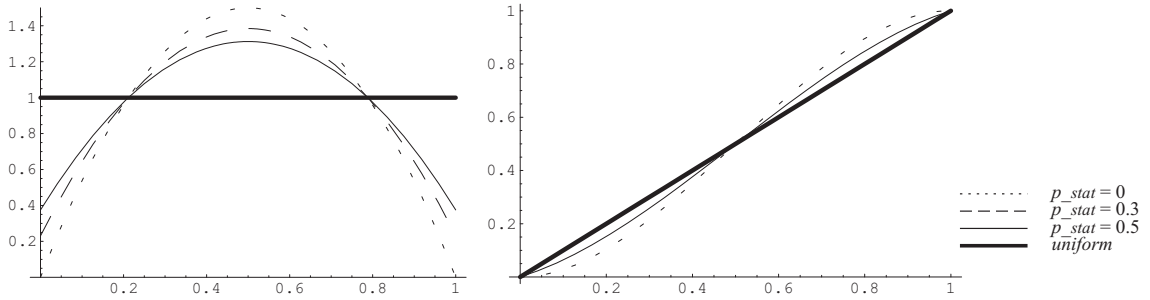


Figure 3: Density and cumulative density function of the one-dimensional node spatial distribution for different values of p_{stat} . The pause time and v are set to 0 and 0.001, respectively.

3 The two-dimensional mobility component

As observed in [4], the mobility component in two dimensions cannot be directly derived from the formula for the one-dimensional case. In fact, the two-dimensional movement is composed by two *dependent* one-dimensional movements. Furthermore, the speed of the mobile node when projected along the x -axis in general is not constant, and is different from the (non-constant) speed along the y -axis. For this reason, we will use a different technique.

Let $P(x, y, d)$ denote the probability that the mobile node is in a square of side d centered in (x, y) , denoted S_d^{xy} . By definition of probability density function, we have:

$$P(x, y, d) = \int_{x-d/2}^{x+d/2} \left(\int_{y-d/2}^{y+d/2} f_M(x, y) dy \right) dx . \quad (1)$$

When d is sufficiently small, f_M can be considered constant in S_d^{xy} , and (1) can be rewritten as:

$$P(x, y, d) = d^2 f_M(x, y) .$$

Hence, we have:

$$f_M(x, y) = \lim_{d \rightarrow 0} \frac{P(x, y, d)}{d^2} .$$

In order to calculate $P(x, y, d)$ we observe that, given the assumption $p_{stat} = t_{pause} = 0$ (we recall that we are investigating the mobility component), the mobile node can be considered as always moving from a starting point S to a destination point D . Denoting by $P_{tr}(x, y, d)$ the probability that the trajectory (i.e., the line that connects S with D) intersect S_d^{xy} , and by l_d^{xy} the length of the segment that intersect

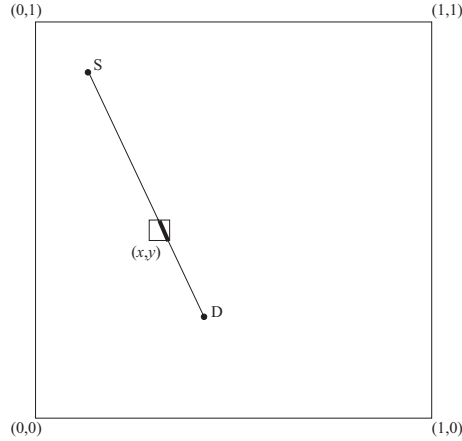


Figure 4: Intersection of the trajectory with S_d^{xy} . The length of the segment that intersect S_d^{xy} (bold line) is denoted l_d^{xy} .

S_d^{xy} (see Figure 4), and by observing that the node moves along the trajectory with constant speed, we have:

$$P(x, y, d) = P_{tr}(x, y, d) \cdot \frac{l_d^{xy}}{L} ,$$

where L is the length of the trajectory. Since we are investigating the asymptotic node spatial distribution, by the law of large numbers we can replace L with the expected distance between two randomly and uniformly distributed points in $[0, 1]^2$, which we denote $E[L]$. Note that $E[L]$ does not depend on the coordinates (x, y) of the point. Similarly, l_d^{xy} can be replaced by the expected length of the intersection of the trajectory with S_d^{xy} , denoted $E[l_d^{xy}]$. Contrary to $E[L]$, $E[l_d^{xy}]$ does depend on the position of the point: intuitively, the closer (x, y) is to the border, the smaller value of l_d^{xy} is expected. Noting that $E[l_d^{xy}]$ is anyway proportional to d , we have:

$$E[l_d^{xy}] = k(x, y) \cdot d .$$

In order to simplify the analysis, we approximate the function $k(x, y)$ with a constant k . As we shall see, this approximation has negligible influence on the quality of our analysis. Summarizing, we have:

$$f_M(x, y) = \lim_{d \rightarrow 0} \frac{P(x, y, d)}{d^2} = \lim_{d \rightarrow 0} \frac{P_{tr}(x, y, d)}{d} \cdot \frac{k}{E[L]} .$$

Up to a constant, we have then reduced the original problem to the problem of determining the probability that a trajectory intersect S_d^{xy} . Observe that it is not necessary to calculate the exact value of k and $E[L]$, since they will be absorbed by the multiplicative constant needed to normalize f_M .

Consider the square S_d^{xy} of side d centered around the point of coordinate (x, y) , and fix an arbitrary starting point $S = (x_s, y_s)$ in the unit square. Since the distribution of the destination point of a trajectory is uniform, the probability that a trajectory starting at S intersects S_d^{xy} is given by the area of the polygon shaded in Figure 5. Observe that the area of the polygon depends on d and on the coordinates of S and S_d^{xy} . Denoting with $A(x, y, x_s, y_s, d)$ this area, we can then calculate $P_{tr}(x, y, d)$ as the integral of $A(x, y, x_s, y_s, d)$ over all possible positions of S , i.e.:

$$P_{tr}(x, y, d) = \iint_{[0,1]^2} A(x, y, x_s, y_s, d) dx_s dy_s ,$$

and f_M can be obtained calculating the limit of the ratio of this integral to d as d goes to 0.

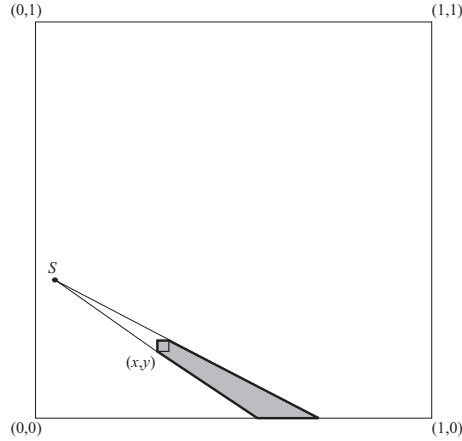


Figure 5: If the starting point of the trajectory is in S , $P_{tr}(x, y, d)$ is given by the area of the shaded polygon, denoted $A(x, y, x_s, y_s, d)$.

Finding the exact expression of $A(x, y, x_s, y_s, d)$ is not straightforward. In fact, the shape of the polygon depends on the relative positions of S and S_d^{xy} . For this reason, given the coordinate of S_d^{xy} , we divide $[0, 1]^2$ into a number of regions, with the property that all the (starting) points in the same region induce polygons with the same shape. This way, we can calculate the partial integral independently on each region, and obtain the overall integral as the sum of the contributes of all the regions.

The division of the unit square into regions for a given position of S_d^{xy} is reported in Figure 6. First, we divide $[0, 1]^2$ into four quadrants Q_1, \dots, Q_4 . Quadrants are separated from each other by strips of width d , obtained by extending the sides of S_d^{xy} to the borders. Each quadrant is then further divided into three subquadrants, obtained by extending the lines that connect the opposite corner of the unit square to opposite vertices of S_d^{xy} . We then have a total of 16 regions. However, it should be noted that the area of some of these regions goes to 0 as d goes to 0, hence their contribution can be omitted when calculating the value of the overall integral. This is the case of the area of the four strips of width d , as well as of the area of Q_{12} and of the corresponding regions in the other quadrants. Then, we can rewrite the overall integral as:

$$\iint_{[0,1]^2} A(x, y, x_s, y_s, d) dx_s dy_s = \sum_{i=1, \dots, 4} \iint_{Q_i} A(x, y, x_s, y_s, d) dx_s dy_s ,$$

where each of the $\iint_{Q_i} A(x, y, x_s, y_s, d) dx_s dy_s$ can be rewritten as:

$$\iint_{Q_{i1}} A(x, y, x_s, y_s, d) dx_s dy_s + \iint_{Q_{i2}} A(x, y, x_s, y_s, d) dx_s dy_s .$$

In the following, we detail the derivation of a partial integral referring to the first quadrant, under the hypothesis that $x \geq y$ and $x \leq 1/2$. The derivation of the partial integrals referring to the other regions can be obtained by similar geometric arguments, and is not reported for the sake of brevity. It should also be observed that, due to the multiple symmetries of the problem, evaluating $f_M(x, y)$ for $(x, y) \in \mathcal{R}$, where $\mathcal{R} = \{(x, y) \in [0, 1]^2 \mid (x \geq y) \wedge (x \leq 1/2)\}$, is sufficient to determine $f_M(x, y)$ in all the unit square by proper variable substitutions.

Let us assume that S is in Q_{11} . Then, $A(x, y, x_s, y_s, d)$ can be calculated as the difference between the area of triangles SR_1R_2 and SP_1P_2 , augmented with half of the area of S_d^{xy} (see Figure 7). Hence, we can write:

$$A(x, y, x_s, y_s, d) = \frac{1}{2} d \left(d - x + x_s + y - y_s + \frac{4y_s^2(x_s - x + y - y_s)}{(2y - 2y_s + d)(2y_s - 2y + d)} \right)$$

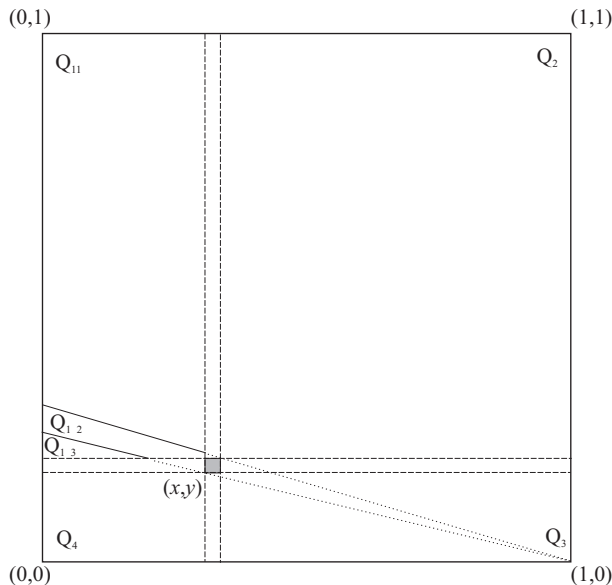


Figure 6: The unit square is divided into quadrants, separated by strips of width d . Every quadrant is then divided into three sub-quadrants, obtained by extending the lines that connect the opposite corner of the unit square to opposite vertices of S_d^{xy} . For clarity, only the division of the first quadrant is reported.

This area must be integrated over Q_{11} . Observing that the line that delimits the lower side of Q_{11} has equation $\bar{y} = m_{11}\bar{x} + q_{11}$, where $m_{11} = \frac{2y+d}{2x-2+d}$ and $q_{11} = \frac{2y+d}{2-2x-d}$, we can write:

$$\int_{Q_{11}} A(x, y, x_s, y_s, d) dx_s dy_s = \int_0^{x-\frac{d}{2}} \left(\int_{m_{11}x_s+q_{11}}^1 \frac{1}{2} d \left(d - x + x_s + y - y_s + \frac{4y_s^2(x_s - x + y - y_s)}{(2y - 2y_s + d)(2y_s - 2y + d)} \right) dy_s \right) dx_s \quad (2)$$

We then evaluate the limit of the ratio of (2) to d as d goes to 0, obtaining:

$$\lim_{d \rightarrow 0} \frac{(2)}{d} = \frac{1}{4} xy \left(\frac{x^2 + x(6y - 7) + 2(3 - 4y + y^2)}{(x - 1)(y - 1)} + 2(x + y) \log \left[\left(\frac{1}{x} - 1 \right) \left(\frac{1}{y} - 1 \right) \right] \right)$$

This term gives the contribution of Q_{11} to the spatial density. The overall density can be calculated by summing the contribution of all the regions. The resulting expression, which we denote $f_{\mathcal{R}}(x, y)$, must then be normalized in such a way that $\int_{\mathcal{R}} f_{\mathcal{R}}(x, y) dx dy = \frac{1}{8}$ (note that the area of \mathcal{R} is $\frac{1}{8}$). After long and tedious calculation which is not reported, we have obtained the following expression for $f_{\mathcal{R}}(x, y)$:

$$f_{\mathcal{R}}(x, y) = 6y + \frac{3}{4}(1 - 2x + 2x^2) \left(\frac{y}{y - 1} + \frac{y^2}{(x - 1)x} \right) + \frac{3}{2} \left((2x - 1)y(1 + y) \log \left(\frac{1 - x}{x} \right) + y(1 - 2x + 2x^2 + y) \log \left(\frac{1 - y}{y} \right) \right)$$

It should be observed that $f_{\mathcal{R}}(x, y)$ is not defined for $x = 0$ or $y = 0$. However, it can be easily extended by continuity to the boundary of \mathcal{R} . Summarizing, we can write:

$$f_M(x, y) = \begin{cases} 0 & \text{if } (x = 0) \text{ or } (y = 0) \\ f_{\mathcal{R}}(x, y) & \text{otherwise} \end{cases}$$

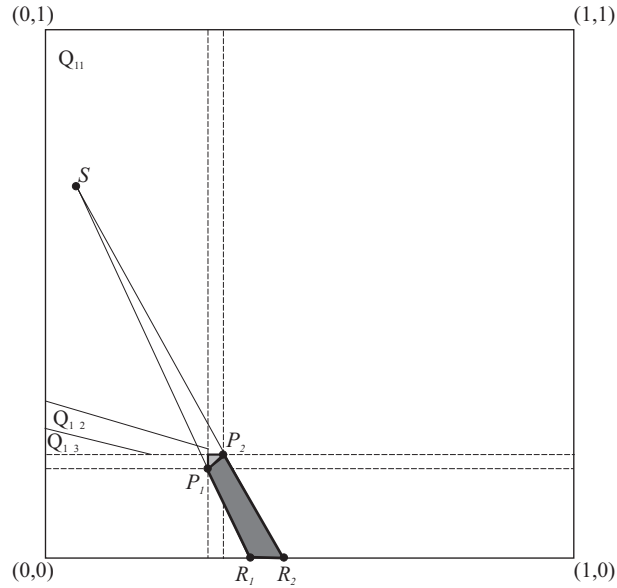


Figure 7: The area $A(x, y, x_s, y_s, d)$ when S is in Q_{11} can be calculated as the difference of the areas of triangles SR_1R_2 and SP_1P_2 (shaded area), plus half of the area of $S_d^{x,y}$ (light shaded area).

We remark that the expression of $f_M(x, y)$ above is valid only for $(x, y) \in \mathcal{R}$. The expression of $f_M(x, y)$ on the remainder of $[0, 1]^2$ can be easily obtained by observing that, given the symmetry of the problem, $f_M(x, y) = f_M(y, x) = f_M(1 - x, y) = f_M(x, 1 - y)$. The 3D and contour plot of $f_M(x, y)$ are reported in Figure 8.

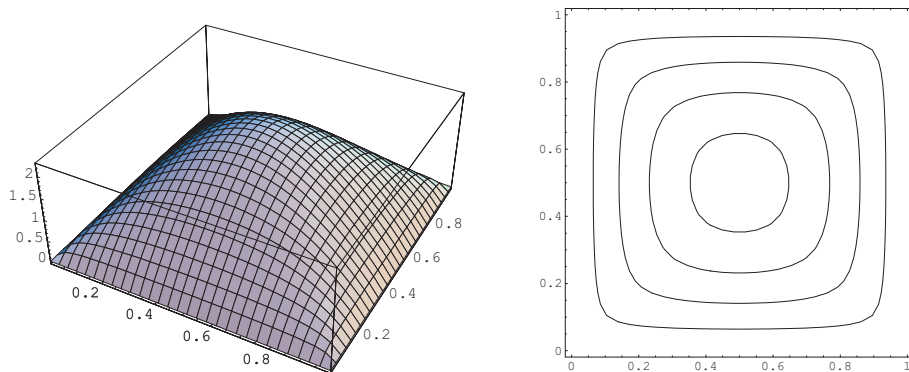


Figure 8: Three-dimensional plot of the mobility component, and contour lines corresponding to the values $f_M(x, y) = .5, 1, 1.5$ and 2 .

4 The generalized random waypoint model

In the previous sections, we have derived the expression of the one and two-dimensional node spatial distribution of the random waypoint model. In the derivation, we have often used the law of large numbers to estimate the asymptotic value of some parameters (e.g., the probability p that a node is resting). Hence, our results can be straightforwardly extended to a generalized random waypoint model, where the pause time and the velocity are chosen according to arbitrary probability distributions with

expectation $E[t_{\text{pause}}]$ and $E[v]$, respectively. We can further generalize the mobility model by allowing the node to be initially placed according to an arbitrary density f_{init} . The generalized random waypoint model can be then defined as follows:

Definition 1 (Generalized random waypoint model). *In the generalized random waypoint model, nodes are initially distributed in $R = [0, 1]^2$ with density f_{init} . A node remains stationary for the entire network operational time with probability p_{stat} . If the node is not stationary, it chooses its destination uniformly at random in R , and moves towards it with a velocity v chosen according to an arbitrary probability distribution \mathcal{P}_v . When the node arrives at destination, it rests for a time t_{pause} , chosen according to an arbitrary probability distribution $\mathcal{P}_{\text{pause}}$. Then, it starts a new movement as described above.*

We believe that this generalization of the random waypoint model allows a more accurate simulation of realistic scenarios. In fact, nodes can be initially placed according to an arbitrary distribution, rather than the uniform distribution. Thus, we can model those situations in which nodes are, for instance, initially concentrated in a subregion of R . More importantly, we do not force a node to have the *same* pause time during the entire network operational time. This is a very unrealistic aspect of the random waypoint model, which is further amplified by the fact that the pause time is assumed to be the same *for all the nodes in the network*. Conversely, we simply assume that the pause time after each movement is chosen according to an arbitrary distribution, *which does not change with time*. Furthermore, this distribution is the same for all the nodes in the network. Then, our probabilistic homogeneity assumption is far less stringent than the equality assumption of the original model. A similar generalization is made for the velocity, which is chosen according to an arbitrary distribution, which does not change with time and is the same for all the nodes in the network.

We can thus state the following theorem, which is the main result of this paper:

Theorem 3. *The asymptotic spatial density function of a node moving in R according to the generalized random waypoint model of parameters f_{init} , p_{stat} , $\mathcal{P}_{\text{pause}}$ and \mathcal{P}_v , is*

$$f(x, y) = \begin{cases} p_{\text{stat}}f_{\text{init}} + (1 - p_{\text{stat}})p + (1 - p_{\text{stat}})(1 - p)f_M(x, y) & \text{if } (x, y) \in [0, 1]^2 \\ 0 & \text{otherwise} \end{cases},$$

where $p = \frac{E[\mathcal{P}_{\text{pause}}]}{E[\mathcal{P}_{\text{pause}}] + \frac{0.521405}{E[\mathcal{P}_v]}}$, $E[\mathcal{P}_{\text{pause}}]$ is the expected pause time, and $E[\mathcal{P}_v]$ is the expected velocity.

Theorem 3 is very important, since it states that as long as the motion pattern has some (probabilistic) homogeneity, the structure of the asymptotic spatial density is composed by three distinct component: the initial component f_{init} , a uniform component generated by the fact that the movement destinations are chosen uniformly in R , and the mobility component f_M . The relative values of these components depend on p_{stat} and on the *expected value* (not on the actual distribution) of the pause time and velocity. Hence, a very large class of mobility models is captured by the formula stated in Theorem 3.

Finally, we want to emphasize the Theorem 3 has also a great practical relevance. So far, the only way to investigate relevant asymptotic properties of mobile networks was to simulate the nodes movement for a very large number of steps. This is done at the expense of considerable computational resources. As a consequence, the size of the mobile system is usually kept small (it is rarely above 100 in existing experimental results). As wireless ad hoc networks will become reality in a near future, their size is likely to grow to as much as thousands of nodes. Hence, the simulation of *large* mobile networks, in which the scalability of the protocols can be carefully investigated, will become an issue. We believe that our characterization of the node spatial distribution of mobile networks be of great help in the simulation of large mobile ad hoc networks. By initially distributing the nodes according to the formula of Theorem 3, we put the system in its asymptotic “steady state”, thus avoiding the large number of mobility steps (in the order of thousands) needed to make the system converge to this “steady state”. Thus, the computational resources can be used to investigate the behavior of the network *after* the “steady state” has been reached, rather than wasted in “calculating” it.

5 Experimental evaluation

In this section we report the result of the experiments that we have performed to validate our theoretical analysis.

For the experiments we have utilized the simulator used in [5] to test the uniformity of the node spatial distribution. The simulator takes as input the number n of nodes to distribute, the number $\#sim$ of simulations to run and the number $\#steps$ of mobility steps to perform for each simulation. The simulator also takes as input the parameters of the random waypoint model, i.e. p_{stat} , t_{pause} (expressed as the number of steps that the node remains stationary between two movements), and the minimum (v_{min}) and maximum (v_{max}) velocity. Nodes are distributed uniformly and independently at random in $[0, 1]^2$; then, they start moving according to the random waypoint mobility model. In order to record the node spatial distribution, we divide $[0, 1]^2$ into a number of square cells of the same side, arranged in a grid fashion. In our experiments, we used a grid of 31×31 cells of side $1/31$. After $\#steps$ steps of mobility, the number of nodes in each cell is recorded. These numbers are accumulated over the $\#sim$ simulations, and are reported as the result of the experiment. If the theoretical analysis is accurate, the 3D plot obtained using these data should closely resemble that obtained by our formula.

In the first experiment, we have evaluated the impact of the approximation that we made in the derivation of f_M (we recall that we have approximated the function $k(x, y)$ in the expression of $E[l_d^{xy}]$ with a constant) on the quality of our analysis. For this reason, we set p_{stat} and t_{pause} to 0, and $v_{min} = v_{max} = 0.01$. We then distributed $n = 961$ nodes in the deployment region, and we performed 10000 simulations of 10000 mobility steps each. These numbers were chosen because they are a good compromise between statistical accuracy (we recall that our simulation corresponds to 9610000 independent experiments) and running time.

The 3D and contour plot resulting from the experiment, which are reported in Figure 9, show a very close resemblance with the plots of f_M . This resemblance is further evidenced by the plots shown in Figure 10. These graphics report two cuts parallel to the x -axis (for $y = 0.5$ and $y \approx 0.21$), and the diagonal cut of the 3D plot. The result of this experimentation shows that the approximation that we made in the derivation of f_M does not affect the quality of the result.

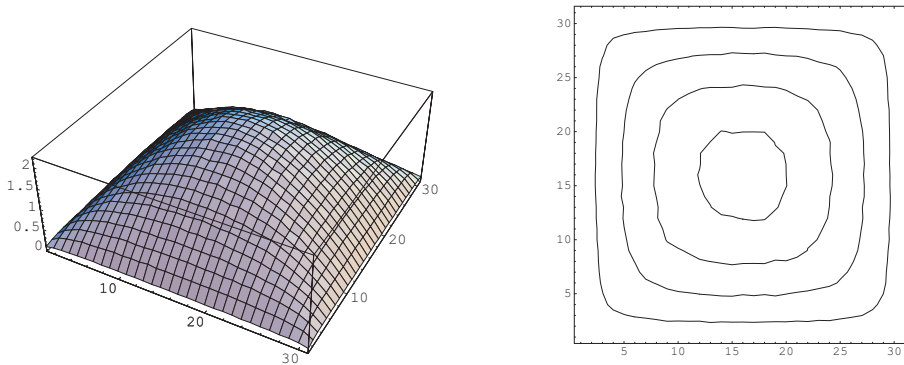


Figure 9: Three-dimensional plot of the experimental data, and contour lines corresponding to the values $f_M(x, y) = .5, 1, 1.5$ and 2 .

In the second experiment, we have verified how well our formula of node spatial distribution fits the experimental data in two different scenarios. Namely, we set $p_{stat} = 0.1$, $t_{pause} = 100$ and $v_{min} = v_{max} = 0.01$ in the first scenario, and $p_{stat} = 0.3$, $t_{pause} = 300$ and $v_{min} = v_{max} = 0.01$ in the second. As in the previous case, we set $n = 961$ and $\#steps = \#sim = 10000$. The results of the experiments are reported in figures 11,12,13,14. As it is seen, our formula is a good fit of the experimental data in both scenarios.

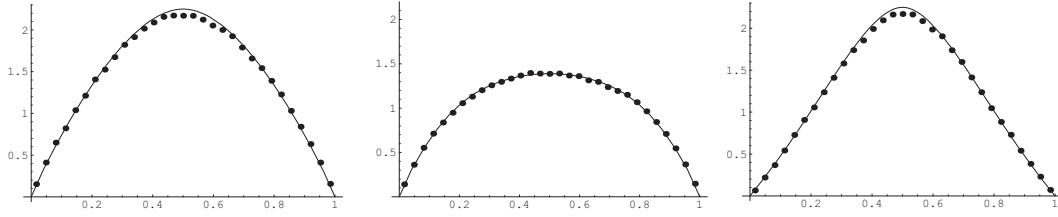


Figure 10: Cuts parallel to the x -axis for $y = 0.5$ (left) and $y \approx 0.21$ (center), and diagonal cut (right) of the 3D plots of the theoretical and experimental node spatial distribution. Experimental data are represented by bold points.

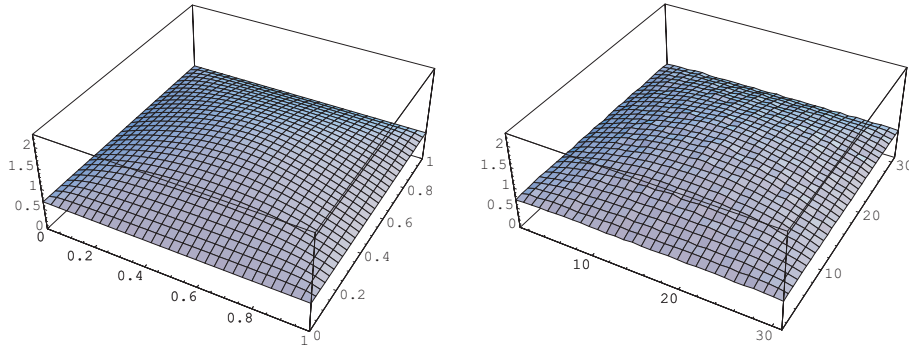


Figure 11: Three-dimensional plot of the theoretical (left) and experimental (right) data for the case $p_{stat} = 0.1$, $t_{pause} = 100$ and $v_{min} = v_{max} = 0.01$.

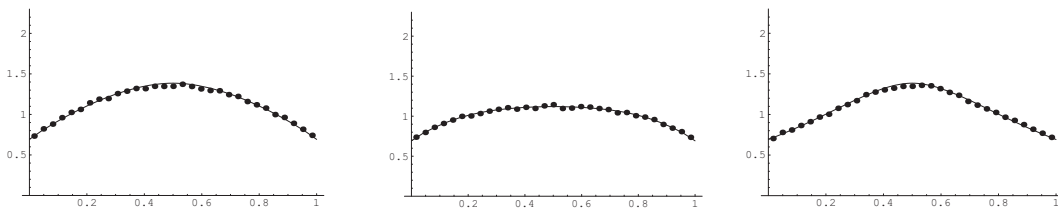


Figure 12: Cuts parallel to the x -axis for $y = 0.5$ (left) and $y \approx 0.21$ (center), and diagonal cut (right) of the 3D plots of the theoretical and experimental node spatial distribution for the case $p_{stat} = 0.1$, $t_{pause} = 100$ and $v_{min} = v_{max} = 0.01$. Experimental data are represented by bold points.

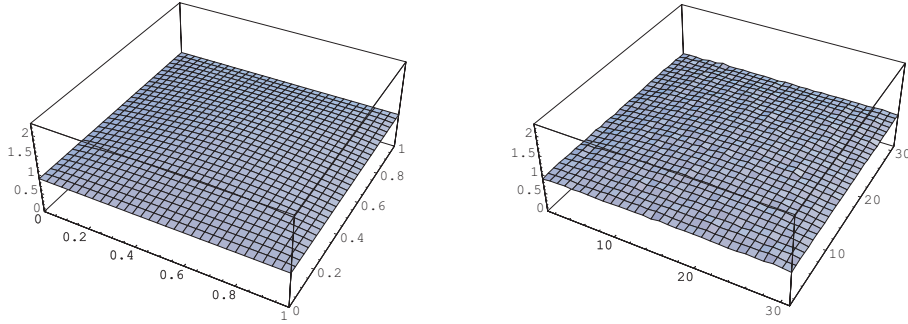


Figure 13: Three-dimensional plot of the theoretical (left) and experimental (right) data for the case $p_{stat} = 0.3$, $t_{pause} = 300$ and $v_{min} = v_{max} = 0.01$.

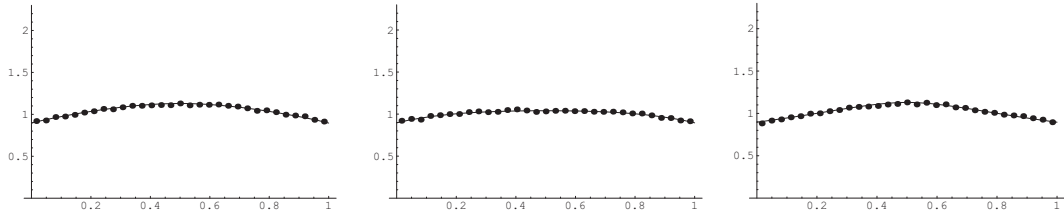


Figure 14: Cuts parallel to the x -axis for $y = 0.5$ (left) and $y \approx 0.21$ (center), and diagonal cut (right) of the 3D plots of the theoretical and experimental node spatial distribution for the case $p_{stat} = 0.3$, $t_{pause} = 300$ and $v_{min} = v_{max} = 0.01$. Experimental data are represented by bold points.

Finally, we have verified the quality of our formula for the generalized random waypoint model. To this purpose, we extended the simulator by allowing nodes to be initially distributed uniformly at random in $[0, 1/2]^2$. Further, the pause time is chosen uniformly at random between a minimum and maximum value for each movement.

We have simulated the following scenario: nodes initially distributed uniformly at random in $[0, 1/2]^2$, $p_{stat} = 0.2$, t_{pause} chosen uniformly at random in the interval $[100, 300]$ at each movement (independently for each node). Finally, the velocity was chosen uniformly at random in the interval $[0.005, 0.015]$. We distributed $n = 961$ nodes, and we performed 10000 simulations of 10000 mobility steps each. The result of the experiment is shown in figures 15 and 16. Also in this case, the formula fits the experimental data very well.

6 Conclusions

In this paper, we have derived an explicit formula of the two-dimensional node spatial distribution generated by a generalization of the random waypoint model. The good quality of our formula has

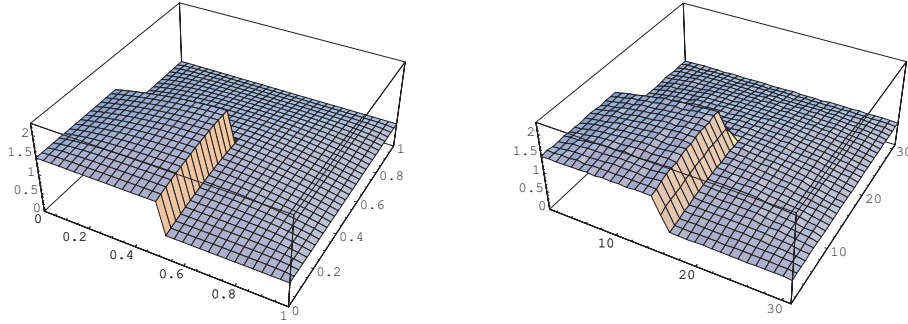


Figure 15: Three-dimensional plot of the theoretical (left) and experimental (right) data for the generalized random waypoint model.

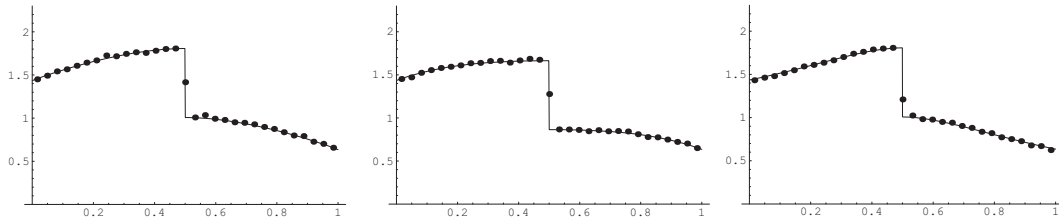


Figure 16: Cuts parallel to the x -axis for $y = 0.5$ (left) and $y \approx 0.21$ (center), and diagonal cut (right) of the 3D plots of the theoretical and experimental node spatial distribution for the generalized random waypoint model. Experimental data are represented by bold points.

been confirmed by extensive experimentation, whose results have shown a very good fit between the experimental and theoretical curves.

Besides its practical relevance, we believe that our result is a starting point in the characterization of fundamental properties of *mobile* ad hoc networks: given the node density derived in this paper, the average route length, or the minimum transmitting range yielding a high probability of connectedness (just to cite two important parameters of ad hoc networks) in presence of mobility can be studied in a theoretical framework. As a further direction of research, we mention extending our analysis to group mobility models.

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