The Critical Transmitting Range for Connectivity in Sparse Wireless Ad Hoc Networks

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Abstract—In this paper, we analyze the critical transmitting range for connectivity in wireless ad hoc networks. More specifically, we consider the following problem: Assume n nodes, each capable of communicating with nodes within a radius of r, are randomly and uniformly distributed in a d-dimensional region with a side of length l; how large must the transmitting range r be to ensure that the resulting network is connected with high probability? First, we consider this problem for stationary networks, and we provide tight upper and lower bounds on the critical transmitting range for one-dimensional networks and nontight bounds for two and three-dimensional networks. Due to the presence of the geometric parameter l in the model, our results can be applied to dense *as well as sparse* ad hoc networks, contrary to existing theoretical results that apply only to dense networks. We also investigate several related questions through extensive simulations. First, we evaluate the relationship between the critical transmitting range and the minimum transmitting range that ensures formation of a connected component containing a large fraction (e.g., 90 percent) of the nodes. Then, we consider the mobile version of the problem, in which nodes are allowed to move during a time interval and the value of r ensuring connectedness for a given fraction of the interval must be determined. These results yield insight into how mobility affects connectivity and they also reveal useful trade offs between communication capability and energy consumption.

Index Terms—Wireless ad hoc networks, sparse ad hoc networks, sensor networks, energy consumption, topology control, critical transmitting range.

1 INTRODUCTION

Wireless ad hoc networks are networks where multiple nodes, each possessing a wireless transceiver, form a network among themselves via peer-to-peer communication. An ad hoc network can be used to exchange information between the nodes and to allow nodes to communicate with remote sites that they otherwise would not have the capability to reach. Wireless ad hoc networks are usually multihop networks because, as opposed to wireless LAN environments, messages typically require multiple hops before reaching a gateway into the wired network infrastructure.

Sensor networks are a particular class of wireless ad hoc networks in which there are many nodes, each containing application-specific sensors, a wireless transceiver, and a simple processor. Potential applications of sensor networks abound, e.g., monitoring of ocean temperature to enable more accurate weather prediction, detection of forest fires occurring in remote areas, and rapid propagation of traffic information from vehicle to vehicle, just to name a few [10], [27], [32], [34], [35].

While the results in this paper apply to wireless ad hoc networks in general, certain aspects of the formulation are specifically targeted to sensor networks. For example, we assume that the initial placement of nodes is random, which could result when sensors are distributed over a region from a moving vehicle such as an airplane. We are also concerned, in part, with minimizing energy consumption, which, although being an important issue in wireless ad hoc networks in general, is vital in sensor networks. Sensor nodes are typically battery-powered and, because replacing or recharging batteries is often very difficult or impossible, reducing energy consumption is the only way to extend network lifetime.

Due to the relatively recent emergence of ad hoc networks, many fundamental questions remain unanswered. We address one of those questions, namely: What are the conditions that must hold to ensure that a deployed network is connected initially and remains connected as nodes migrate? We address this question, and a number of related ones, in probabilistic terms, i.e., we evaluate the probabilities of various events related to network connectedness. More specifically, we assume that n nodes are independently and uniformly distributed in a deployment region $R = [0, l]^d$, with $d = 1, 2, 3$, and that all the nodes have the same transmitting range r. The goal is to determine the *critical transmitting range* for connectivity, i.e., the minimum value of r, which generates communication graphs that are connected with high probability (w.h.p.).[1] Determining the critical transmitting range for connectivity is essential to minimize energy consumption since transmitting power is proportional to the square (or, depending on environmental conditions, to a higher power) of the transmitting range.

The question of how many nodes are necessary to ensure connectedness w.h.p. for a given transmitting range (which is the reverse of the question above) is very important for the planning and design of sensor networks. In fact, in

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1. A formal definition of the term *with high probability* will be given in Section 4.

sensor networks, the individual unit should cost as little as possible, and inexpensive transceivers, which might not allow the transmitting range to be adjusted, are likely to be used [28].

Overall, the results presented in this paper are useful guidelines in the design of wireless ad hoc and sensor networks: Given the value of l (which is known, at least with a certain approximation, to the network designer) and n (or r), we can set the transmitting range r to the minimum value (or, disperse the minimum number n of nodes) that ensures connectedness w.h.p.

In many applications of wireless ad hoc networks, the nodes are mobile. This complicates analysis of network characteristics because the network topology is constantly changing in this situation. In this work, we consider networks both with and without mobility. We present analytical results that apply to networks without mobility and confine ourselves to simulation results for networks with mobility, due to the intractability of analysis with existing mathematical methods.

The first analytical result in this paper concerns one-dimensional networks (i.e., nodes are placed along a line of length l). We show that the communication graph that results when all the nodes have the same transmitting range r is connected w.h.p. if $rn \geq 2l \ln l$, while it is not connected w.h.p. if $rn < (1-\epsilon) l \ln l$, for some constant $0 < \epsilon < 1$. This closes a gap between lower and upper bounds on the product rn that were established in earlier versions of the paper [30], [31]. Next, we consider two and three-dimensional networks. We generalize the sufficient condition for connectedness w.h.p. to the two and three-dimensional case, while we give a necessary condition for connectedness w.h.p. that is weaker than in the one-dimensional case.

Besides analytical results, in this paper, we present a considerable body of simulation results. These results demonstrate convincingly that significant reductions of transmitting range (and, therefore, significant reductions in energy consumption as well) can be achieved by either connecting a large percentage (but not all) of the nodes for stationary networks or allowing temporary disconnections for mobile networks. The results also show that mobility comes with a cost in terms of transmitting range and energy consumption, i.e., the transmitting range required to maintain connectedness continuously during a long simulation in a highly mobile network is approximately 10 percent higher than that required to achieve connectedness in a stationary network of the same size and having the same number of nodes. However, simulations in which different mobility parameters were varied demonstrate that, for surprisingly large ranges of some parameter values, mobile networks are effectively stationary as far as the connectedness property is concerned, meaning that the transmitting range necessary for continuous connectedness is essentially identical to that necessary for connectedness in a similar stationary network.

2 Related Work

Until recently, only a few papers considered the probabilistic modeling of the communication graph properties of wireless ad hoc networks.

The main difficulty that arises in this context is that the well-established model of random graph theory [3], [20] cannot be used. In fact, a fundamental assumption in this model is that the probability of edge occurrences in the graph are independent, which is not the case in wireless ad hoc networks. As an example, consider three nodes u, v, w such that $\delta(u,v) < \delta(u,w)$, where $\delta(x,y)$ denotes the distance between x and y. With common wireless technologies that use omni-directional antennas, if u has a link to w, then it has also a link to v. Hence, the occurrences of edges (u,v) and (u,w) are correlated.

A more recent theory, which is still in development, is the theory of *geometric random graphs* (GRG). In the theory of GRG, a set of n points is distributed according to some density in a *d*-dimensional region R, and some property of the resulting node placement is investigated. For example, the longest nearest-neighbor link, the longest edge of the Euclidean Minimum Spanning Tree (MST), and the total cost of the MST have been investigated. For a survey of GRG, the reader is referred to [8].

Some of these GRG results can be applied in the study of connectivity in ad hoc networks. For example, consider a set N of points distributed in the deployment region. It is known that the longest edge of the MST built on N equals the critical transmitting range for connectivity [23]. Hence, results concerning the asymptotic distribution of the longest MST edge [22], [23] can be used to characterize the critical transmitting range, as has been done in [21].

Another notable result of the theory of GRG is that, under the assumption of uniformly distributed points, the longest nearest-neighbor link and the longest MST edge have the same value (asymptotically as $n \to \infty$). In terms of the resulting communication graph, this means that connectivity occurs (asymptotically) when the last isolated node disappears from the graph. This observation can be generalized to the case of *k*-connectivity: When the minimum node degree becomes *k*, the graph becomes *k*-connected [24]. This result, which has been used in [1] to characterize the *k*-connectivity of dense ad hoc networks, reveals an interesting analogy with nongeometric random graphs, which display the same behavior.

Although interesting, the theory of GRG can be used only to derive results concerning *dense* ad hoc networks. In fact, a standard assumption in this theory is that the deployment region R is fixed, and the asymptotic behavior of r as n grows to infinity is investigated, i.e., the node density is assumed to grow to infinity. A similar limitation applies to the model of Gupta and Kumar [12]. In their case, R is the disk of unit area, and the authors show that, if the units' transmitting range is set to $r = \sqrt{\frac{\log n + c(n)}{\pi n}}$, then the resulting network is connected w.h.p. if and only if $c(n) \to \infty$. This result is obtained making use of the theory of continuum percolation [19], which is also used in [9] to investigate the connectivity of hybrid ad hoc networks.

Given the discussion above, the applicability of existing theoretical results concerning connectivity in ad hoc networks to realistic scenarios could be impaired. In fact, it is known that real wireless networks cannot be too dense, due to the problem of spatial reuse: When a node is transmitting, all the nodes within its transmitting range must be silent, in order not to corrupt the transmission. If the node density is very high, many nodes must remain silent when a node is transmitting, and the overall network capacity is compromised [13].

In order to circumvent this problem, we add the size of the deployment region as a parameter of the model, and characterize the critical transmitting range as the size goes to infinity. The critical coverage range,[2] which is closely related to the critical transmitting range, has been investigated in [25] for the case of nodes distributed in a square with a side of length l according to a Poisson process of fixed density. The critical transmitting range for Poisson distributed points in a line of length l is derived in [26]. However, these results are also difficult to apply in real scenarios since, in a Poisson process, the actual number of deployed nodes is a random variable itself. Hence, only the *expected* number of deployed nodes can be controlled.

In this paper, we consider a model similar to that of [25], [26], but under the assumption that a fixed number n of nodes are uniformly distributed in the deployment region $R = [0, l]^d$. Furthermore, we consider also the case of a three-dimensional deployment region. In our analysis, the node density $\frac{n}{l^d}$ might either converge to 0, or to a constant $c > 0$, or diverge as the size of the deployment region grows to infinity, depending on the relative values of r, n, and l. Thus, our results can be applied both to dense, *as well as sparse*, ad hoc networks.

To conclude this section, we mention a more general connectivity problem for ad hoc networks, called the *range assignment problem*. In this version of the problem, nodes are not all forced to have the same transmitting range, and the goal is to find a range assignment that generates a (strongly) connected communication graph while minimizing some measure of energy consumption. The solution of the range assignment problem can be seen as the optimal result of the execution of a topology control protocol.[3] Thus, the investigation of the range assignment problem gives hints on the best possible energy savings achievable by any topology control protocol. It has been shown that determining an optimal range assignment is solvable in polynomial time in the one-dimensional case, while it is NP-hard (i.e., computationally infeasible) in the two and three-dimensional cases [5], [15]. A constrained version of this problem has been investigated in [2], [4].

3 Preliminaries

A d-dimensional mobile wireless ad hoc network is represented by a pair $M_d = (N, P)$, where N is the set of nodes, with $|N| = n$, and $P : N \times T \to [0, l]^d$, for some $l > 0$, is the placement function. The placement function assigns to every element of N and to any time $t \in T$ a set of coordinates in the d-dimensional cube of side l, representing the node's physical position at time t. The choice of limiting the admissible physical placement of nodes to a bounded region of $\mathbb{R}^d$ of the form $[0, l]^d$, for some $l > 0$, is realistic and will ease the probabilistic analysis of Section 4. If the physical node placement does not vary with time, the network is said to be *stationary*, and function P can be represented simply as $P : N \to [0, l]^d$.

A *range assignment* for a d-dimensional network $M_d = (N, P)$ is a function $RA : N \to (0, r_{max}]$ that assigns to every element of N a value in $(0, r_{max}]$, representing its transmitting range. Parameter r_{max} is called the *maximum transmitting range* of the nodes in the network and depends on the features of the radio transceivers equipping the mobile nodes. We assume that all the nodes are equipped with transceivers having the same features; hence, we have a single value of r_{max} for all the nodes in the network.

In this paper, we are mostly concerned with range assignments in which all the nodes have the same transmitting range r, called *homogeneous* range assignments. With this assumption, the *communication graph* of M_d induced at time t, denoted $G_{M(t)}$, is defined as $G_{M(t)} = (N, E(t))$, where the edge $(u, v) \in E(t)$ if and only if v is at distance at most r from u at time t. If $(u, v) \in E(t)$, node v is said to be a *neighbor* of u at time t. $G_{M(t)}$ corresponds to a point graph as defined in [33]. Although quite simplistic, the point graph model is widely used in the analysis of ad hoc networks. If the radio coverage area is not regular, as it is likely to be the case in real-life scenarios, the results presented in this paper are still useful since the transmitting range can be thought of as the radius of the largest circular subarea of the actual area of coverage. In this case, there could exist nodes that are connected in reality that would not be connected considering the circular region; thus, the actual probability of connectedness could be higher compared to our results.

In the next section, we consider probabilistic solutions to the following problem for stationary ad hoc networks:

Definition 1 (Minimum Transmitting Range (MTR)). *Suppose n nodes are placed in $R = [0, l]^d$; what is the minimum value of r such that the resulting communication graph is connected?*

Observe that, when dealing with the magnitude of l, the choice of unit is important. In the following, we assume that r and l are measured using the same arbitrary unit, which is therefore canceled out when discussing the relative sizes of r and l.

Given the number of nodes, minimizing r while maintaining a connected network is of primary importance if energy consumption is to be reduced. In fact, the energy consumed by a node for communication is directly dependent on its transmitting range. Furthermore, a small value of r reduces the interferences between node transmissions, thus increasing the network capacity [13]. Observe that we could just as easily have stated the problem as one of finding the minimum number of nodes to ensure connectedness given a fixed transmitting range. In fact,

2. Network coverage is defined as follows: Every node covers a circular area of radius r_c, and the monitored area R is *covered* if every point of R is at a distance at most r_c from at least one node. The goal is to find the critical value of r_c that ensures coverage w.h.p.

3. A topology control protocol is an algorithm in which nodes adjust their transmitting ranges in order to achieve a desired topological property, e.g., connectedness, while reducing energy consumption.

our solutions typically specify requirements on the product of n and r^d that ensures connectedness. These solutions can, therefore, be used to solve either MTR, as specified above, or the alternate formulation where the number of nodes is the primary concern.

It should be observed that the solution to MTR depends on the information we have about the physical node placement. If the node placement is known in advance, the minimum value of r ensuring connectedness can be easily determined (it is the longest edge of the MST). Unfortunately, in many realistic scenarios of ad hoc networks, the node placement cannot be known in advance, for example, because nodes are spread from a moving vehicle (airplane, ship, or spacecraft). If nodes' positions are not known, the minimum value of r ensuring connectedness in all possible cases is $r \approx l\sqrt{d}$, which accounts for the fact that nodes could be concentrated at opposite corners of the placement region. However, this scenario is very unlikely in most realistic situations. For this reason, we study MTR under the assumption that nodes are distributed independently and uniformly at random in the placement region.

In the following, we will use the standard notation regarding the asymptotic behavior of functions, which we recall. Let f and g be functions of the same parameter x. We have:

1. $f(x) = O(g(x))$ if there exist constants C and x_0 such that $f(x) \leq C \cdot g(x)$ for any $x \geq x_0$;
2. $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$;
3. $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $f(x) = \Omega(g(x))$;
4. $f(x) = o(g(x))$ if $\frac{f(x)}{g(x)} \to 0$ as $x \to \infty$;
5. $f(x) \ll g(x)$ or $g(x) \gg f(x)$ if $f(x) = o(g(x))$.

In the next section, we will improve the results of [30], [31] for the one-dimensional case by means of a more accurate analysis of the conditions leading to disconnected communication graphs. The analysis will use some results of the occupancy theory [16], which are presented next.

The occupancy problem can be described as follows: Assume we have C cells and n balls to be thrown independently in the cells. The allocation of balls into cells can be characterized by means of random variables describing some property of the cells. The occupancy theory is aimed at determining the probability distribution of such variables as n and C grow to infinity (i.e., the *limit distribution*). The most studied random variable is the number of empty cells after all the balls have been thrown, which will be denoted $\mu(n, C)$ in the following.

Under the assumption that the probability for any particular ball to fall into the ith cell is $1/C$ for $i = 1, \dots, C$ (*uniform allocation*), the following results have been proven:[4]

1. $P(\mu(n, C) = 0) = \sum_{i=0}^{C} \binom{C}{i} (-1)^i \left(1 - \frac{i}{C}\right)^n$,
2. $E[\mu(n, C)] = C\left(1 - \frac{1}{C}\right)^n$, and
3. $$Var[\mu(n, C)] = C(C-1)\left(1 - \frac{2}{C}\right)^n + C\left(1 - \frac{1}{C}\right)^n - C^2\left(1 - \frac{1}{C}\right)^{2n},$$

where $E[\mu(n, C)]$ and $Var[\mu(n, C)]$ denote the expected value and the variance of $\mu(n, C)$, respectively. The asymptotic behaviors of $P(\mu(n, C) = k)$, $E[\mu(n, C)]$, and $Var[\mu(n, C)]$ depend on the relative magnitudes of n and C as they grow to infinity. The following theorems have been proven.

Theorem 1. *For every n and C, $E[\mu(n, C)] \leq Ce^{-\alpha}$, where $\alpha = n/C$. Furthermore, if $n, C \to \infty$ in such a way that $\alpha = o(C)$, then:*

- $E[\mu(n, C)] = Ce^{-\alpha} - \frac{\alpha}{2} e^{-\alpha} + O\left(\frac{\alpha(\alpha+1)e^{-\alpha}}{C}\right)$ *and*
- $$Var[\mu(n, C)] = Ce^{-\alpha}(1 - (1+\alpha)e^{-\alpha}) + O\left(\alpha(1+\alpha)e^{-\alpha}\left(e^{-\alpha} + \frac{1}{C}\right)\right).$$

Theorem 2. *If $n = \Theta(C \log C)$, the limit distribution of the random variable $\mu(n, C)$ is the Poisson distribution of parameter λ, where $\lambda = \lim_{n,C\to\infty} E[\mu(n, C)]$.*

4 The Critical Transmitting Range in Stationary Networks

Consider the probability space $(\Omega_l, \mathcal{F}_l, P_l)$, where $\Omega_l = [0, l]^d$, with $d = 1, 2, 3$, $\mathcal{F}_l$ is the family of all closed subsets of Ω_l, and P_l is a probability distribution on Ω_l. In this paper, we assume that P_l is the uniform distribution on Ω_l. Under this setting, nodes in N can be modeled as independent random variables taking value (according to the uniform distribution) in $[0, l]^d$, which will be denoted as $Z_1, \dots, Z_n$.

We say that an event V_k, describing a property of a random structure depending on a parameter k, holds w.h.p., if $P(V_k) \to 1$ as $k \to \infty$. In the following, we consider the asymptotic behavior of the event $CONN_l$ on the random structures $(\Omega_l, \mathcal{F}_l, P_l)$ as $l \to \infty$. Informally speaking, event $CONN_l$ corresponds to all the values of the random variables $Z_1, \dots, Z_n$ for which the communication graph is connected.

4.1 The One-Dimensional Case

The following upper and lower bounds on the magnitude of rn ensuring connectedness w.h.p. have been derived in [31].

Theorem 3. *Suppose n nodes are placed in $R = [0, l]$ according to the uniform distribution. If $rn \in \Theta(l \log l)$, then the communication graph is connected w.h.p., while it is not connected w.h.p. if $rn \in O(l)$.*

Observe that the gap between the upper and lower bounds provided by Theorem 3 is considerable (in the order of $\log l$). Furthermore, Theorem 3 gives only asymptotic

4. All the results presented in this section are taken from [16].

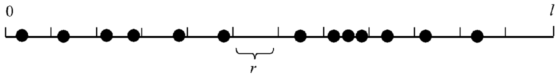

Fig. 1. Node placement generating a disconnected communication graph.

results, and gives no clue, for instance, on the actual multiplicative factor needed to ensure connectedness w.h.p. Thus, its usefulness in a realistic setting is limited. In this section, we derive a more precise characterization of the critical transmitting range in one-dimensional networks, providing explicit values to the multiplicative constants.

We start with the following theorem, which gives a more precise sufficient condition for connectedness w.h.p. than that provided by Theorem 3.

Theorem 4. *Assume that n nodes, each with transmitting range r, are distributed uniformly and independently at random in $R=[0,l]$ and assume that $rn=kl\ln l$ for some constant $k>0$. Further, assume that $r=r(l)\ll l$ and $n=n(l)\gg 1$. If $k>2$, or $k=2$ and $r=r(l)\gg 1$, then $\lim_{l\to\infty} P(CONN_l)=1$.*

Proof. See Appendix. □

Observe that the conditions on the magnitude of $r=r(l)$ and $n=n(l)$ in the statement of Theorem 4 are not restrictive. In fact, if $r=\Omega(l)$, then every node is able to transmit directly to most of the other nodes, and connectedness is ensured independently of n. The condition $n=n(l)\gg 1$ is a straightforward consequence of the first condition since otherwise the probability of connectedness would be negligible.

Note that the value of k established in Theorem 4 is the same as that obtained in [26] when nodes are distributed with Poisson density λ, where $\lambda = n/l$. Hence, the sufficient conditions for connectivity w.h.p. in the cases of Poisson and uniformly distributed nodes are the same.

Let us now consider the necessary condition for connectedness w.h.p. The bound of Theorem 3 is obtained by analyzing the probability of existence of an isolated node. In fact, the existence of an isolated node implies that the resulting communication graph (which is a point graph [33]) is disconnected. However, the class of disconnected point graphs is much larger than the class of point graphs containing at least one isolated node. For this reason, the bounds established in [31] are not tight. In [31], it is conjectured that the upper bound stated in Theorem 3 is actually tight. In what follows, we prove that this conjecture is true. The result derives from a more accurate approximation of the class of disconnected point graphs, which is based on occupancy theory.

In order to derive the lower bound, we consider the following subdivision of the placement region into cells. We assume that a line of length l is subdivided into $C=l/r$ segments of equal length r. With this subdivision, if there exists an empty cell c_i separating two cells c_{i-1}, c_{i+1} that each contains at least one node, then the nodes in c_{i-1} are unable to communicate to those in c_{i+1}, and the resulting communication graph is disconnected (see Fig. 1). The following lemma, whose immediate proof is omitted, establishes a sufficient condition for the communication graph to be disconnected.

Lemma 1. *Assume that n nodes are placed in $[0,l]$, and that the line is divided into $C=l/r$ segments of equal length r. Assign to every cell c_i, for $i=0,\dots,C-1$, a bit b_i, denoting the presence of at least one node in the cell. Without loss of generality, assume $b_i=0$ if c_i is empty, and $b_i=1$ otherwise. Let $B=\{b_0\dots b_{C-1}\}$ be the string obtained by concatenating the bits b_i, for $i=0,\dots,C-1$. If B contains a substring of the form $\{10^*1\}$, where 0^* denotes that one or more 0s may occur, then the resulting communication graph is disconnected.*

Observe that the condition stated in Lemma 1 is sufficient, but not necessary to produce a disconnected graph. In fact, there exist node placements such that B does not contain any substring of the form $\{10^*1\}$, but the resulting communication graph is disconnected.

Let us denote with $DISCONN_l$ and $E_l^{10^*1}$ the events corresponding to all the values of the random variables $Z_1,\dots,Z_n$ such that the resulting communication graph is disconnected, or a substring of the form $\{10^*1\}$ occurs in B, respectively. The subscript l indicates that we are considering these events in the case that the length of the line is l. Since $CONN_l=\Omega_l - DISCONN_l$ and $E_l^{10^*1}\subset DISCONN_l$, it is immediate that a necessary condition for connectedness w.h.p. is that $\lim_{l\to\infty} P(E_l^{10^*1})=0$.

In order to evaluate $\lim_{l\to\infty} P(E_l^{10^*1})$, we decompose the event $E_l^{10^*1}$ by conditioning on the disjoint events $\{\mu(n,C)=h\}$, for $h=0,\dots,C$, i.e.,

$$P\big(E_l^{10^*1}\big)=\sum_{h=0}^{C} P\big(E_l^{10^*1}|\{\mu(n,C)=h\}\big)\cdot P(\mu(n,C)=h).$$

Observe that, when l grows to infinity, $P(E_l^{10^*1})$ is defined as the sum of an infinite number of nonnegative terms $t_1,t_2,\dots$. Clearly, if there exists at least one term $t_{\bar h}$ such that $\lim_{l\to\infty} t_{\bar h}=\sigma>0$, then $\lim_{l\to\infty} P(E_l^{10^*1})\geq\sigma>0$. In what follows, we prove that, if $rn=(1-\epsilon)l\ln l$ and $r=\Theta(l^\epsilon)$, for some $0<\epsilon<1$, then $\lim_{l\to\infty} t_{\bar h}=\sigma>0$, where $\bar h=\lceil E[\mu(n,C)]\rceil$. Thus, in these conditions, the communication graph *is not* connected w.h.p.

We start with a lemma that characterizes the asymptotic behavior of $P\big(E_l^{10^*1}|\{\mu(n,C)=h\}\big)$ as l goes to infinity.

Lemma 2. *If $0<h\ll C$ and $r=r(l)\ll l$, then*

$$\lim_{l\to\infty} P\big(E_l^{10^*1}|\{\mu(n,C)=h\}\big)=1.$$

Proof. See Appendix. □

Let us set $\bar h=\lceil E[\mu(n,C)]\rceil$. By Lemma 2, if $0<\bar h\ll C$ and $\lim_{l\to\infty} P(\mu(n,C)=\bar h)=\sigma>0$, then the communication graph is not connected w.h.p. The following lemma establishes the asymptotic value of $P(\mu(n,C)=\bar h)$ in the hypothesis that $rn=(1-\epsilon)l\ln l$ and $r=\Theta(l^\epsilon)$, for some $0<\epsilon<1$.

Lemma 3. *Assume that n nodes, each with a transmitting range of r, are distributed uniformly and independently at random in $R=[0,l]$, and assume that $rn=(1-\epsilon)l\ln l$ and $r=\Theta(l^\epsilon)$, for some $0<\epsilon<1$. Then, $\lim_{l\to\infty} P(\mu(n,C)=\bar h)=\sigma>0$.*

Proof. See Appendix. □

We are now ready to state the necessary condition for connectedness w.h.p.

Theorem 5. *Assume that n nodes, each with a transmitting range of r, are distributed uniformly and independently at random in $R = [0, l]$, and assume that $rn = (1 - \epsilon) l \ln l$ for some $0 < \epsilon < 1$. If $r = r(l) \in \Theta(l^\epsilon)$, then the communication graph is not connected w.h.p.*

Proof. The proof follows immediately by Lemmas 2 and 3, and by observing that in the hypotheses of the theorem, we have

$$\bar{h} = \lceil E[\mu(n, C)] \rceil \approx \frac{l^\epsilon}{r} \ll C,$$

$r \approx l^\epsilon \ll l$, and $n = n(l) \gg 1$. □

Observe that Theorem 5 holds only when $r = r(l)$ is $\Theta(l^\epsilon)$, for some $0 < \epsilon < 1$. Although this expression covers a wide range of functions for r, many other interesting functions (for instance, functions including logarithmic terms) are not considered. When r is not of the form $\Theta(l^\epsilon)$, the following weaker result holds [30].

Theorem 6. *Assume that n nodes, each with a transmitting range of r, are distributed uniformly and independently at random in $R = [0, l]$ and assume that $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $rn \ll l \ln l$, then the communication graph is not connected w.h.p.*

We summarize the analysis above in the following theorem, which is the main result of this section.

Theorem 7. *Assume that n nodes, each with a transmitting range of r, are distributed uniformly and independently at random in $R = [0, l]$ and assume that $rn = kl \ln l$ for some constant $k > 0$. Further, assume that $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $k > 2$, or $k = 2$ and $r = r(l) \gg 1$, then the communication graph is connected w.h.p. If $k \leq (1 - \epsilon)$ and $r = r(l) \in \Theta(l^\epsilon)$ for some $0 < \epsilon < 1$, then the communication graph is not connected w.h.p. If r is not of the form $\Theta(l^\epsilon)$, but $rn \ll l \ln l$, then the communication graph is not connected w.h.p.*

In words, Theorem 7 states that setting $k \geq 2$ guarantees connectedness w.h.p., while a value of k smaller than 1 implies that the communication graph is not connected w.h.p. Hence, the asymptotic behavior of $P(CONN_l)$ for $1 \leq k < 2$ is not known. This result is somewhat weaker than that presented in [26] for the case of Poisson distributed nodes, where it is shown that, if $k < 2$, the graph is disconnected w.h.p. This more accurate result is derived from the nature of the Poisson distribution, whose asymptotic behavior can be analyzed more easily with respect to the case of uniformly distributed nodes.

The result stated in Theorem 7, for random distribution of nodes, can be compared to the transmitting ranges necessary with worst-case and best-case placements. Consider the case where the number of nodes is linear with the length of the line, l. In the worst case, nodes are clustered at either end of the line and the transmitting range must be $\Omega(l)$ for the network to be connected. In the best-case placement, nodes are equally spaced at intervals of l/n, meaning that a constant transmitting range is sufficient. Theorem 7's result yields a transmitting range of $\Omega(\log l)$ with random placement. Thus, there is a substantial reduction in transmitting range from the worst case, but also a significant increase compared to the best-case.

4.2 The Two and Three-Dimensional Cases

In this section, we provide necessary and sufficient conditions for connectedness w.h.p. in the cases of two and three-dimensional networks.

We start with the following theorem, which is a direct generalization of Theorem 4.

Theorem 8. *Assume that n nodes, each with a transmitting range of r, are distributed uniformly and independently at random in $R = [0, l]^d$, for $d = 2, 3$, and assume that $r^d n = kl^d \ln l$ for some constant $k > 0$, with $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $k > d \cdot k_d$, or $k = d \cdot k_d$ and $r = r(l) \gg 1$, then the communication graph is connected w.h.p., where $k_d = 2^d d^{d/2}$.*

Proof. The proof is similar to that of Theorem 4. In this case, the deployment region R is subdivided into nonoverlapping d-dimensional cells of side $\frac{r}{2\sqrt{d}}$. □

Unfortunately, generalizing the necessary condition of Theorem 5 to the two and three-dimensional case is not straightforward. In fact, in these cases, the conditions for the graph to be disconnected are more difficult to analyze. For instance, a "hole" in one dimension (as in the case of the $E_l^{10^*1}$ event of the previous section) is not sufficient to cause disconnectedness because there could exist paths that "go around the hole" using other dimensions, thereby maintaining connectivity. Thus, we are only able to state the following weaker necessary condition for connectedness, which is obtained by analyzing the probability of an isolated node.

Theorem 9. *Suppose n nodes are placed in $R = [0, l]^d$, with $d = 2, 3$, according to the uniform distribution. Further, assume that $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $r^d n \in O(l^d)$, then the communication graph is not connected w.h.p.*

Proof. See Appendix. □

5 Simulation Results for Stationary Networks

In this section, we present results of the simulation of stationary ad hoc networks. The goals of the simulations were:

- to validate the quality of the analytical results of the previous section;
- to investigate stronger necessary conditions for connectedness w.h.p. in the two and three-dimensional cases; and
- to investigate the relationship between the critical transmitting range and the minimum transmitting range, which ensures (w.h.p.) the formation of a connected component that includes a large fraction (e.g., 90 percent) of the nodes.

The simulator distributes n nodes in $[0, l]^d$ according to the uniform distribution, then generates the communication graph assuming that all nodes have the same transmitting range r. Parameters n, l, d, and r are given as input to the

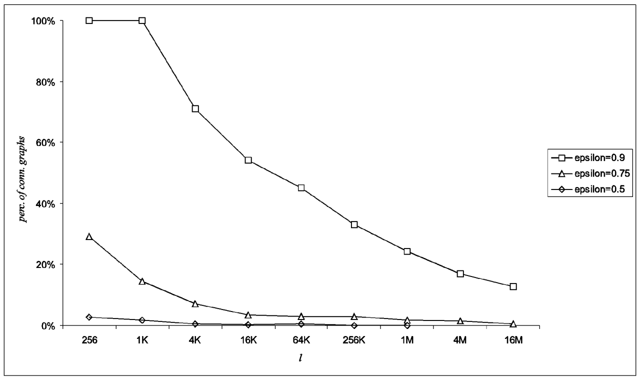

Fig. 2. Percentage of connected graphs for increasing values of l. Parameters r and n were set to l^ϵ and $(1-\epsilon)l^{(1-\epsilon)}\ln\, l$, respectively.

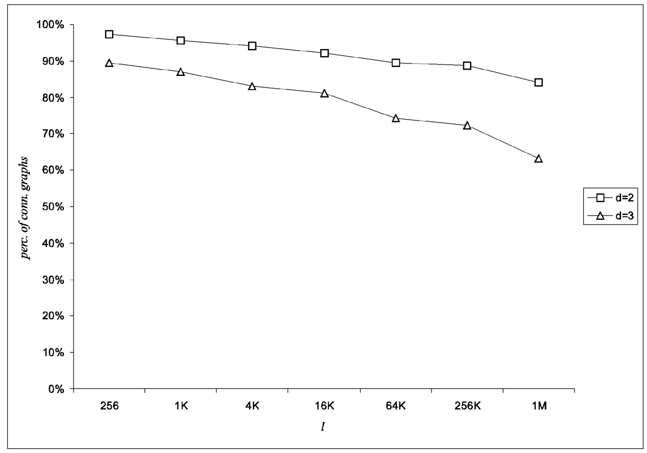

Fig. 3. Percentage of connected graphs for increasing values of l. Parameter n was set to $\sqrt{l}$. Parameter r was set to $2l^{3/4}$ for $d=2$ and $1.5l^{5/6}$ for $d=3$.

simulator, along with the number $\sharp iter$ of iterations to run. The simulator returns the percentage of connected graphs generated and the average number of neighbors of a node (i.e., the average degree of the communication graph). The average is evaluated over all iterations, including those that yielded a disconnected graph.

5.1 Validating the Theoretical Analysis

The first set of simulations was aimed at validating the theoretical results of Section 4.

In the case of one-dimensional networks, Theorem 7 states that, if $rn = kl\ln l$, then the communication graph is connected w.h.p. if $k \geq 2$, and it is not connected w.h.p. if $k \leq (1-\epsilon)$, for some $0 < \epsilon < 1$, where $r = r(l) \in \Theta(l^\epsilon)$. In order to validate this result, we have performed several simulations. In each simulation, we set $r = l^\epsilon$ for $\epsilon = 0.5,\ 0.75$, and 0.9, and we varied l from 256 to 16,777,216 (16M).

First, we have verified the sufficient condition for connectedness, setting n to $2l^{(1-\epsilon)}\ln l$, and performing experiments for increasing values of l. For every value of l, the percentage of connected networks generated was always 100 percent. To verify the necessary condition, we set n to $(1-\epsilon)l^{(1-\epsilon)}\ln l$ and repeated the simulations. The results are shown in Fig. 2. Also, in this case, the "quality" of Theorem 7 was confirmed: For every value of ϵ, the percentage of connected graphs decreases as l increases.

It should be emphasized that the necessary condition of Theorem 7 holds for very different "regimes" of r and n, depending on the value of ϵ: When ϵ is close to 0, r grows very slowly and n grows very fast as l increases; when ϵ is close to 1, the situation is reversed. Table 1 illustrates some of these regimes by showing the values of r, n, and l for Fig. 2, which was generated using medium to high values of ϵ. Due to limitations on the size of n in the simulator, we were able to validate the theorem only for $\epsilon \geq 0.5$.

TABLE 1
Values of r and n for Increasing Values of l

	$\epsilon = 0.5$		$\epsilon = 0.75$		$\epsilon = 0.9$	
l	r	n	r	n	r	n
256	16	44	64	6	147	1
1K	32	111	181	10	512	1
4K	64	266	512	17	1783	2
16K	128	621	1448	27	6208	3
64K	256	1420	4K	44	21619	3
256K	512	3194	11585	71	75281	4
1M	1K	7098	32K	111	256K	6

For two and three-dimensional networks, we first verified the "quality" of Theorem 9, which states that, if the order of magnitude of the product $r^d n$ is at most l^d, then the communication graph is not connected w.h.p. To this end, we have simulated several "disconnected scenarios" for increasing values of l. Namely, we considered values of l ranging from 256 to 1,048,576 (1M) and, for every value of l, we chose r and n in such a way that $r^d n = l^d$ and we ran 250 simulations. Two choices for n were considered: $n = \sqrt{l}$ and $n = l/(\log_2 l)^2$, thus obtaining values of n ranging from 16 to 1,024 and from 4 to 2,621, respectively.[5]

The results of these simulations fully agreed with the theoretical result of Theorem 9: The percentages of connected graphs generated were always quite low, and tend to decrease as l increases. These results are not shown because the percentage of connected graphs was quite close to 0 for all simulation runs. We have also considered the impact of a multiplicative factor to the product $r^d n$ on the percentages of connected graphs generated. In particular, we set $n = \sqrt{l}$ and $r = 2l^{3/4}$ for $d=2$ (thus, $r^2 n = 4l^2$), and $n = \sqrt{l}$ and $r = 1.5l^{5/6}$ for $d=3$ (thus, $r^3 n = 3.375 l^3$). Although showing higher percentages of connected graphs with respect to the previous simulations, the asymptotic behavior was confirmed: As l increases, the percentage of connected graphs decreases (see Fig. 3).

In the second experiment, we have investigated whether better lower bounds on the critical transmitting range can be experimentally achieved. We ran simulations for values of l ranging from 256 to 4,194,304 (4M), with values of n ranging from 16 to 2,048 and from 4 to 8,666. The larger value of l (and, consequently, of n) was needed in order to better investigate the asymptotic behavior. For every simulation, we set r in such a way that $r^d n = l^d \log_2 l$. With

5. In the latter case, the simulation for $n=4$ was not considered, due to its scarce significance.

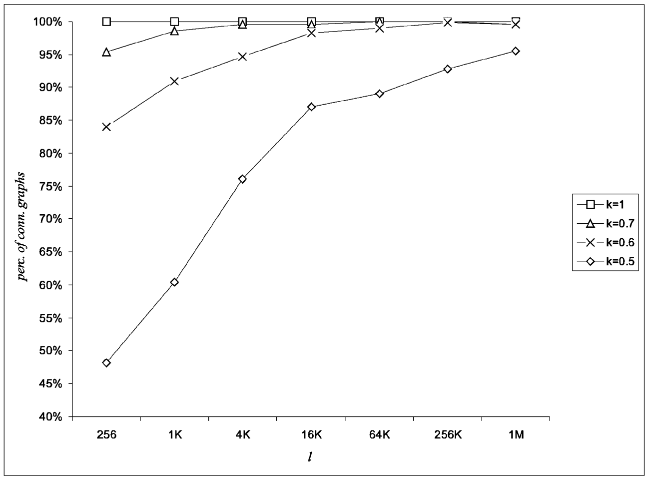

Fig. 4. Percentage of connected graphs for increasing values of l in two-dimensional networks. Parameters n and r were set to $\sqrt{l}$ and $kl^{3/4}\sqrt{\log_2 l}$, respectively.

these values, 100 percent of the graphs were connected for all simulation runs. We also set the transmitting range to $r' = kr$, for values of k ranging from 0.5 to 0.9 in steps of 0.1. As shown in Figs. 4 and 5, the results showed that a $l^d \log_2 l$ bound is sufficient to ensure increasing percentages of connected graphs. Note that, for $d=3$ (Fig. 5), when the multiplicative constant on r gets small ($k=0.5$), the percentages of connected graphs are low but the asymptotic trend is still increasing.

Our results provide precise values of the product of n and r^d that will generate connected graphs w.h.p. Among other uses, a network designer can employ this information to determine how large a transmitting range or how many nodes are required for a specific application. Table 2 reports, for $d = 2$ and fixed l, the specific values of n and r that yield a percentage of connected graphs above 99 percent (the value of the transmitting range is expressed as a fraction of l). These data can be directly applied in the network design process

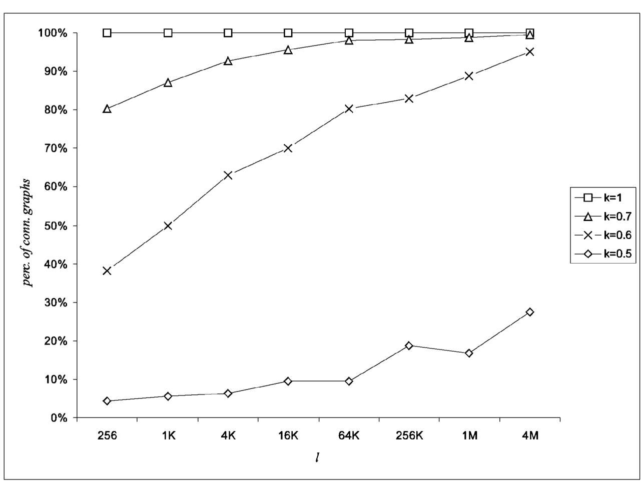

Fig. 5. Percentage of connected graphs for increasing values of l in three-dimensional networks. Parameters n and r were set to $\sqrt{l}$ and $kl^{5/6}\sqrt[3]{\log_2 l}$, respectively.

TABLE 2
Values of the Transmitting Range r
(Expressed as a Fraction of l) Ensuring Connectedness W.H.P.

n	r	n	r
10	0.57748	75	0.24694
20	0.44280	80	0.24194
25	0.40287	90	0.23191
30	0.37059	100	0.22217
40	0.32423	250	0.15756
50	0.29684	500	0.13127
60	0.27637	750	0.11923
70	0.25557	1000	0.11516

and can give a feel for the relative magnitude of transmitting range necessary for different values of n.

It is also useful to give a feeling for how large is the gap between transmitting ranges that provide connectedness w.h.p. and those that do not. For example, when $d=2$, $l=65,536$, and $n=256$, we have that a value of the transmitting range equal to $2l^{3/4}=8,192$ is not sufficient to generate graphs which are connected w.h.p.: Only 89 percent of the graphs generated are connected (see Fig. 3). Conversely, a value of r equal to $0.6l^{3/4}\sqrt{\log_2 l}=9,830$ provides 99 percent of connected graphs, and, by Theorem 8, guarantees connectedness w.h.p.

To summarize, the results of our simulations of two and three-dimensional networks provide strong evidence to support the conjecture that a value of $r^d n$ in the order of $l^d \log l$ is necessary and sufficient to provide connectedness w.h.p.

5.2 Connectedness vs. Energy Cost

In this set of simulations, we investigated the minimum transmitting range that, w.h.p., ensures either a connected communication graph or the formation of a connected component that includes a large fraction (e.g., 90 percent) of the nodes. The rationale for this investigation is to see whether weaker requirements on graph connectedness may achieve considerable reductions of the transmitting range (i.e., of the energy cost).

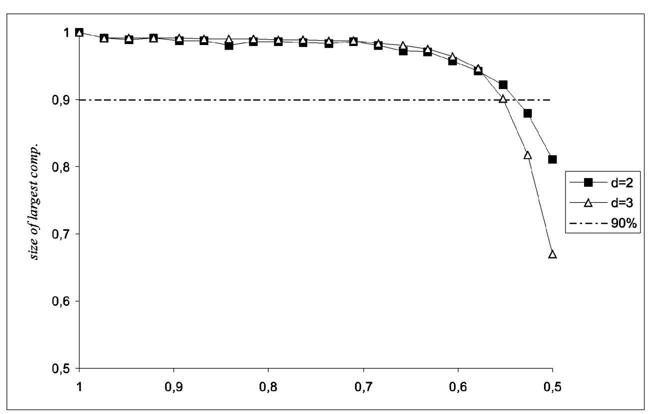

Fig. 6. Average size of the largest connected component expressed as a fraction of the total number of nodes. The x-axis reports the ratio $r/\bar{r}$. Parameters in this experiment were $l = 16,384$, $n = \sqrt{l} = 128$, $\bar{r} = 1,430$ for $d = 1$, 3,800 for $d = 2$, and 6,500 for $d = 3$.

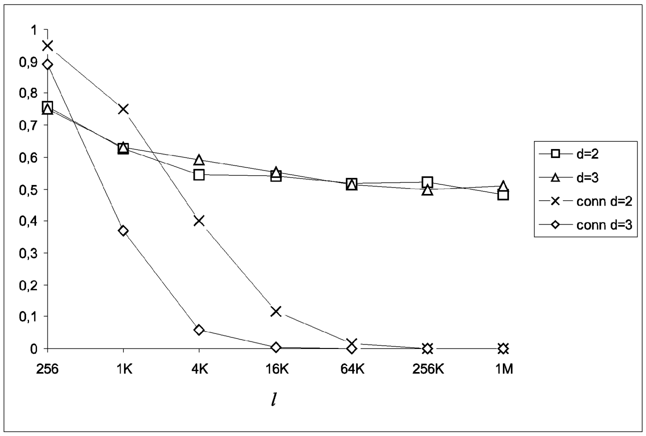

Fig. 7. Value of ratio $r'/\bar{r}$ (y-axis) for increasing values l. Also shown is the fraction of connected graphs when the transmitting range was set to r'.

We ran 250 iterations for every simulation. First, we set $l=16,384$, $n=\sqrt{l}=128$, and, for every dimension, we experimentally determined the minimum value $\bar{r}$ of the transmitting range yielding 100 percent of connected graphs. These values are 1,430 for $d=1$, 3,800 for $d=2$, and 6,500 for $d=3$. Starting from $\bar{r}$, we decreased the value of the transmitting range r until $r=\bar{r}/2$, and we evaluated the average size of the largest connected component. The result of this experiment is shown in Fig. 6. A similar experiment, which confirmed the behavior displayed in Fig. 6, was conducted setting $l=1,048,576$ and $n=1,024$. As can be seen, in two and three-dimensional networks, connectedness can be traded off with energy cost: As r decreases, the size of the largest connected component decreases smoothly. When $r=\bar{r}/2$, the average size of the largest component in two-dimensional networks is $0.81n$ when $l=16,384$ and $0.94n$ when $l=1,048,576$, while, in three-dimensional networks, we have $0.67n$ and $0.87n$, respectively. Two and three-dimensional networks display similar behaviors for values of r as low as $0.6\bar{r}$, while a somewhat higher connectedness for two-dimensional networks arises for lower values of r. This tradeoff has potential primarily in two and three-dimensional networks because most disconnections in the $d=1$ case split the network into at least two moderately-sized components, thereby eliminating the possibility of having a single component with a very large fraction of the nodes. For this reason, we confine our results in this section to $d=2,3$.

The phenomenon outlined by our experimental analysis is coherent with a theoretical result from the theory of GRG (which, we recall, can be applied only to *dense* ad hoc networks) concerning two and three-dimensional networks, namely, that connectivity occurs (asymptotically) when the last isolated node disappears from the graph [24]. The results of our simulations clearly show that when the graph is disconnected, but r is close to $\bar{r}$, there exists a very large connected component (the *giant component* in random graph terminology); thus, in this regime, disconnection is caused by few isolated nodes. This seems to indicate that, also in case of *sparse* two and three-dimensional networks, connectivity occurs (asymptotically) when the last isolated node disappears from the communication graph.

We also evaluated the ratio between $\bar{r}$ and the minimum value r' of the transmitting range such that the average size of the largest connected component is at least $0.9n$, for values of l ranging from 256 to 1,048,576 (1M). The number of nodes was set to $\sqrt{l}$. The result of this experiment is shown in Fig. 7. Two and three-dimensional networks display similar behaviors: As l increases, the ratio $r'/\bar{r}$ tends to "converge" to 0.5. The figure also displays the fraction of connected graphs when $r=r'$. As can be seen, this fraction drops to zero as l increases. Thus, for a large value of l, halving $\bar{r}$ produces disconnected graphs w.h.p., but the average size of the largest connected component is approximately $0.9n$. This means that considerable energy savings can be achieved if connecting 90 percent of the nodes is acceptable. For many applications, substantially increasing the energy in order to connect the remaining 10 percent of the nodes is not worthwhile.

6 The Critical Transmitting Range in Mobile Networks

In this section, we consider the mobile version of MTR, which can be formulated as follows:

Definition 2 (Minimum Transmitting Range Mobile (MTRM)). *Suppose n nodes are placed in $R=[0,l]^d$ and assume that nodes are allowed to move during a time interval $[0,T]$. What is the minimum value of r such that the resulting communication graph is connected during some fraction, f, of the interval?*

A formal analysis of MTRM is much more complicated than that of MTR and is beyond the scope of this paper. In this section, we study MTRM by means of extensive simulations. The goal is to study the relationship between the value of r ensuring connected graphs in the stationary case (denoted $r_{stationary}$) and the values of the transmitting range ensuring connected graphs during some fraction of the operational time.

In this paper, we focus on the transmitting ranges needed to ensure connectedness during 100 percent, 90 percent, and 10 percent of the simulation time (denoted r_{100}, r_{90}, and r_{10}, respectively). These values are chosen as indicative of three different dependability scenarios that the ad hoc network must satisfy. In the first case, the network is used for safety-critical or life-critical applications (e.g., systems to detect physical intrusions in a home or business). In the second case, temporary network disconnections can be tolerated, especially if this is counterbalanced by a significant decrease of the energy consumption with respect to the case of continuous connectedness. In the latter case, the network stays disconnected most of the time, but temporary connection periods can be used to exchange data among nodes. This could be the case of wireless sensor networks used for environmental monitoring.

We also consider the value of the transmitting range ensuring that the average size of the largest connected component is a given fraction of the total number of nodes in the network. Table 3 summarizes the values of the transmitting range considered in our simulations. The rationale for this investigation is that the network designer

TABLE 3
Values of the Transmitting Range Considered in Our Simulations

$r_{stationary}$	t.r. ensuring connectedness in the stationary case
r_{100}	t.r. ensuring connectedness during 100% of sim. time
r_{90}	t.r. ensuring connectedness during 90% of sim. time
r_{10}	t.r. ensuring connectedness during 10% of sim. time
r_0	largest t.r. yielding no connected graphs
r_{l90}	t.r. ensuring avg. size of largest conn. comp. is $0.9n$
r_{l75}	t.r. ensuring avg. size of largest conn. comp. is $0.75n$
r_{l50}	t.r. ensuring avg. size of largest conn. comp. is $0.5n$

could be interested in maintaining only a certain fraction of the nodes connected, if this would result in significant energy savings. Further, considering that, in many scenarios (e.g., wireless sensor networks), the cost of a node is very low, it could also be the case that dispersing twice as many nodes as needed and setting the transmitting ranges in such a way that half of the nodes remain connected is a feasible and cost-effective solution.

In all the simulations reported herein, we set $d=2$, as the two-dimensional setting is an appropriate model for many applications of wireless ad hoc networks.

6.1 Mobility Models

To generate the results of this section, we extended the simulator used in the previous section for the stationary case by implementing two mobility models. The initial communication graph is generated as in the stationary case. Then, the nodes start moving according to the selected mobility model (all the nodes use the same mobility model). For each mobility step, the simulator checks for graph connectedness and, in case the graph is not connected, evaluates the size of the largest connected component. At the end of the simulation, the percentage of connected graphs, the minimum, and the average size of the largest connected component (averaged over the runs that yield a disconnected graph) are reported.

The first mobility model implemented in the simulator is the classical random waypoint model [14], and is used to model intentional movement: Every node chooses uniformly at random a destination in $[0, l]^2$, and moves toward it along a straight line with a velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. When it reaches the destination, it remains stationary for a predefined pause time t_{pause}, and then it starts moving again according to the same rule. In the simulator, t_{pause} is expressed as the number of mobility steps for which the node must remain stationary, and velocity is normalized with respect to the mobility step. We have also included a further parameter in the model, namely, the probability $p_{stationary}$ that a node remains stationary during the entire simulation time. Hence, only $(1 - p_{stationary})n$ nodes (on the average) will move. Introducing $p_{stationary}$ in the model accounts for those situations in which some nodes are not able to move. For example, this could be the case when sensors are spread from a moving vehicle and some of them remain entangled, say, in a bush or tree. This can also model a situation where two types of nodes are used, one type that is stationary and another type that is mobile.

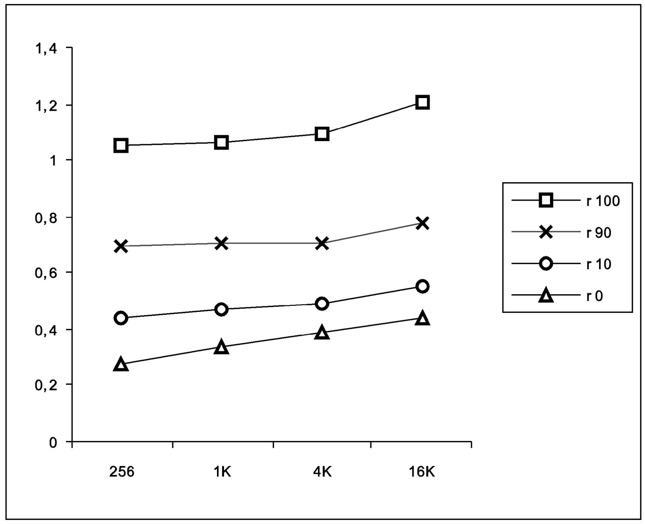

Fig. 8. Values of the ratio $r_x/r_{stationary}$ (y-axis) for increasing values l in the random waypoint model.

The second mobility model resembles Brownian (i.e., nonintentional) motion. Mobility is modeled using parameters $p_{stationary}$, p_{pause}, and m. Parameter $p_{stationary}$ is defined as above. Parameter p_{pause} is the probability that a node remains stationary at a given step. This parameter accounts for heterogeneous mobility patterns, in which nodes may move at different times. Intuitively, the higher the value of p_{pause}, the more heterogeneous the mobility pattern is. However, values of p_{pause} close to 1 result in an almost stationary network. If a node is moving at step i, its position in step $i+1$ is chosen uniformly at random in the square of side m centered at the current node location. If the chosen position is out of the boundaries of the deployment region, a new position is generated until a location inside R is found. Parameter m models, to a certain extent, the velocity of the nodes: The larger m is, the more likely it is that a node moves far away from its position in the previous step.

6.2 Simulation Results for Increasing System Size

In the first set of simulations, we have investigated the value of the ratio of r_{100} (respectively, of r_{90} and r_{10}) to $r_{stationary}$ for values of l ranging from 256 to 16,384. We also considered the largest value r_0 of the transmitting range that yields no connected graphs. In both mobility models, n was set to $\sqrt{l}$. The value of $r_{stationary}$ is obtained from the simulation results for the stationary case of the previous section, while those for r_{100}, r_{90}, r_{10}, and r_0 are averaged over 50 simulations of 10,000 steps of mobility each.

First, we considered the random waypoint model, with parameters set as follows: $p_{stationary}=0$, $v_{min}=0.1$, $v_{max}=0.01l$, and $t_{pause}=2,000$. This setting models a homogeneous mobility scenario in which all nodes are moving. The values of the ratios are reported in Fig. 8. Fig. 9 reports the same graphic obtained for the Brownian-like model, with $p_{stationary}=0.1$, $p_{pause}=0.3$, and $m=0.01l$. This is a more heterogeneous mobility scenario in which a small percentage of the nodes remain stationary.

The graphics show the same qualitative behavior: As l increases, the ratio of the different transmitting ranges for

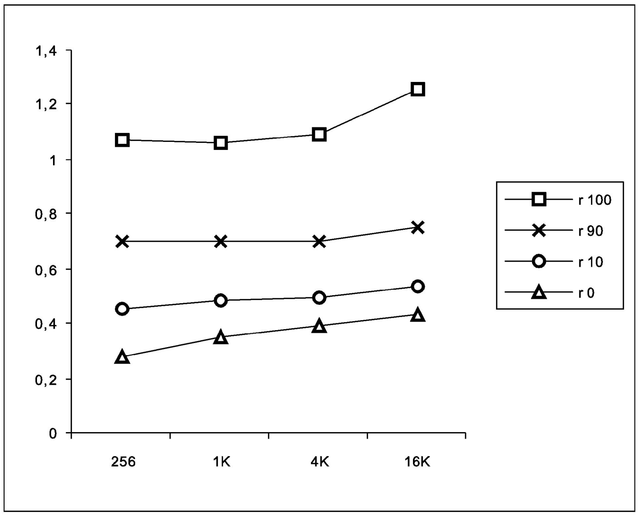

Fig. 9. Values of the ratio $r_x/r_{stationary}$ (*y*-axis) for increasing values l (*x*-axis) in the Brownian-like model.

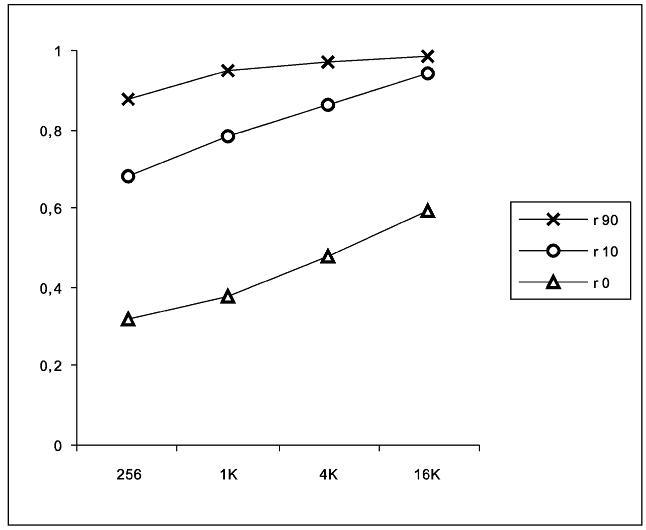

Fig. 10. Average size of the largest connected component expressed as a fraction of n (*y*-axis) for increasing values of l (*x*-axis) in the random waypoint model.

mobility to $r_{stationary}$ tends to increase, and this increasing behavior is more pronounced for the case of r_{100}. However, even when l is large, a modest increase to $r_{stationary}$ (about 21 percent in the random waypoint and about 25 percent in the Brownian-like model) is sufficient to ensure connectedness during the entire simulation time. Comparing the results for the two mobility models, we can see somewhat higher values of the ratios for the Brownian-like model, especially for the case of r_{100}. This seems to indicate that more homogeneous mobility patterns help in maintaining connectedness. However, it is surprising that the results for the two mobility models are so similar. This indicates that it is more the existence of mobility rather than the precise details of how nodes move that is significant, at least as far as network connectedness is concerned.

The graphics reported in Figs. 8 and 9 also show that r_{90} is far smaller than r_{100} (about 35-40 percent smaller) in both mobility models, independently of the system size. Hence, substantial energy savings can be achieved under both models if temporary disconnections can be tolerated. When the requirement for connectedness is only 10 percent of the operational time, the decrease in the transmitting range is about 55-60 percent, enabling further energy savings. However, if r is reduced to about 25 percent to 40 percent of $r_{stationary}$, the network becomes disconnected during the entire simulation time.

We have also investigated the average size of the largest connected component when the transmitting range is set to r_{90}, r_{10}, and r_0. Once again, the results of the simulations were almost independent of the mobility model used. For this reason, we only report the results obtained with the random waypoint model (Fig. 10). The graphic shows that the ratio of the average size of the largest connected component to n increases as l increases. When the transmitting range is set to r_{90} and l is sufficiently large, this ratio is very close to 1 (about 0.98 in both mobility models). This means that during the short time in which the network is disconnected, a vast majority of its nodes forms a large connected component. Hence, on the average, disconnection is caused by only a few isolated nodes (as it was in the stationary case). This fact is confirmed by the plot for r_{10}: Even when the network is disconnected most of the time, a large connected component (of average size about $0.9n$ for large values of l) still exists. However, if the transmitting range is further decreased to r_0, the size of the largest connected component drops to about $0.5n$.

We also considered the value of the transmitting range ensuring that the average size of the largest connected component is at least $0.9n$, $0.75n$, and $0.5n$, respectively, during the entire simulation. The corresponding values of the transmitting range are denoted r_{l90}, r_{l75}, and r_{l50}. The mobility parameters and n were set as above. The rationale for this investigation is that the network designer could be interested in maintaining only a certain fraction of the nodes connected if this would result in significant energy savings.

The value of the ratio of r_{l90}, r_{l75}, and r_{l50} to $r_{stationary}$ for increasing values of l in the random waypoint model is shown in Fig. 11. Simulation results have shown that, while $r_{l90}/r_{stationary}$ tends to decrease with increasing values of l,

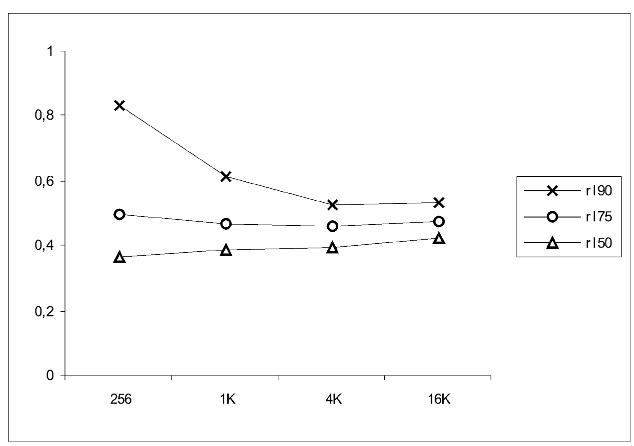

Fig. 11. Values of the ratio of r_{l90}, r_{l75}, and r_{l50} to $r_{stationary}$ (*x*-axis) for increasing values of l (*y*-axis) in the random waypoint model.

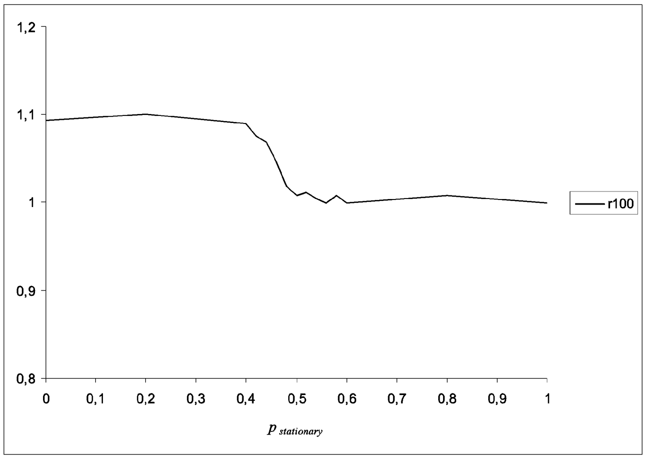

Fig. 12. Values of the ratio $r_{100}/r_{stationary}$ (y-axis) for different values of $p_{stationary}$ in the random waypint model.

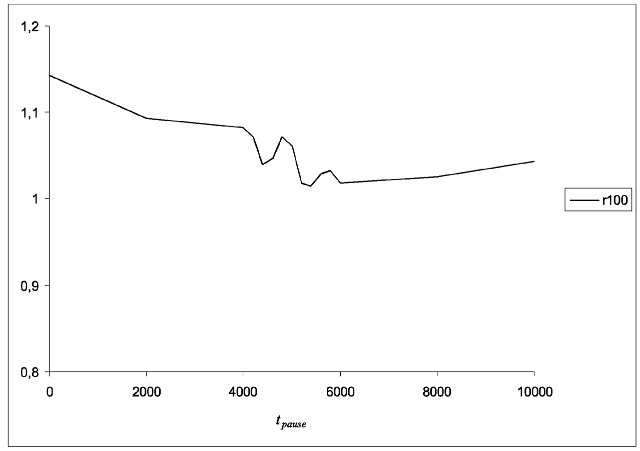

Fig. 13. Values of the ratio $r_{100}/r_{stationary}$ (y-axis) for different values of t_{pause} in the random waypoint model.

converging to about 0.52, the ratios $r_{l75}/r_{stationary}$ and $r_{l50}/r_{stationary}$ are almost independent of l. In particular, $r_{l75}/r_{stationary}$ is about 0.46 and $r_{l50}/r_{stationary}$ is about 0.4. Further, the relative differences between the three ratios decrease for an increasing value of l. This indicates that, while for small networks (few nodes distributed in a relatively small region), the energy needed to maintain 90 percent of the nodes connected is significantly higher than that required to connect 50 percent of the nodes (r_{l50} is less than half of r_{l90} for $l=256$), for large networks the savings are not as great if the requirement for connectivity is only 50 percent of the nodes (r_{l50} is 20 percent smaller than r_{l90} for $l=16,384$).

6.3 Simulation Results for Different Mobility Parameters

A second set of simulations was done to investigate the effect of different choices of the mobility parameters on the value of r_{100}. We considered the random waypoint model with $l=4,096$ and $n=\sqrt{l}=64$. The default values of the mobility parameters were set as above, i.e., $p_{stationary}=0$, $v_{min}=0.1$, $v_{max}=0.01l$, and $t_{pause}=2,000$. Then, we varied the value of one parameter, leaving the others unchanged.

Fig. 12 reports the value of r_{100} for values of $p_{stationary}$ ranging from 0 (no stationary nodes) to 1 (corresponding to the stationary case) in steps of 0.2. Simulation results show a sharp drop of r_{100} in the interval 0.4-0.6: For $p_{stationary}=0.4$, r_{100} is about 10 percent larger than $r_{stationary}$, while for $p_{stationary}=0.6$ and for larger values of $p_{stationary}$, we have $r_{100}\approx r_{stationary}$. To investigate this drop more closely, we performed further simulations by exploring the interval 0.4-0.6 in steps of 0.02. As shown in Fig. 12, there is a distinct threshold phenomenon: When the number of stationary nodes is about $n/2$ or higher, the network can be regarded as practically stationary from a connectedness point of view. This result is very interesting since it seems to indicate that a certain number (albeit a rather large fraction) of stationary nodes would significantly increase network connectedness. With more than $n/2$ mobile nodes, the network quickly becomes equivalent to one in which all nodes are mobile.

The effect of t_{pause} on r_{100} is shown in Fig. 13. Increasing values of t_{pause} tend to decrease the value of r_{100}, although the trend is not as pronounced as in the case of $p_{stationary}$. A threshold phenomenon seems to exist in the interval 4,000 - 6,000 in this case also. However, further simulations in this interval have shown that, although the trend can be observed, no sharp threshold actually exists. We believe that the rationale for this is the following: While the value of $p_{stationary}$ has a direct impact on the "quantity of mobility" (which can be informally understood as the percentage of stationary nodes with respect to the total number of nodes), the effect of the pause time is not so direct. In fact, in the random waypoint model, the "quantity of mobility" depends heavily on the node destinations, which are chosen uniformly at random: Even if the pause time is long and the velocity is moderate, a node could be "mobile" for a long time if its destination is very far from its initial location. So, an increased pause time tends to render the system more stationary, but in a less direct way than $p_{stationary}$.

We have also evaluated the impact of different values of v_{max} on the value of r_{100}. The simulation results, which are not reported, have shown that r_{100} is almost independent of the value of v_{max}: Except for low velocities (v_{max} below $0.1l$), r_{100} is slightly above $r_{stationary}$. This quite surprising result could be due to the apparently counterintuitive fact that the "quantity of mobility" is only marginally influenced by the value of v_{max}, and a larger value of v_{max} tends to decrease the "quantity of mobility." In fact, the larger v_{max} is, the more likely it is that nodes arrive quickly at destination and remain stationary for $t_{pause}=2,000$ steps.

7 Conclusions

In this paper, we have analyzed the critical transmitting range for connectivity in both stationary and mobile wireless ad hoc networks.

For stationary networks, we have provided both analytical and experimental results. We have proven tight bounds on the critical transmitting range for the one-dimensional case, and given less precise bounds in the case of two and three-dimensional networks. The most notable aspect of our analysis is that, contrary to the case of existing theoretical results, it can be applied to both dense and sparse ad hoc networks. We have also presented the results of extensive simulations, which have shown that a stronger necessary condition for connectedness w.h.p. than that proved in the paper is likely to hold in two and three-dimensional networks. Furthermore, we have investigated the relationship between the critical transmitting range and the minimum transmitting range that ensures the formation of a connected component containing a large fraction (e.g., 90 percent) of the nodes. The results of this investigation have shown that in two and three-dimensional networks, network "connectedness" and energy cost can be traded off: Reducing the transmitting range, we obtain progressively "less connected" graphs. This behavior is not displayed in one-dimensional networks, where a modest decrease on the transmitting range over the minimum required for connectedness w.h.p. can cause the formation of several connected components of relatively small size.

We have also investigated the critical transmitting range in two-dimensional mobile networks through extensive simulations. We have considered two mobility patterns (random waypoint and Brownian-like) to model both intentional and nonintentional movements. Simulation results have shown that considerable energy savings can be achieved if temporary disconnections can be tolerated or if connectedness must be ensured only for a large fraction of the nodes. Regarding the influence of mobility patterns, simulation results have shown that connectedness is only marginally influenced by whether motion is intentional or not, but it is rather related to the "quantity of mobility," which can be informally defined as the percentage of stationary nodes with respect to the total number of nodes. For example, when about $n/2$ nodes are static, the network can be regarded as stationary from a connectivity point of view. Further investigation in this direction is needed and is a matter of ongoing research.

APPENDIX

Proof of Theorem 4. Let $[0,l]$ be subdivided into $C=\frac{2l}{r}$ nonoverlapping segments (cells) of length $\frac{r}{2}$. It is immediate that, if every segment contains at least one node, then the resulting communication graph is connected. Let $\mu(n,C)$ be the random variable denoting the number of empty cells. Since $\mu(n,C)$ is a nonnegative integer random variable, then

$$P(\mu(n,C)>0) \leq E[\mu(n,C)],$$

where $E[\mu(n,C)]$ is the expected value of $\mu(n,C)$ ([20, pp. 10-11]). We have [16]:

$$E[\mu(n,C)] = C\left(1-\frac{1}{C}\right)^n.$$

We want to investigate the asymptotic value of $E[\mu(n,C)]$ as $l\to\infty$, which, given the hypotheses $r=r(l)\ll l$ and $n=n(l)\gg 1$, is equivalent to the asymptote as $C,n\to\infty$. Taking the logarithm, we obtain:

$$\ln E[\mu(n,C)] = \ln C + n\ln\left(1-\frac{1}{C}\right) = \ln\frac{2l}{r} + n\ln\left(1-\frac{r}{2l}\right). \tag{1}$$

The Taylor series expansion of the $\ln$ part of the second term of (1) yields:

$$\ln\left(1-\frac{r}{2l}\right) = -\frac{r}{2l} - \frac{r^2}{8l^2} - \frac{r^3}{24l^3} - \cdots < -\frac{r}{2l}.$$

Thus, we obtain the following upper bound:

$$\ln E[\mu(n,C)] < \ln\frac{2l}{r} - \frac{nr}{2l}. \tag{2}$$

Substituting the expression $rn=kl\ln l$ into inequality (2), we obtain:

$$\ln E[\mu(n,C)] < \ln\frac{2l}{r} - \frac{k\ln l}{2} = \ln\frac{2}{rl^{k/2-1}}.$$

If $k>2$, or if $k=2$ and $r=r(l)\gg 1$, then it is easily seen from this expression that $\lim_{n,C\to\infty}\ln E[\mu(n,C)] = -\infty$. Therefore,

$$\lim_{n,C\to\infty} E[\mu(n,C)] = 0$$

and $\lim_{l\to\infty} P(\mu(n,C)=0) = 1$. It follows that each cell contains at least one node w.h.p., which implies $\lim_{l\to\infty} P(CONN_l) = 1$. □

Proof of Lemma 2. Consider the complementary event of $E_l^{10^*1}$, i.e., $E_l^1=\Omega_l - E_l^{10^*1}$. It can be easily seen that E_l^1 corresponds to all the values of the random variables $Z_1,\ldots,Z_n$ such that the 1-bits in B are consecutives. Given the hypothesis of independence of the random variables $Z_1,\ldots,Z_n$, when exactly h cells out of C are empty (i.e., h bits in B are 0), $P\big(E_l^1|\{\mu(n,C)=h\}\big)$ corresponds to the ratio of all configurations of $(C-h)$ consecutive 1-bits over all possible configurations of h 0-bits in C positions, i.e.,

$$P\big(E_l^1|\{\mu(n,C)=h\}\big) = \frac{h+1}{\binom{C}{h}}.$$

Since $C=l/r$ and $r\ll l$, we have:

$$\begin{aligned}\lim_{l\to\infty} P\big(E_l^{10^*1}|\{\mu(n,C)=h\}\big) &= 1 - \lim_{l\to\infty} P\big(E_l^1|\{\mu(n,C)=h\}\big)\\ &= 1 - \lim_{C\to\infty}\frac{h+1}{\binom{C}{h}}.\end{aligned}$$

We can rewrite the last limit as:

$$\lim_{C\to\infty}\frac{h+1}{\binom{C}{h}} = \lim_{C\to\infty}\frac{(h+1)!}{C(C-1)\ldots(C-h+1)}.$$

Since $h\ll C$, we have:

$$\lim_{C\to\infty}\frac{(h+1)!}{C(C-1)\ldots(C-h+1)} = \lim_{C\to\infty}\frac{(h+1)!}{C^h}.$$

Taking the logarithm, we obtain:

$$\begin{aligned}\lim_{C\to\infty}\ln\frac{(h+1)!}{C^h} &= \lim_{C\to\infty}\ln(h+1)! - h\ln C\\ &= \lim_{C\to\infty} h\ln h - h\ln C = \lim_{C\to\infty} h(\ln h - \ln C).\end{aligned}$$

Since $0 < h \ll C$, we conclude that

$$\lim_{C\to\infty} h(\ln h - \ln C) = -\infty,$$

hence,

$$\lim_{C\to\infty} \frac{h+1}{\binom{C}{h}} = 0,$$

and the lemma is proved. □

Proof of Lemma 3. Proceeding as in the proof of Theorem 4 and observing that $rn = (1-\epsilon) l \ln l$ and $r = \Theta(l^\epsilon)$ implies $n = n(l) \gg 1$, we obtain:

$$\ln E[\mu_0(n, C)] \approx \ln \frac{l^\epsilon}{r},$$

hence $E[\mu_0(n, C)] \approx \frac{l^\epsilon}{r}$. Given the hypothesis $r = \Theta(l^\epsilon)$, we have that $\lim_{l\to\infty} E[\mu_0(n, C)] = c$, for some constant $c > 0$. Since $rn = (1-\epsilon) l \ln l$ for some $0 < \epsilon < 1$, we are in the hypothesis of Theorem 2, and the limit distribution of the random variable $\mu(n, C)$ is the Poisson distribution of parameter $\lambda = \lim_{l\to\infty} E[\mu_0(n, C)] = c$ (see Theorem 2). Hence,

$$\lim_{l\to\infty} P(\mu(n, C) = \bar{h}) = \left(\frac{c}{e}\right)^c \cdot \frac{1}{c!} > 0.$$

□

Proof of Theorem 9. We report the proof for the case $d = 2$. The proof for the case $d = 3$ is similar.

Consider the event $ISOLATED_i$, corresponding to all the values of the random variables $Z_1, \dots, Z_n$ such that node i is isolated in the communication graph, for $1 \le i \le n$. It is immediate that a necessary condition for connectedness w.h.p. is that $\lim_{l\to\infty} P(ISOLATED_i) = 0$. Considering that node i is isolated if none of the remaining $n-1$ nodes is within its transmitting range, we have:

$$\left(1 - \frac{\pi r^2}{l^2}\right)^{n-1} \le P(ISOLATED_i) \le \left(1 - \frac{\pi r^2}{4l^2}\right)^{n-1},$$

where the upper and lower bounds account for the fact that node i is in the corner or at a distance of at least r from the border of the deployment region, respectively. Hence, the asymptotic behavior of $P(ISOLATED_i)$ is given by $\lim_{l\to\infty}\left(1 - \frac{cr^2}{l^2}\right)^n$, for some constant $c > 0$. Taking the logarithm we have:

$$\lim_{l\to\infty} \ln\left(1 - \frac{cr^2}{l^2}\right)^n = \lim_{l\to\infty} n \ln\left(1 - \frac{cr^2}{l^2}\right).$$

Considering that $r = r(l) \ll l$ and using the Taylor expasion, we can rewrite the last term as $\lim_{l\to\infty} -\frac{cr^2 n}{l^2}$. Since $r^2 n \in O(l^2)$, we have two cases:

- $r^2 n = \Theta(l^2)$; in this case, we have

$$\lim_{l\to\infty} -\frac{cr^2 n}{l^2} = -c',$$

for some $c' > 0$. It follows that

$$\lim_{l\to\infty} P(ISOLATED_i) = e^{-c'} > 0.$$

- $r^2 n = o(l^2)$; in this case, we have $\lim_{l\to\infty} -\frac{cr^2 n}{l^2} = 0$. It follows that $\lim_{l\to\infty} P(ISOLATED_i) = 1$. □

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References

[1] C. Bettstetter, "On the Minimum Node Degree and Connectivity of a Wireless Multihop Network," *Proc. ACM MobiHoc '02,* pp. 80-91, 2002.
[2] D.M. Blough, M. Leoncini, G. Resta, and P. Santi, "On the Symmetric Range Assignment Problem in Wireless Ad Hoc Networks," *Proc. IFIP Conf. Theoretical Computer Science,* pp. 71-82, Aug. 2002.
[3] B. Bollobas, *Random Graphs.* London: Academic Press, 1985.
[4] G. Calinescu, I.I. Mandoiu, and A. Zelikovsky, "Symmetric Connectivity with Minimum Power Consumption in Radio Networks," *Proc. Second IFIP Conf. Theoretical Computer Science,* pp. 119-130, Aug. 2002.
[5] A.E.F. Clementi, P. Penna, and R. Silvestri, "Hardness Results for the Power Range Assignment Problem in Packet Radio Networks," *Proc. Second Int'l Workshop Approximation Algorithms for Combinatorial Optimization Problems (RANDOM/APPROX '99),* pp. 197-208, 1999.
[6] A.E.F. Clementi, P. Penna, and R. Silvestri, "The Power Range Assignment Problem in Radio Networks on the Plane," *Proc. XVII Symp. Theoretical Aspects of Computer Science (STACS '00),* pp. 651-660, 2000.
[7] A.E.F. Clementi, A. Ferreira, P. Penna, S. Perennes, and R. Silvestri, "The Minimum Range Assignment Problem on Linear Radio Networks," *Proc. Eighth European Symp. Algorithms (ESA 2000),* pp. 143-154, 2000.
[8] J. Diaz, M.D. Penrose, J. Petit, and M. Serna, "Convergence Theorems for Some Layout Measures on Random Lattice and Random Geometric Graphs," *Combinatorics, Probability, and Computing,* no. 6, pp. 489-511, 2000.
[9] O. Dousse, P. Thiran, and M. Hasler, "Connectivity in Ad Hoc and Hybrid Networks," *Proc. IEEE Infocom '02,* 2002.
[10] D. Estrin, R. Govindan, J. Heidemann, and S. Kumar, "Next Century Challenges: Scalable Coordination in Sensor Networks," *Proc. MOBICOM '99,* pp. 263-270, 1999.
[11] W. Feller, *An Introduction to Probability Theory and Its Applications.* New York: John Wiley and Sons, 1950.
[12] P. Gupta and P.R. Kumar, "Critical Power for Asymptotic Connectivity in Wireless Networks," *Stochastic Analysis, Control, Optimization and Applications,* Boston: Birkhauser, pp. 547-566, 1998.
[13] P. Gupta and P.R. Kumar, "The Capacity of Wireless Networks," *IEEE Trans. Information Theory,* vol. 46, no. 2, pp. 388-404, 2000.
[14] D.B. Johnson and D.A. Maltz, "Dynamic Source Routing in Ad Hoc Wireless Networks," *Mobile Computing,* Boston: Kluwer Academic, pp. 153-181, 1996.
[15] L.M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc, "Power Consumption in Packet Radio Networks," *Theoretical Computer Science,* vol. 243, pp. 289-305, 2000.
[16] V.F. Kolchin, B.A. Sevast'yanov, and V.P. Chistyakov, *Random Allocations.* Washington, D.C.: H. Winston and Sons, 1978.
[17] L. Li, J.H. Halpern, P. Bahl, Y. Wang, and R. Wattenhofer, "Analysis of a Cone-Based Distributed Topology Control Algorithm for Wireless Multi-Hop Networks," *Proc. ACM ACM Symp. Principles of Distributed Computing (PODC '01),* 2001.
[18] A.W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications.* New York: Academic Press, 1979.
[19] R. Meester and R. Roy, *Continuum Percolation.* Cambridge, Mass.: Cambridge Univ. Press, 1996.

[20] E.M. Palmer, *Graphical Evolution.* New York: John Wiley and Sons, 1985.
[21] P. Panchapakesan and D. Manjunath, "On the Transmission Range in Dense Ad Hoc Radio Networks," *Proc. IEEE SPCOM '01,* 2001.
[22] M.D. Penrose, "A Strong Law for the Longest Edge of the Minimal Spanning Tree," *The Annals of Probability,* vol. 27, no. 1, pp. 246-260, 1999.
[23] M.D. Penrose, "The Longest Edge of the Random Minimal Spanning Tree," *The Annals of Applied Probability,* vol. 7, no. 2, pp. 340-361, 1997.
[24] M.D. Penrose, "On *k*-Connectivity for a Geometric Random Graph," *Random Structures and Algorithms.* vol. 15, no. 2, pp. 145-164, 1999.
[25] T.K. Philips, S.S. Panwar, and A.N. Tantawi, "Connectivity Properties of a Packet Radio Network Model," *IEEE Trans. Information Theory,* vol. 35, no. 5, pp. 1044-1047, 1989.
[26] P. Piret, "On the Connectivity of Radio Networks," *IEEE Trans. Information Theory,* vol. 37, no. 5, pp. 1490-1492, 1991.
[27] G.J. Pottie and W.J. Kaiser, "Wireless Integrated Network Sensors," *Comm. ACM,* vol. 43, no. 5, pp. 51-58, May 2000.
[28] R. Ramanathan and R. Rosales-Hain, "Topology Control of Multihop Wireless Networks Using Transmit Power Adjustment," *Proc. IEEE Infocom '00,* pp. 404-413, 2000.
[29] V. Rodoplu and T.H. Meng, "Minimum Energy Mobile Wireless Networks," *IEEE J. Selected Areas in Comm.,* vol. 17, no. 8, pp. 1333-1344, 1999.
[30] P. Santi and D.M. Blough, "An Evaluation of Connectivity in Mobile Wireless Ad Hoc Networks," *Proc. Int'l Conf. Dependable Systems and Networks (DSN '02),* pp. 89-98, 2002.
[31] P. Santi, D.M. Blough, and F. Vainstein, "A Probabilistic Analysis for the Range Assignment Problem in Ad Hoc Networks," *Proc. ACM MobiHoc '01,* pp. 212-220, 2001.
[32] L. Schwiebert, S.K.S. Gupta, and J. Weinmann, "Research Challenges in Wireless Networks of Biomedical Sensors," *Proc. ACM Mobicom '01,* pp. 151-165, 2001.
[33] A. Sen and M.L. Huson, "A New Model for Scheduling Packet Radio Networks," *Proc. IEEE Infocom '96,* pp. 1116-1124, 1996.
[34] M.B. Srivastava, R. Muntz, and M. Potkonjak, "Smart Kindergarten: Sensor-Based Wireless Networks for Smart Developmental Problem-Solving Environments," *Proc. ACM Mobicom '01,* pp. 132-138, 2001.
[35] D.C. Steere, A. Baptista, D. McNamee, C. Pu, and J. Walpole, "Research Challenges in Environmental Observation and Forecasting Systems," *Proc. ACM Mobicom '00,* pp. 292-299, 2000.
[36] Y. Xu, J. Heidemann, and D. Estrin, "Geography-Informed Energy Conservation for Ad Hoc Routing," *Proc. ACM Mobicom '01,* pp. 70-84, 2001.

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