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Optimal Capacity of p-Persistent CSMA Protocols

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***Abstract*—In this letter we deal with the characterization and computation of the p value, say $p_{\rm opt}$, corresponding to the maximum protocol capacity in p-persistent carrier-sense multiaccess (CSMA) protocols. The contribution of this letter is twofold. First, we give an analytical justification, and a numerical validation of a heuristic formula widely used in the literature to characterize the $p_{\rm opt}$. Second, we provide closed formulas for the $p_{\rm opt}$, and we show that the optimal capacity state, given the message length distribution, is characterized by an invariant figure: the $Mp_{\rm opt}$ product.**

***Index Terms*—Capacity, carrier sense multiaccess, wireless networks.**

I. Introduction

CARRIER-SENSE multiaccess (CSMA)-BASED access schemes have been usually adopted for wireless multiaccess networks due to the inherent flexibility of random access protocols. Recently, the performance analysis of p-persistent CSMA protocols have gathered a renewed interest since the behavior of the IEEE 802.11 MAC protocol [7] can be closely studied by a p-persistent CSMA model (see [1] and [2]). Due to the limited wireless channel bandwidth, a significant performance index for wireless (LANs) is the protocol capacity, i.e., the maximum channel utilization achievable by the access scheme. In [3] it was shown that the channel utilization in p-persistent CSMA protocols is strongly affected by the adopted p value. Specifically, small p values cause large delays due to collisions, while large p values degrade the protocol capacity forcing the channel to be idle. A tradeoff between small and large values is therefore necessary. In this letter we prove that this tradeoff problem reduces to identify the p value that balances the time wasted in collisions with the time spent listening to the channel. It is worth pointing out that a similar balancing equation was already proposed for optimizing the performance of the Slotted-Aloha [4] and p-persistent IEEE 802.11 [2] protocols. However, in previous papers the use of this balancing equation was motivated only by simple heuristic considerations. In particular, in [2] it was analyzed the p-persistent IEEE 802.11 protocol where the messages length were geometrically distributed, and it was numerically shown that the balance between collisions' durations and idle times is a valid approximation of the optimal capacity state. On the other hand, in this work we analytically investigate the optimal capacity state to formally prove that for the family of p-persistent CSMA protocols, independently of the message length distribution, the balance between collisions' durations and idle times is asymptotically exact (i.e., it is exact for a large number of active stations). Finally, the proposed balancing equation is also exploited to derive approximated closed formulas for the $p_{\rm opt}$ value, that constitute a very compact and powerful characterization of the maximum protocol capacity in p-persistent CSMA protocols.

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II. Protocol Model

We consider a system with M active stations accessing a slotted multiaccess channel. The random access protocol for controlling this channel can be either a Slotted-Aloha or a p-persistent CSMA algorithm. In the first case (i.e., Slotted-Aloha), the stations transmit constant-length messages with length l that exactly fits in a slot of length t_{slot}. In the second case (i.e., CSMA), the message length is a random variable L with average l. To simplify the presentation we will assume that the L values always correspond to an integer number of slots. In both cases (i.e., slotted-Aloha and CSMA), when a transmission attempt is completed (successfully, or with a collision), each network station with packets ready for transmission (hereafter backlogged station) will start a transmission attempt with probability p. To study the channel utilization, ρ, for p-persistent CSMA protocols we observe the channel between two consecutive successful transmissions. Let us denote with t_i the time between the $(i-1)$th and the ith successful transmission, also referred to as the ith virtual transmission time, and with s_i the duration of the ith successful transmission. Hence, the channel utilization can be expressed as

$$\rho = \lim_{n\to\infty} \frac{s_1+s_2+\cdots+s_n}{t_1+t_2+\cdots+t_n}. \quad (1)$$

By denoting with $E[S]$ the average duration of a successful transmission (i.e., l according to our protocol model) and with $E[T]$ the average time between two consecutive successful transmissions, and by assuming that both $E[S]$ and $E[T]$ exist and are finite, then (1) can be written as

$$\rho = \lim_{n\to\infty} \frac{\dfrac{s_1+s_2+\cdots+s_n}{n}}{\dfrac{t_1+t_2+\cdots+t_n}{n}} = \frac{E[S]}{E[T]}. \quad (2)$$

The $E[T]$ formula can be obtained by considering the behavior of the p-persistent CSMA protocols. Specifically, by denoting with N_i^c the number of collisions that occur during t_i, the following relationship holds:

$$t_i = \sum_{j=1}^{N_i^c}[\mathrm{Idle}_{i,j} + \mathrm{Coll}_j] + \mathrm{Idle}_{i,j} + L_i$$

where, with reference to t_i, $\mathrm{Idle}_{i,j}$ is the duration of the jth (possible null) idle time that precedes the channel busy period (either collision or success), Coll_j is the duration of the jth collision

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given that a collision occurs, and L_i is the length of the successful transmission. Hence

$$E[T] = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} \sum_{j=1}^{N_i^c} [\text{Idle}_{i,j} + \text{Coll}_j] + \text{Idle}_{i,j} + L_i}{n}. \quad (3)$$

With routine algebraic manipulations (3) can be rewritten as

$$E[T] = (E[N_c]+1)\cdot E[\text{Idle}] + E[N_c]\cdot E[\text{Coll}|\text{Coll}] + E[S], \quad (4)$$

where $E[N_c]$ is the average number of collision in a virtual transmission time, and $E[\text{Coll}|\text{Coll}]$ is the average duration of a collision, given that a collision occurs. The unknown quantities in (4) are derived in Lemma 1 of [2] under the assumptions: 1) all the stations adopt a p-persistent CSMA algorithm to access the channel; 2) all the stations operate in saturation conditions, i.e., they have always a message waiting to be transmitted; and 3) the message lengths L_i are random variables identically and independently distributed.

As shown in [2], the channel utilization is a function of the protocol parameter p, the number M of active stations and the message length distribution. $E[S]$ is a constant value, given the message length distribution. The protocol capacity, say ρ_{MAX}, can thus be obtained by finding the p value, say p_{opt}, that minimizes $E[T]$

$$\min_{p\in[0,1]} \{(E[N_c]+1)\cdot E[\text{Idle}] + E[N_c]\cdot E[\text{Coll}|\text{Coll}]\}. \quad (5)$$

For instance, for the Slotted-Aloha access scheme the p_{opt} value is calculated by considering in (5) constant length messages which transmission occupies one t_{slot}. Hence, by solving (5) we obtain that $p_{\text{opt}} = 1/M$ and $\rho_{\text{MAX}} \underset{M\to\infty}{\longrightarrow} e^{-1}$ (see also [5]). Unfortunately, from (5) it is not possible to derive an exact closed formula for the p_{opt} value in the case of a general message-length distribution. Equation (5) can be adopted to numerically derive the optimal capacity state in an off-line analysis, but it is necessary to derive a simple, yet approximate, relationship to characterize the value corresponding to the optimal capacity.

III. A Balancing Equation to Derive a Quasi-Optimal Capacity State

Lemma 1 below shows that, asymptotically, in p-persistent CSMA protocols the optimal capacity state is characterized by the balancing between collisions' durations and idle times.

Lemma 1: For $M \to \infty$ and $l > 1$, the p value that satisfies (5) can be obtained by solving the following equation:

$$E[\text{Idle}] = E[\text{Coll}|N_{\text{tr}} > 1] \quad (6)$$

where $E[\text{Coll}|N_{\text{tr}} > 1]$ is the average duration of a collision given that at least a transmission occurs.

Proof: The proof is derived by observing that for p values close to the optimal value: *i*) given a collision, the probability that more than two stations collide is negligible (as shown in [6]), hence $E[\text{Coll}|\text{Coll}] = \overline{C} \cong E[\max\{L_1, L_2\}]$; *ii*) for $\overline{C} > 1$ it holds $p_{\text{opt}} < 1/M$. In fact, the p value is a decreasing function of the collision length and when $\overline{C} = 1$ (as in the Slotted-Aloha) the $p_{\text{opt}} = 1/M$. Hereafter, we assume that $Mp < 1$. Under the assumption *i*), substituting the expressions of $E[\text{Idle}]$ and $E[Nc]$ (see [2]) in (5), after some algebraic manipulations, it follows that the p_{opt} value is derived by solving

$$\min_{p\in[0,1]} \left\{ \frac{\overline{C} - (1-p)^M(\overline{C}-1)}{Mp(1-p)^{M-1}} \right\} = \min_{p\in[0,1]} \left\{ F(p, M, \overline{C}) \right\}. \quad (7)$$

Taking the derivative of $F(p, M, \overline{C})$ with respect to p, and imposing it equal to 0, we obtain the following equation:

$$\{(1-p)^M + p(1-p)^{M-1}\} = \overline{C}\left\{ p(1-p)^{M-1} - [1-(1-p)^M + (M-1)\frac{p}{1-p} \right\}. \quad (8)$$

The p_{opt} value is the solution of (8). First we analyze the left hand side (LHS) of (8). It is easy to observe that the LHS of (8) is equal to $(1-p)^{M-1}$ that tends to $(1-p)^M$ if M is sufficiently large. Furthermore, $E[\text{Idle}]\cdot P_{N_{\text{tr}}\geq 1} = (1-p)^M$, i.e., the probability that at least a station is transmitting. Under the condition $Mp < 1$, the right hand side (RHS) of (8) can be expressed as

$$\left\{ (1-p)^{M-1} + \left[1-(1-p)^M\right] + (M-1)\frac{p}{1-p} \right\} = \frac{(M+2)(M-1)}{2}p^2 + O\left((Mp)^3\right). \quad (9)$$

By indicating with $P_{\text{Coll}|N_{\text{tr}}\geq 1}$ the collision probability conditioned to have at least one transmitting station, it holds that

$$P_{\text{Coll}|N_{\text{tr}}\geq 1}\cdot P_{N_{\text{tr}}\geq 1} = 1 - \left[(1-p)^M + Mp(1-p)^{M-1}\right] = \frac{M(M-1)}{2}p^2 + O\left((Mp)^3\right). \quad (10)$$

It is worth noting the similarity between the RHS of (10), and the RHS of (9). Specifically, the RHS of (9) can be written as

$$\left[\frac{(M-1)}{2}p^2\right](M+1) + \left((Mp)^3\right) \underset{M\to\infty}{\longrightarrow} \left[\frac{(M-1)}{2}p^2\right]M + \left((Mp)^3\right) = P_{\text{Coll}|N_{\text{tr}}\geq 1}\cdot P_{N_{\text{tr}}\geq 1}. \quad (11)$$

Hence, it follows that (8) can be rewritten as:

$$E[\text{Idle}]P_{N_{\text{tr}}\geq 1} = \overline{C}P_{\text{Coll}|N_{\text{tr}}\geq 1}P_{N_{\text{tr}}\geq 1}. \quad (12)$$

By dividing all the terms in (12) by $P_{N_{\text{tr}}\geq 1}$, and substituting the $\overline{C}$ approximation with $E[\text{Coll}|\text{Coll}]$, (12) becomes $E[\text{Idle}] = E[\text{Coll}|N_{\text{tr}} > 1]$, and this concludes the proof. ■

Lemma 1 shows that, asymptotically in p-persistent CSMA protocols the optimal capacity state is characterized by the balancing between collisions' durations and idle times. To verify the existence of this relationship for small and medium M values, we numerically solved both (5) and (6) for a wide range of M values, and several message-length distributions. Specifically, in Fig. 1 we show, for several average message lengths, the relative error[1] between the p_{opt} value, and the p value that solve (6), say p_E. The shown results refer only to a geometric message-length distribution, however similar results have been obtained also for the deterministic and bimodal distributions. Fig. 1 also shows the relative error between the ρ_{MAX} and the channel utilization measured when all the stations adopt the p_E value. Results presented in the figures

[1]The relative error is the difference between the exact value and its approximation, normalized to the exact value.

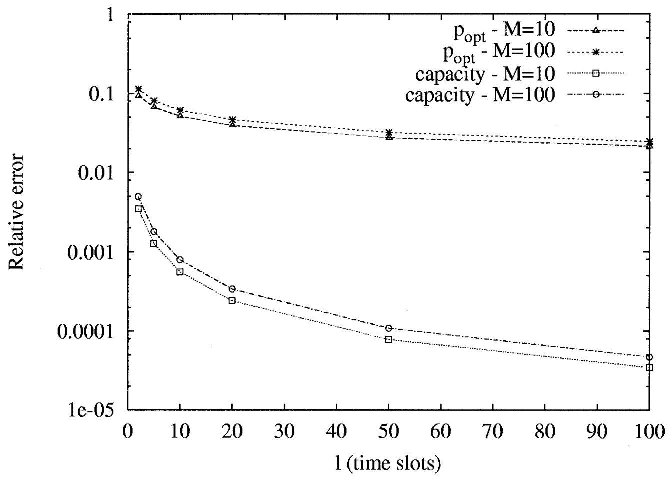

Fig. 1. Relative errors related to $p_{\rm opt}$ and $\rho_{\rm MAX}$ approximations for a geometric message-length distribution.

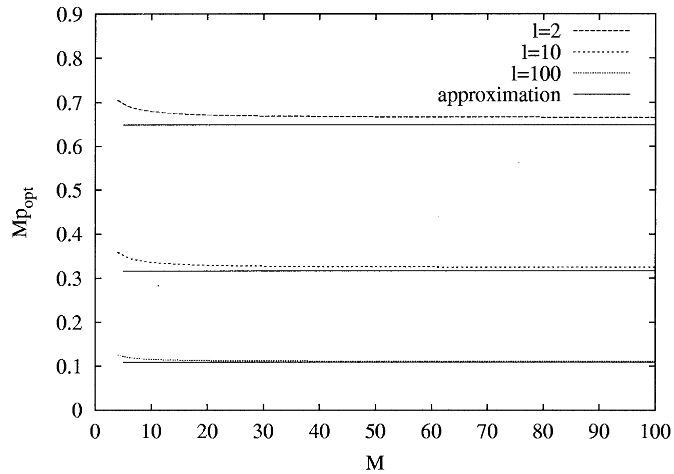

Fig. 2. The $Mp_{\rm opt}$ product for a geometric message-length distribution.

indicate that (6) provides accurate estimates also for small-medium M values. Specifically, (6) provides an approximation of the protocol capacity with a relative error that is always lower than 1%. Furthermore, ρ_{MAX} shows a low sensitiveness to the deviations of the p value from $p_{\rm opt}$. In fact, the relative error related to the ρ_{MAX} approximation is always a magnitude lower than the relative error related to the $p_{\rm opt}$ approximation. Finally, the relative errors related to the $p_{\rm opt}$ and $\rho_{\rm MAX}$ approximations rapidly decreases when the message length and the network population increase. In the following we further elaborate (6) to derive a closed formula for the $p_{\rm opt}$ value.

Lemma 2: In a M-station network that adopts a p-persistent CSMA access scheme, in which the message are i.i.d. random variables, if the stations operate in asymptotic conditions and the $Mp \ll 1$ (i.e., the Mp value is much lower than 1),[2] the $p_{\rm opt}$ value is

$$p_{\rm opt} \cong \frac{\sqrt{1-2(\overline{C}-1)\frac{M-1}{M}}-1}{(M-1)(\overline{C}-1)}. \tag{13}$$

Proof: We assume that the $p_{\rm opt}$ value is identified by (6). By assuming that collisions involve exactly two stations, under the condition $Mp \ll 1$, (6) reduces to

$$\frac{M(M-1)}{2}(\overline{C}-1)p^2 + Mp - 1 \approx 0. \tag{14}$$

By solving (14) we obtain (13) and this concludes the proof of the lemma. ■

Proposition 1: For large M and large $\overline{C}$, $p_{\rm opt}$ has the following limiting expressions:

$$p_{\rm opt} \underset{M\to\infty}{\longrightarrow} \frac{\sqrt{1+2(\overline{C}-1)}-1}{M(\overline{C}-1)} \tag{15a}$$

$$p_{\rm opt} \underset{M\to\infty \text{ and } \overline{C}\to\infty}{\longrightarrow} \frac{1}{M\sqrt{\overline{C}}}. \tag{15b}$$

[2]This is as more correct as bigger is the average message length.

Proof: Formulas (15a) and (15b) are obtained taking the limits of (13) ■

Formulas (15a) and (15b) indicate that, when $M \to \infty$, the $Mp_{\rm opt}$ product mainly depends on the average collision length but not on the network population size. To confirm this indication and to validate the above formulas, in Fig. 2 we plot the $Mp_{\rm opt}$ product versus the number of stations in the network, for the geometric message-length distribution.[3] In the figure we report both the exact $Mp_{\rm opt}$ value obtained by the numerical solution of (5) and its approximation provided by (15a). The numerical results are aligned with all previous observations and confirm the accuracy of (15a). To conclude, it is worth pointing out that (13), together with either (15a) or (15b), might be used to define an optimal tuning of the p-persistent CSMA protocols. However, in a wireless environment it is difficult to have a precise knowledge of the number of stations having packets to transmit. The interested reader is referred to [6], where a simple feedback-based algorithm that keep $E[\text{Idle}] = E[\text{Coll}|N_{\rm tr} \geq 1$ without any knowledge of the number of active station, and/or the message length distribution, has been proposed and evaluated.

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[3]Similar results have been obtained also for deterministic and bimodal distributions.