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Topology Control in Wireless Ad Hoc and Sensor Networks

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Topology control is one of the most important techniques used in wireless ad hoc and sensor networks to reduce energy consumption, which is essential to extend the network operational time. The goal of this technique is to control the topology of the graph representing the communication links between network nodes, with the purpose of maintaining some global graph property (e.g., connectivity) while reducing energy consumption, which is strictly related to the nodes' transmitting range. In this paper, we state several problems related to topology control in wireless ad hoc and sensor networks, and we survey state-of-the-art solutions which have been proposed to tackle them. We also outline several directions for further research, which we hope will motivate researchers to undertake additional studies on this field.

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1. INTRODUCTION

Recent emergence of affordable, portable, wireless communication and computation devices, and concomitant advances in the communication infrastructure, have resulted in the rapid growth of mobile wireless networks. On one hand, this has led to the exponential growth of cellular networks, which are based on the combination of wired and wireless technologies. On the other hand, this has renewed the interest of the scientific and industrial community in the more challenging scenario in which a group of mobile units equipped with radio transceivers communicate without the assistance of any fixed infrastructure.

Networks composed by mobile, untethered units communicating with each other via radio transceivers, typically along multi-hop paths, have been called *ad hoc networks* in the literature[1]. Ad hoc networks can be used wherever a wired backbone is infeasible and/or economically not convenient, e.g., to provide communications

[1]Sometimes, ad hoc networks are also called *packet radio networks*, which is the name used in the early papers in the field.

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during emergencies, special events (expos, concerts, and so on) or in hostile environments.

Wireless sensor networks (WSN for short) are a special class of ad hoc networks. In a WSN, the interconnected units are battery-operated microsensors, each of which is integrated in a single package with low-power signal processing, computation, and a wireless transceiver. Sensor nodes collect the data of interest (e.g., temperature, pressure, soil makeup, and so on), and transmit them, possibly compressed and/or aggregated with those of neighboring nodes, to the other nodes. This way, every node in the network acquires a global view of the monitored area, which can be accessed by the external user connected to the WSN. Potential application of sensor networks abound: they can be used to monitor remote and/or hostile geographical regions, to trace animals movement, to improve weather prediction, and so on. Examples of scenarios where WSN can be used are described in [Estrin et al. 1999; Heinzelman et al. 1999; Khan et al. 2000; Mainwaring et al. 2002; Pottie and Kaiser 2000; Schwiebert et al. 2001; Srivastava et al. 2001; Steere et al. 2000].

The following aspects, which have to be carefully taken into account in the design stage, are peculiar of wireless ad hoc networks:

– *energy conservation*: contrary to the case of wired networks, units in ad hoc networks are typically equipped with limited energy supplies. Hence, one of the primary goals of the design is to use this limited energy as efficiently as possible. Energy efficiency is especially important in WSN, where replacing/refilling sensor batteries is in general infeasible. If energy conservation techniques are used at different levels of the wireless architecture, the functional lifetime of both individual units and the network can be extended considerably.

– *unstructured and time-varying network topology*: nodes in the network may in principle be arbitrarily placed in the deployment region; hence, the graph representing the communication links between the nodes is usually unstructured. Furthermore, due to node mobility and/or failure, the network topology may vary with time. As a consequence, determining the appropriate value of fundamental network parameters (e.g., the critical transmitting range for connectivity) is a difficult task.

– *low-quality communications*: communication on wireless channels is in general much less reliable than in wired channels. Furthermore, the quality of communication is strongly influenced by environmental factors, which can be time-varying. Considering that ad hoc networks, and especially WSN, are likely to be deployed in hostile environments, low communication quality is to be expected in general, with non-negligible off-service time intervals.

In case of WSN, the following aspects must also be considered:

– *operation in hostile environments*: as in many scenarios WSN are expected to operate in hostile environments, sensors must be explicitly designed to work under extreme conditions, which may make individual unit failure quite a common event. Hence, resilience to sensor faults must be explicitly addressed at different network layers.

– *data processing*: given the energy constraints and the expected poor communication quality, sensed data must be compressed and/or aggregated with data of neighboring sensors.

– *scalability*: depending on the scenario considered, WSN might be composed by several thousands of sensors. Thus, the scalability of the proposed solutions is an important issue.

Several solutions have been proposed in the literature that address at least some of the issues raised above. In particular, great efforts have been devoted to the design of energy-efficient and mobility resilient routing, broadcast and multicast protocols [Basagni et al. 1999; Gerla and Tsai 1995; Ko and Vaidya 1998; Murthy and Garcia-Luna-Aceves 1996; Rajaraman 2002].

Routing and broadcast protocols are usually concerned with energy-efficient message delivery on a given communication graph, which is considered as an input to the protocol. However, contrary to the case of wired networks, the network topology in wireless networks is not fixed, and can be changed by varying the nodes' transmitting range. So, further energy can be saved if the network topology used to route/broadcast messages is energy-efficient itself. The goal of *topology control* is to dynamically change nodes' transmitting range in order to maintain some property (e.g., connectivity) of the communication graph, while reducing the energy consumed by node transceivers (which is strictly related to the transmitting range). Since transceivers are one of the primary source of energy consumption in the wireless unit, especially in WSN, topology control mechanisms are fundamental to achieve a good network energy efficiency.

Besides reducing energy consumption, topology control has the positive effect of reducing contention when accessing the wireless channel. In general, when the nodes' transmitting ranges are relatively short, many nodes can transmit simultaneously without interfering with each other, and the network capacity is increased. Ideally, the nodes' transmitting range should be set to the minimum value such that the graph that represents the communication links between units is connected. How to compute this value, under different hypotheses on the initial node distribution, presence and type of mobility, and so on, is the matter of this survey.

The rest of this paper is organized as follows. In Section 2 we introduce a simplified but widely accepted model of wireless ad hoc network, which will be used in the rest of the paper. In Section 3, we review the probabilistic theories that have been used in the derivation of theoretical results concerning topology control. In Section 4, we introduce several problems related to topology control in stationary networks, and we survey state-of-the-art solutions which have been proposed to tackle them. In Section 5, we will discuss how does node mobility affect the picture drawn in Section 4. Finally, in Section 6 we outline several directions for further research.

2. A WIRELESS AD HOC NETWORK MODEL

In this section, we introduce a simplified but widely accepted model of wireless ad hoc network, which will be used in the definition of the various problems related to topology control considered in the literature.

A d-dimensional mobile wireless ad hoc network, with $d = 1, 2, 3$, is represented by a pair $M_d = (N, P)$, where N is the set of nodes, with $|N| = n$, and $P : N \times T \rightarrow [0, l]^d$, for some $l > 0$, is the placement function. The placement function assigns to every element of N and to any time $t \in T$ a set of coordinates in the d-dimensional cube of side l, representing the node's physical position at time t. The choice of limiting the admissible physical placement of nodes to a bounded region of $\mathbb{R}^d$ of the form $[0,l]^d$, for some $l > 0$, is realistic and eases the treatment of some of the problems considered in the following.

Node $i \in N$ is said to be *stationary* if its physical placement does not vary with time. If all the nodes are stationary, the network is said to be stationary, and function P can be represented simply as $P : N \rightarrow [0,l]^d$.

A *range assignment* for a d-dimensional network $M_d = (N, P)$ is a function $RA : N \rightarrow (0, r_{max}]$ that assigns to every element of N a value in $(0, r_{max}]$, representing its transmitting range. Parameter r_{max} is called the *maximum transmitting range* of the nodes in the network and depends on the features of the radio transceivers equipping the mobile nodes. A common assumption is that all the nodes are equipped with transceivers having the same features; hence, we have a single value of r_{max} for all the nodes in the network.

It is known [Pahlavan and Levesque 1995] that the power p_i required by node i to correctly transmit data to node j must satisfy inequality

$$\frac{p_i}{\delta_{i,j}^{\alpha}} \geq \beta \ , \tag{1}$$

where $\alpha \geq 2$ is the *distance-power gradient*, $\beta \geq 1$ is the *transmission quality* parameter, and $\delta_{i,j}$ is the Euclidean distance between the nodes. While the value of β is usually set to 1, the value of α depends on environmental conditions. In the ideal case, we have $\alpha = 2$; however, α is typically 4 in realistic situations. A value of α in the interval $[2, 6]$ is commonly accepted. Given the formula above, we can define the *energy cost* of a range assignment RA as $c(RA) = \sum_{i \in N} (RA(i))^{\alpha}$.

Formula (1) holds for free-space environments with non-obstructed line of sight, and it does not consider the possible occurrence of reflections, scattering and diffraction caused by buildings, terrain, and so on. Although more complicated formulae of the radio signal attenuation with distance are known, such as that recently derived in [Bruck et al. 2002], inequality (1) is widely accepted in the ad hoc networks community.

Given a network $M_d = (N, P)$ and a range assignment RA, the *communication graph* induced by RA on M_d at time t is defined as the directed graph $G_t = (N, E(t))$, where the directed edge $[i, j]$ exists if and only if $RA(i) \geq \delta_{P(i,t),P(j,t)}$. In words, the directed edge $[i, j]$ exists if and only if nodes i and j are at distance at most $RA(i)$ at time t. In this case node j is said to be a *neighbor* of i. A range assignment RA is said to be *connecting at time t* if the resulting communication graph at time t is strongly connected. If the network is stationary, we simply say that the range assignment RA is connecting. A range assignment in which all the nodes have the same transmitting range r, for some $0 < r \leq r_{max}$, is called *r-homogeneous* range assignment[2]. Observe that the communication graph generated by a homogeneous range assignment can be considered as undirected, since $[i, j] \in E(t) \Leftrightarrow [j, i] \in E(t)$.

In general, the range assignment may vary with time in order to ensure target

[2]When the value of r is not relevant, the r-homogeneous range assignment is simply called homogeneous range assignment.

properties (e.g., strong connectivity, a given network diameter $h < n$) of the communication graph. Hence, a sequence of range assignments $RA_{t_1}, RA_{t_2}, \ldots$ can be defined, where RA_{t_i} is the range assignment at time t_i, and the transition between range assignments is determined by the topology control mechanism.

The communication graph as defined here is essentially the *point graph* model introduced in [Sen and Huson 1996]. This model, under the assumption the nodes' position are chosen according to some probability distribution, is known as Random Geometric Graph (RGG) in the applied probability community (see Section 3 for details). The main weakness of the point graph model is the assumption that the radio coverage area is a perfect circle. This assumption is quite realistic in open air flat environments, but it is critical in indoor or urban scenarios, where the presence of objects, walls, buildings, and so on, renders the radio coverage area extremely irregular. Further, the area and shape of the radio coverage is influenced by weather conditions and by the interference with pre-existing infrastructure (e.g., power lines, base stations, and so on). Including all these details in the network model would make it extremely complicated and scenario-dependent, hampering the derivation of meaningful and sufficiently general analytical results. For this reason, the point graph model described above, although quite simplistic, is widely used in the analysis of ad hoc networks.

Before ending this Section, we want to emphasize that the results obtained using the point graph model could be useful, at least to some extent, also in scenarios where the radio coverage is known to be irregular. For example, suppose we want to characterize a minimum common value r of the transmitting range such that the resulting communication graph is connected. This value has been characterized, under some probabilistic assumptions on the node spatial distribution and using the point graph model, in several recent papers [Bettstetter 2002b; Gupta and Kumar 1998; Panchapakesan and Manjunath 2001; Philips et al. 1989; Piret 1991; Sanchez et al. 1999; Santi et al. 2001; Santi and Blough 2003]. The value of r thus obtained can be thought of as the radius of the largest circular sub-area of the actual area of coverage. In this case, there could exist nodes that are connected in reality that would not be connected considering the circular region; thus, the actual probability of connectivity could be higher compared to the analytical result. Clearly, the analytical characterization of r becomes less and less significant as the actual radio area coverage is more and more irregular.

3. PROBABILISTIC TOOLS

Some of the analytical results presented in this paper are based on a probabilistic approach. In this Section, we survey the probabilistic theories that have been used to derive them.

The main difficulty that arises in the probabilistic analysis of wireless ad hoc networks is that the well-established theory of random graphs [Bollobás 1985; Palmer 1985] cannot be used. In fact, a fundamental assumption in this model is that the probability of edge occurrences in the graph are independent, which is not the case in wireless ad hoc networks. As an example, consider three nodes i, j, k such that $\delta_{i,j} < \delta_{i,k}$. With common wireless technologies that use omni-directional antennas, if i has a link to k, then it has also a link to j. Hence, the occurrences of edges

(i,j) and (i,k) are correlated.

In order to circumvent this problem, Chlamtac and Faragó [Chlamtac and Faragó 1999] introduced the *Random Network* (RN) model as a generalization of the uniform random graph model, in which graphs are selected according to a more general probability distribution. We recall that in the uniform random graph model, each element of a given class of graphs with n vertices is assigned an equal probability of being chosen. Examples of uniform models are random graphs with a given number m, with $0 \leq m \leq \binom{n}{2}$, of edges, random trees, random k-regular graphs, and so on. In the RN model, graphs can be chosen according to an arbitrary non-degenerate distribution, where a non-degenerate distribution is a distribution that does not concentrate (in a probabilistic sense) on a class of graphs of relatively small size. Based on the RN model and using the theory of Kolmogorov complexity, Chlamtac and Faragó analyzed the performance of a randomized distributed algorithm aimed at connecting clusterheads in a Virtual Cellular Architecture. The authors claim that the RN model, relying on an arbitrary non-degenerate probability distribution, can account for correlations between edges, which were not allowed in the uniform model. Unfortunately, the actual probability distribution of the graphs describing ad hoc networks might be degenerate. In fact, the actual distribution is the uniform distribution over the class of point graphs, which is degenerate if the size of the class of point graphs is relatively small as compared to the class of all possible graphs. Since the class of point graphs has not yet been characterized, its size is unknown, and determining whether this distribution is degenerate or not is still an open problem.

A more recent theory, which is still in development, is the theory of *geometric random graphs* (GRG). In the theory of GRG, a set of n points is distributed according to some density in a d-dimensional region R, and some property of the resulting node placement is investigated. For example, the longest nearest neighbor link [Penrose 1999a], the longest edge of the Euclidean Minimum Spanning Tree (MST) [Penrose 1999b; 1997], and the total cost of the MST have been investigated [Aldous and Steele 1992; Steele 1988; Yukich 2000]. For a survey of GRG, the reader is referred to [Diaz et al. 2000]. Recently, several papers [Bettstetter 2002b; Blough et al. 2002; Panchapakesan and Manjunath 2001; Santi 2002] have used the theory of GRG to analyze fundamental properties (typically, connectivity) of wireless ad hoc networks.

Two others theories have been used in the probabilistic analysis of ad hoc networks: the theory of *continuum percolation* and the *occupancy theory*.

In the theory of continuum percolation [Meester and Roy 1996], nodes are assumed to be distributed with Poisson density λ in $\mathbb{R}^2$, and two nodes are connected to each other if the distance between them is at most r. It has been proved that for each $\lambda > 0$, there exists at most one infinite-order component with high probability. However, the existence of an infinite-order component is not sufficient to ensure the connectivity of the network. In fact, there could exist (infinitely many) nodes which do not belong to the giant component, thus leading to a disconnected communication graph. Hence, the quality of connectivity is related to the fraction θ of nodes belonging to the giant component, which in turn depend on the *percolation probability*. The percolation probability is the probability that an arbitrary node belongs to a connected component of infinite order. The main result of the theory of continuum percolation is that there exists a finite, positive value λ_c of λ, called *critical density*, under which the percolation probability is zero, and above which it is non zero. However, no explicit expression of the percolation probability is known to date. The theory of continuum percolation have been used in [Dousse et al. 2002; Gupta and Kumar 1998] to analyze the connectivity of ad hoc networks.

In the occupancy theory [Kolchin et al. 1978], it is assumed that n balls are thrown independently into C cells. The allocation of balls into cells can be characterized by means of random variables describing some property of the cells. The occupancy theory is aimed at determining the probability distribution of such variables as n and C grow to infinity (i.e., the *limit distribution*). The most studied random variable is the number of empty cells after that all the balls have been thrown, which we denote $\mu(n, C)$. Of course, the limit distribution of $\mu(n, C)$ depends on the relative magnitude of n and C, i.e., on the asymptotic behavior of $\rho = n/C$. Depending on the asymptotic behavior of ρ, five domains such that $n, C \to \infty$ for which the limit distribution of $\rho(n, C)$ is different have been determined. Depending on the domain, the limit distribution can be either Poisson or normal with different parameters. The occupancy theory can be used to analyze connectivity in ad hoc networks by subdividing the deployment region R into equal subregions (cells) of size $\approx r^d$, and by determining under which conditions all the cells are filled with at least one node (ball). This technique has been used in [Santi and Blough 2003; 2002].

4. STATIONARY NETWORKS

In this Section, we will consider several problems related to topology control in stationary ad hoc networks. The generalization of some of these problems to the more complicated scenario of mobile networks is presented in Section 5.

4.1 The critical transmitting range

First, we consider the following problem concerning homogeneous range assignments:

Definition 4.1 CTR. Suppose n nodes are placed in $R = [0, l]^d$, with $d = 1, 2, 3$; what is the minimum value of r such that the r-homogeneous range assignment is connecting?

The minimum value of r such that the r-homogeneous range assignment is connecting is known as the *critical transmitting range* in the literature.

The motivation for studying CTR (Critical Transmitting Range) stems from the fact that in many situations dynamically adjusting node transmitting range is not feasible. In fact, inexpensive radio transceivers might not allow the transmission range to adjusted [Ramanathan and Rosales-Hain 2000]. This type of transceiver is likely to be used in WSN, where the cost of the individual sensor should be as little as possible. In this scenario, setting the same transmitting range r for all the units is a reasonable choice, and the only option to reduce power consumption and increase network capacity is to set r to the minimum possible value that ensures connectivity.

Characterizing the critical transmitting range helps the system designer to answer fundamental questions, such as: given a number of nodes n to be deployed in a region R, which is the minimum value of the transmitting range that ensures network connectivity? Conversely, for a given transmitter technology, how many nodes must be distributed over a given region to ensure network connectivity?

The solution to CTR depends on the information we have about the physical placement of nodes. If the node placement is known in advance, the critical transmitting range can be easily determined (it is the longest edge of the Euclidean MST [Penrose 1997; Sanchez et al. 1999]). Unfortunately, in many realistic scenarios of ad hoc networks the node placement cannot be known in advance. For example, in WSN sensors could be spread from a moving vehicle (airplane, ship, or spacecraft). If nodes' positions are not known, the minimum value of r ensuring connectivity in all possible cases is $r \approx l\sqrt{d}$, which accounts for the fact that nodes could be concentrated at opposite corners of the deployment region. However, this scenario is very unlikely in most realistic situations. For this reason, CTR has been studied under the assumption that nodes are distributed in R according to some probabilistic distribution. In this case, the goal is to characterize the minimum value of r which provides connectivity with high probability (w.h.p.), i.e., with probability that tends to 1 as the number of nodes (or the side l of the deployment region) increases.

The probabilistic theory that is most suited to the analysis of CTR is the theory of geometric random graphs (see Section 3). Since the critical transmitting range is the longest MST edge, probabilistic solutions to CTR can be derived by using results concerning the asymptotic distribution of the longest MST edge [Penrose 1999b; 1997]. This approach has been used in [Panchapakesan and Manjunath 2001] to prove that, under the hypothesis that nodes are uniformly distributed in $[0,1]^2$, the critical transmitting range for connectivity w.h.p. is $r = c\sqrt{\frac{\log n}{n}}$, for some constant $c > 0$.

A notable result of the theory of GRG is that, under the assumption of uniformly distributed points and $d \geq 2$, the longest nearest neighbor link and the longest MST edge have the same value (asymptotically, as $n \to \infty$) [Penrose 1999b]. In terms of the resulting communication graph, this means that connectivity occurs (asymptotically) when the last isolated node disappears from the graph. This observation can be generalized to the case of k-connectivity: when the minimum node degree becomes k, the graph becomes k-connected [Penrose 1999a]. This result, which has been used in [Bettstetter 2002b] to characterize the k-connectivity of ad hoc networks, reveals an interesting analogy with non-geometric random graphs, which display the same behavior (known as the *giant component* phenomenon).

Although interesting, the theory of GRG can be used only to derive results concerning *dense* ad hoc networks. In fact, a standard assumption in this theory is that the deployment region R is fixed, and the asymptotic behavior of r as n grows to infinity is investigated, i.e., the node density is assumed to grow to infinity. A similar limitation applies to the model of Gupta and Kumar [Gupta and Kumar 1998]. In their case, R is the disk of unit area, and the authors show that if the units' transmitting range is set to $r = \sqrt{\frac{\log n + c(n)}{\pi n}}$, then the resulting network is connected w.h.p. if and only if $c(n) \to \infty$. This result is obtained making use of the theory of continuum percolation [Meester and Roy 1996], which is also used in [Dousse et al. 2002] to investigate the connectivity of hybrid ad hoc networks, in which base stations can be used to improve connectivity.

Given the discussion above, the applicability of theoretical results concerning connectivity in ad hoc networks to realistic scenarios could be impaired. In fact, it is known that real wireless networks cannot be too dense, due to the problem of spatial reuse: when a node is transmitting, all the nodes within its transmitting range must be silent, in order not to corrupt the transmission. If the node density is very high, many nodes must remain silent when a node is transmitting, and the overall network capacity is compromised [Gupta and Kumar 2000].

In order to circumvent this problem, other authors have characterized the critical transmitting range in the more general model in which the side l of the deployment region is a further parameter, and n and r can be arbitrary functions of l. In this case, the critical transmitting range is analyzed asymptotically, as $l \to \infty$. Note that using this model, the node density might either converge to 0, or to a constant $c > 0$, or diverge as the size of the deployment region grows to infinity. Thus, results based on this framework can be applied to dense, as well as *sparse*, ad hoc networks.

The critical coverage range[3], which is closely related to the critical transmitting range, has been investigated in [Philips et al. 1989] for the case of nodes distributed in a square with side of length l according to a Poisson process of fixed density. The critical transmitting range for Poisson distributed points in a line of length l is derived in [Piret 1991]. However, also these results are difficult to be applied in real life scenarios, since in a Poisson process the actual number of deployed nodes is a random variable itself. Hence, only the *expected* number of deployed nodes can be controlled.

The critical transmitting range for connectivity in sparse ad hoc networks have been analyzed in [Santi et al. 2001; Santi and Blough 2003; 2002] using the occupancy theory. It has been proved that, under the assumption that n nodes are distributed uniformly at random in $R = [0, l]^d$, the r-homogeneous range assignment is connecting w.h.p. if $r = l \sqrt[d]{c \frac{\log l}{n}}$, for some constant $c > 0$. The authors also prove that, if $r \in O\left(l \sqrt[d]{\frac{1}{n}}\right)$, then the r-homogeneous range assignment is not connected w.h.p.

Besides analytical characterization, the critical transmitting range has been investigated from a more practical viewpoint. In [Narayanaswamy et al. 2002], Narayanaswamy et al. presented a distributed protocol that attempts to determine the minimum common transmitting range needed to ensure network connectivity. The authors show that setting the transmitting range to this value has the beneficial effects of maximizing network capacity, reducing the contention to access the wireless channel, and extending network lifetime. In [Bettstetter 2002a], Bettstetter analyzed network connectivity under the assumption that some of the nodes have transmitting range r_1, and the remaining have transmitting range $r_2 \neq r_1$. In [Santi

[3] Network coverage is defined as follows: every node covers a circular area of radius r_c, and the monitored area R is *covered* if every point of R is at distance at most r_c from at least one node. The goal is to find the critical value of r_c that ensures coverage w.h.p..

and Blough 2003], Santi and Blough investigated through simulation the trade off between the transmitting range and the size of the largest connected component in the communication graph. The experimental results presented in [Santi and Blough 2003] show that, in two and three-dimensional networks, the transmitting range can be reduced significantly if weaker requirements on connectivity are acceptable: halving the critical transmitting range, the largest connected component has average size of $0.9n$, approximately. This means that a considerable amount of energy is spent to connect relatively few nodes. This behavior is not displayed in case of one-dimensional networks, in which a modest decrease of the transmitting range below the critical value split the network into at least two connected components of moderate size. Quite interestingly, the experimental analysis of [Santi and Blough 2003] is coherent with the theoretical result of the theory of GRG (which, we recall, can be applied only to dense ad hoc networks) concerning the giant component phenomenon occurring in two and three-dimensional networks. This seems to indicate that, also in the case of sparse ad hoc networks, connectivity occurs (asymptotically) when the last isolated node disappears from the communication graph.

4.2 Non-homogeneous topology control

In the previous Section, we have analyzed the problem of determining a minimum common value for the transmitting range that generates a connected communication graph, under the hypothesis that only probabilistic information about node positions is available. In this Section, we survey the considerable body of results obtained for the more general problem in which nodes are allowed to have different transmitting ranges. As in the case of homogeneous topology control, in this Section we only report results concerning the stationary case. Non-homogeneous topology control techniques for mobile networks will be discussed in Section 5.

4.2.1 *The range assignment problem.* The problem of assigning transmitting range to nodes in such a way that the resulting communication graph is strongly connected and the energy cost is minimum is called the *range assignment problem* (RA), and was first studied in [Kirousis et al. 2000]. More formally, the problem is defined as follows.

Definition 4.2 RA. Let $N=\{u_1,\ldots,u_n\}$ be a set of points in the d-dimensional space ($d=1,2,3$), denoting the positions of the network nodes. Determine a connecting range assignment RA such that $c(RA) = \sum_{u_i \in N}(RA(u_i))^\alpha$ is minimum.

The computational complexity of RA has been analyzed in [Kirousis et al. 2000]. The problem is solvable in polynomial time (more specifically, in time $O(n^4)$) in the one-dimensional case (i.e., nodes in a line), while it is shown to be NP-hard in the case of three-dimensional networks. In a later paper, Clementi et al. [Clementi et al. 1999] have shown that RA is NP-hard also in the two-dimensional case. Thus, computing the optimal range assignment in two and three-dimensional networks is a virtually impossible task. However, the optimal solution can be approximated within a factor of 2 using the range assignment generated as follows [Kirousis et al. 2000]: let T be the MST built on N, where the weight of edge (u_i,u_j) is the power $\delta^\alpha_{u_i,u_j}$ needed to transmit a message between u_i and u_j; for every node $u_i \in N$, define

$RA(u_i)$ as the maximum of distances δ_{u_i,u_j}, for all nodes u_j which are neighbors of u_i in T. In the following, we will denote this range assignment with RA_{MST}.

Several variants of RA have been considered in the literature. In [Clementi et al. 1999; 2000; Clementi et al. 2000; Kirousis et al. 2000], the focus is on a constrained version of RA, in which the additional requirement of diameter at most h, for some constant $h < n$, is imposed on the communication graph. However, we believe this version of the problem is less interesting from a practical point of view. In fact, imposing a topology which is "too connected" would often cause communication interference to occur even between nodes that are far apart, thus decreasing the network capacity. This phenomenon is confirmed by theoretical as well as experimental results [Grossglauser and Tse 2001; Gupta and Kumar 2000; Li et al. 2001], which show that the communication graph in wireless ad hoc networks should be as sparse as possible, while preserving connectivity.

Two important variants of RA which have been recently studied are based on the concept of *symmetry* of the communication graph. In general, the communication graph generated by a range assignment is not symmetric, i.e., it might contain unidirectional links. Although implementing wireless unidirectional links is technically feasible (see [Bao and Garcia-Luna-Aceves 2001; Kim et al. 2001; Pearlman et al. 2000; Prakash 2001; Ramasubramanian et al. 2002] for unidirectional link support at different layers), the actual advantage of using unidirectional links is questionable. For example, in [Marina and Das 2002] Marina and Das have shown that the high overhead needed to handle unidirectional links in routing protocols outweighs the benefits that they can provide, and better performance can be achieved by simply avoiding them. The high overhead is due to the fact that low level protocols, such as the MAC (Medium Access Control) protocol, are naturally designed to work under the symmetric assumption. For instance, the MAC protocol defined in the IEEE 802.11 standard [IEEE 1999] is based on RTS - CTS message exchange: when node u_i wishes to send a message to one of its neighbors u_j (at this level, communication is only between immediate neighbors), it sends a RTS (Request To Send) to u_j, and waits for a CTS (Clear To Send) message from u_j. If the CTS message is not received within a certain time, then message transmission is aborted and it is tried again after a backoff interval. Hence, for the protocol to work u_i must be within the transmitting range of u_j, i.e., the range assignment must be symmetric.

The symmetric range assignment problem have been independently defined and studied in [Blough et al. 2002; Calinescu et al. 2002]. In [Blough et al. 2002], two different symmetric restrictions of RA are considered:

Definition 4.3 WSRA. Let $N = \{u_1, \ldots, u_n\}$ be a set of points in the d-dimensional space ($d = 1, 2, 3$), denoting the positions of the network nodes, and let G_S be the subgraph of the communication graph G obtained by deleting unidirectional links. Determine a connecting range assignment RA such that G_S is connected and $c(RA) = \sum_{u_i \in N} (RA(u_i))^\alpha$ is minimum.

Definition 4.4 SRA. Let $N = \{u_1, \ldots, u_n\}$ be a set of points in the d-dimensional space ($d = 1, 2, 3$), denoting the positions of the network nodes. A range assignment RA is said to be symmetric if it generates a communication graph which contains only bidirectional links, i.e., $RA(u_i) \geq \delta_{u_i,u_j} \Leftrightarrow RA(u_j) \geq \delta_{u_i,u_j}$. Determine a connecting symmetric range assignment RA such that $c(RA) = \sum_{u_i \in N}(RA(u_i))^\alpha$ is minimum.

In SRA (Symmetric Range Assignment), it is required that the communication graph contains only bidirectional links. This requirement is weakened in WSRA (Weakly Symmetric Range Assignment), in which unidirectional links may exist, but they are not essential for connectivity. The motivation for studying WSRA stems from the observation that what is really important in the design of ad hoc networks is the existence of a connected backbone of symmetric edges. In other words, there could exist further edges for which symmetry is not guaranteed, but these links can be ignored without compromising network connectivity.

In [Blough et al. 2002], it is shown that SRA remains NP-hard in two and three-dimensional networks. Hence, imposing (weak) symmetry does not reduce the computational complexity of the problem. The authors of [Blough et al. 2002] have also investigated the relations between the energy cost of the optimal solutions of RA, WSRA and SRA. Denoting with c_{RA}, c_{WS} and c_S these costs, respectively, we have $c_{WS} - c_{RA} \in O(1)$, and $c_S - c_{RA} \in \Omega(n)$. In words, this means that the requirement for weak symmetry has only a marginal effect on the energy cost of the range assignment, while it eases significantly the integration of topology control mechanisms with existing higher level protocols (e.g., routing). On the other hand, imposing the stronger requirement of symmetry incurs a considerable additional energy cost. Overall, we can conclude that weak symmetry is a desirable property of the range assignment.

In [Calinescu et al. 2002], Calinescu et al. introduce two polynomial approximation algorithms for WSRA, which improve on the approximation ratio of 2 previously known[4]. The first algorithm has an approximation ratio of $1 + \ln 2 \approx 1.69$, while the second, which is more computationally efficient, has an approximation ratio of $\frac{15}{8}$. These ratios have been recently improved to $\frac{5}{3} + \epsilon$, for any positive constant ϵ, and to $\frac{11}{6}$, respectively [Althaus et al. 2003]. Further, the authors of [Althaus et al. 2003] present an exact branch and cut algorithm for solving WSRA based on a new integer linear program formulation of the problem. Experimental results show that the branch and cut algorithm solves instances with up to 35-40 nodes (with randomly generated positions) in 1 hour. Most importantly, the experimental results show that the average improvement of the exact solution over RA_{MST}, which can be easily calculated, is in the range 4–6%. This means that the average case approximation ratio of RA_{MST} is much smaller than the worst case ratio of 2.

4.2.2 *Minimum energy unicast and broadcast.*

4.2.2.1 **Unicast**. In the previous Section, the emphasis was on finding a range assignment that generates a connected topology of minimum energy cost. Another branch of research focused on computing topologies which have energy-efficient paths between potential source-destination pairs. More specifically, the following problem has been considered (see [Li et al. ; Rajaraman 2002]).

[4]It can be easily observed that the RA_{MST} range assignment used in [Kirousis et al. 2000] to approximate RA within a factor of 2 is weakly symmetric. This observation has been used in [Blough et al. 2002] to prove that $c_{WS} - c_{RA} \in O(1)$.

Let G be the communication graph obtained when all the nodes transmit at maximum power, and assume G is connected. Every edge (u_i, u_j) in G is weighted with the power $\delta^\alpha_{u_i,u_j}$ needed to transmit a message between u_i and u_j. Given any path $P = u_1, u_2, \ldots, u_k$ in G, the *power cost* of P is defined as the sum of the power costs of the single edges, i.e., $pc(P) = \sum_{i=1}^{k-1} \delta^\alpha_{u_i,u_{i+1}}$. Let $pc_G(u, v)$ denote the minimum of $pc(P)$ over all paths P that connect nodes u and v in G. The path in G connecting u and v and consuming the minimum power $pc_G(u, v)$ is called the *minimum-power path* between u and v. Let G' be an arbitrary subgraph of G. The *power stretch factor* of G' with respect to G is the maximum over all possible node pairs of the ratio between the minimum-power path in G' and in G. Formally, $\rho_{G'} = \max_{u,v \in N} \frac{pc_{G'}(u,v)}{pc_G(u,v)}$.

The power stretch factor is a generalization of the concept of *distance stretch factor*, which is well known in computational geometry. Another similar concept is the *hop stretch factor*, which measures the ratio of the hop-counts rather than that of power or distance.

In general, we would like to identify a subgraph G' (also called *routing graph* in the following) of the communication graph G which has a low power stretch factor, and which is sparser than the original graph. The routing graph can be used to compute routes between nodes, with the guarantee that the power needed to communicate along these routes is "almost minimal". The advantage of using G' instead of G is that computing the optimal routes in G' is easier than in G, and that a sparse communication graph requires little maintenance in presence of node mobility.

Given the communication graph G, the problem of computing a subgraph G' with low power stretch factor has been widely studied in the literature. Ideally, the routing graph should have the following features:

a. constant power stretch factor, i.e., $\rho_{G'} \in O(1)$. Using the terminology of geometric graphs, G' should be a *power spanner* of G.

b. linear number of edges; in other words, G' should be *sparse*.

c. bounded node degree.

d. easily computable in a distributed and localized fashion. By localized, we mean that every node should be able to compute the set of its neighbors in G' using only information provided by its neighbor nodes in G.

Property *a.* ensures that the routes calculated on G' are at most a constant factor away from the energy-optimal routes. Property *b.* eases the task of finding routes in G', and of maintaining the routing graph in presence of node mobility. The requirement of bounded node degree is motivated by the fact that nodes with high degree are likely to be bottlenecks in the communication graph. Finally, property *d.* is fundamental for a fast and effective computation of the routing graph in a real wireless ad hoc network.

Several routing graphs that satisfy some of the requirements above have been proposed in the literature. Most of them are based on subgraphs of G which have been shown to be good distance spanners. In fact, it can be easily seen that if a subgraph G' is a distance spanner of graph G, then it is also a power spanner of G (note that the reverse implication in general is not true). Thus, the considerable

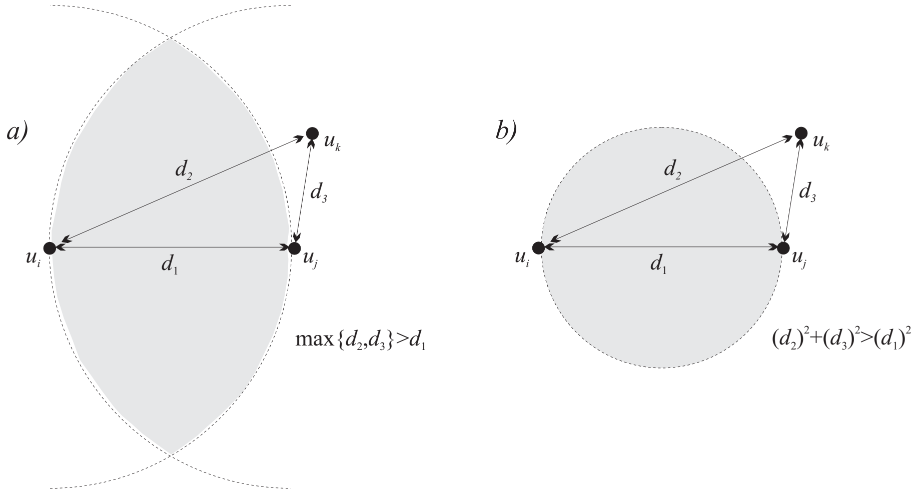

Fig. 1. Edges in the Relative Neighborhood Graph (left) and in the Gabriel Graph (right).

body of research devoted to distance spanners in computational geometry can be used to design good routing graphs.

The following geometric graphs have been considered in the literature:

Definition 4.5. Let N be a set of points in the Euclidean two-dimensional space;

RNG. the Relative Neighborhood Graph of N has an edge between two nodes u_i and u_j if there is no node u_k such that $\max\{\delta_{u_i,u_k}, \delta_{u_j,u_k}\} \leq \delta_{u_i,u_j}$ (see Figure 1–a);

GG. the Gabriel Graph of N has an edge between two nodes u_i and u_j if there is no node u_k such that $\delta^2_{u_i,u_k} + \delta^2_{u_j,u_k} \leq \delta^2_{u_i,u_j}$; in words, $(u_i, u_j) \in GG(N)$ if and only if the disk obtained using $\overline{u_i u_j}$ as diameter does not contain any node from N (see Figure 1–b);

DG. the Delaunay Graph of N is the unique triangulation such that the circumcircle of every triangle contains no points of N in its interior;

YG. the Yao Graph of N of parameter c, for any integer $c \geq 6$, is denoted YG_c, and is defined as follows. At each node $u_i \in N$, any c equally separated rays originated at u_i define c equal cones. In each cone, choose the shortest directed edge $(u_i, u_j) \in G$, if any, and add the correspondent directed edge in YG_c. If we add the reverse directed link from u_j to u_i, we obtain the Reverse Yao Graph. If we ignore the direction of edges, we have the Undirected Yao Graph.

Note that in general the DG of a set of points may include edges much longer than the maximum node transmitting range. For this reason, a restricted version of DG has been introduced in [Gao et al. 2001], in which a limit on the maximum edge length is imposed. We denote the restricted DG graph of a set of points N with $RDG(N)$.

The graphs defined above are called *proximity graphs*, since the set of neighbors of any node u in the computed graph can be calculated based on the position of the neighbor nodes in the original graph. Thus, proximity graphs satisfy requirement *d.* above.

The following relationships between proximity graphs have been proved [Goodman and O'Rourke 1997; Li et al.]: for any set of points N, $RNG(N) \subseteq GG(N)$, and $RNG(N) \subseteq YG_c(N)$, for any $c \geq 6$. Furthermore, $MST(N)$ is contained in $RNG(N)$, $GG(N)$, $DG(N)$ and $YG_c(N)$, for any $c \geq 6$.

The distance stretch factor, the power stretch factor, and the maximum node degree of the proximity graphs defined above have been analyzed in [Gao et al. 2001; Li et al.], and are reported in Table I. As it is seen, the Gabriel Graph is energy-optimal, since it has a power stretch factor of 1.

	Distance	Power	Degree
RNG	$n-1$	$n-1$	$n-1$
GG	$\frac{4\pi\sqrt{2n-4}}{3}$	1	$n-1$
RDG	$\frac{1+\sqrt{5}}{2}\pi$	$\left(\frac{1+\sqrt{5}}{2}\pi\right)^{\alpha}$	$\Theta(n)$
YG_c	$\frac{1}{1-2\sin\frac{\pi}{c}}$	$\frac{1}{1-\left(2\sin\frac{\pi}{c}\right)^{\alpha}}$	$n-1$

Table I. Distance stretch factor, power stretch factor, and maximum node degree of different proximity graphs.

All the graphs defined above have been shown to be sparse, which implies that they have a constant *average* node degree. However, the *maximum* node degree is not constant in any of the considered graphs. For this reason, several variants of these proximity graphs have been proposed, with the purpose of bounding the maximum node degree [Wang et al.]. Unfortunately, it has been shown that no geometric graph with constant node degree contains the minimum-power path for any pair of nodes [Wang et al.]. Thus, no energy-optimal spanner with a constant bounded maximum node degree exists. To date, the routing graph with constant maximum node degree which has the best power stretch factor is the YG_c^* graph of [Wang et al.], which is obtained by pruning properly the YG_c graph, with $c > 6$. The YG_c^* graph has power stretch factor of $\left(\frac{1}{1-(2\sin\frac{\pi}{c})^{\alpha}}\right)^2$, and maximum node degree of $(c+1)^2 - 1$. For example, setting $c = 8$ and $\alpha = 2$, we obtain a power stretch factor of 5.82 with a bound on the maximum node degree of 81.

4.2.2.2 **Broadcast**. Another relevant problem that has been considered in the literature is the determination of energy-efficient *broadcast graphs*. Here, the emphasis is on the one-to-all communication scheme typical of broadcast, rather than on point–to–point communications.

Similarly to the case of unicast, the concept of *broadcast stretch factor* can be defined. More precisely, let us consider a connected communication graph G. Any broadcast generated by node u can be seen as a directed spanning tree T rooted at u, which we call a *broadcast tree*. The power cost of the broadcast tree T is defined as follows. Denoting with $pc_T(v)$ the power consumed by node v to broadcast the message along T, we have that $pc_T(v) = 0$ for any leaf node of T, and $pc_T(v) = \max_{(v,w)\in T} \delta_{v,w}^{\alpha}$ otherwise. Thus, the total power needed to broadcast the message along the broadcast tree T is $pc(T) = \sum_{v\in N} pc_T(v)$. The tree in G rooted at u and consuming the minimum power is called the *minimum-power broadcast tree* of u. Let G' be an arbitrary subgraph of G. The *broadcast stretch factor* of G' with respect to G is the maximum over all possible nodes of the ratio between the minimum-power broadcast tree in G' and in G. Formally, $\beta_{G'} = \max_{u \in N} \frac{pc_{G'}(u)}{pc_G(u)}$, where $pc_{G'}(u)$ and $pc_G(u)$ denote the cost of the minimum-power broadcast tree of u in G' and in G, respectively.

As in the case of unicast, the goal is to find sparse broadcast spanners[5] that can be computed in a distributed and localized fashion. Unfortunately, this task is more difficult than in the case of unicast.

The problem of computing the minimum-power broadcast tree rooted at a node u has been proved to be NP-hard [Cagali et al. 2002; Liang 2002], under the hypothesis that nodes can transmit at different power levels $P = \{p_1, \ldots, p_k\}$, where the p_i are arbitrary power levels and k is an arbitrary positive constant. Thus, the task of finding the energy-optimal broadcast tree of a given communication graph G is virtually impossible in any realistic scenario.

In [Wieselthier et al. 2000], Wieselthier et al. introduce three greedy heuristics for the minimum-power broadcast problem based on the construction of the MST, and evaluate them by means of simulation. The broadcast stretch factor of the graphs generated by these heuristics are formally derived in [Wan et al. 2002], in which it is shown that the MST has constant broadcast stretch factor c, for some $6 \leq c \leq 12$. Thus, the MST is a broadcast spanner of the original graph. Unfortunately, the construction of the MST, as well as of the other graphs proposed in [Wieselthier et al. 2000], requires global information, which can be a major difficulty in implementing it in a real ad hoc network. To date, no distributed and localized algorithm that constructs a broadcast spanner is known.

Before ending this Section, we want to outline the similarities between the range assignment problem discussed in Section 4.2.1 and the problem of energy-efficient broadcast. Suppose G is the complete graph on the set of points N. In the RA problem, the goal is to find the energy-optimal range assignment that generates a connected communication graph. Suppose an arbitrary node $u \in N$ wants to broadcast a message m, and let RA be the optimal range assignment. A very simple broadcast scheme is the following. Node u transmits m at distance $RA(u)$, and every other node v, upon receiving m for the first time, re-transmits it at distance $RA(v)$. It is immediate that, after all nodes in N have transmitted the message once, m has been broadcast in all the network. Thus, the energy cost of RA is an upper bound to the power cost of any broadcast tree in G. We recall that the energy cost of the optimal range assignment (and of the optimal weakly symmetric range assignment) differs from the cost of the MST at most for a factor 2. Since the MST is a broadcast spanner of G, this implies that the communication graph generated by the optimal (weakly symmetric) range assignment is a broadcast spanner of G. Unfortunately this does not help very much, since computing this graph in two and three-dimensional networks is NP-hard.

4.2.3 *Topology control protocols.* In Sections 4.2.1 and 4.2.2, we have reviewed several problems related to energy-efficient communication in wireless ad hoc net-

[5] A subgraph G' of graph G is a *broadcast spanner* of G if it has $O(1)$ broadcast stretch factor.

works. In this Section, we survey state-of-the-art topology control protocols, which allow nodes to dynamically adjust their transmitting ranges in an attempt to minimize some measure of energy consumption.

Ideally, a topology control protocol should be fully distributed and localized. As discussed above, these requirements are vital for an effective implementation of the protocol, especially in presence of node mobility. Another aspect to be considered in topology control is the "quality" of the information needed by the protocol. In general, there is a trade off between information quality and energy consumption: the more accurate is the information required (e.g., exact node positions), the more energy savings can be achieved. However, the price to be paid (in terms of additional hardware on the nodes, or of additional messages to be exchanged) to obtain high quality information must be carefully considered. For example, suppose protocol P_1 is based on positional information, and protocol P_2 is based on distance estimation. Clearly, the cost of implementing P_2 in a real network is significantly lower than that required by P_1, since the hardware needed to estimate distance between nodes is much cheaper than that required to estimate the node position. So, if the energy savings provided by protocol P_1 are not considerably higher than those achieved by P_2, a solution based on protocol P_2 may be preferable in practice.

In [Rodoplu and Meng 1999], Rodoplu and Meng presented a distributed topology control algorithm that leverages on position information (provided by low-power GPS receivers) to build a topology that is proved to minimize the energy required to communicate with a given master node. Unfortunately, the protocol relies on global knowledge and specialized hardware (the GPS receiver), which makes it infeasible in many application scenarios. In [Li and Wan 2001], Li and Wan described a more efficient implementation of the protocol which, however, computes only an approximation of the minimum energy topology.

In [Ramanathan and Rosales-Hain 2000], Ramanathan and Rosales-Hain considered the problem of minimizing the maximum of node transmitting ranges while achieving connectedness. They also considered the stronger requirement of bi-connectivity of the communication graph. They present centralized topology control algorithms based on distance estimation that provide the optimal solution for both versions of the problem. The range assignment returned by the algorithm has the additional property of being per-node minimal, i.e., no transmitting range can be reduced further without impairing connectivity (or bi-connectivity).

In [Wattenhofer et al. 2001], Wattenhofer et al. introduced a distributed topology control protocol based on directional information, called CBTC (Cone Based Topology Control). The basic idea is the same of the Yao graph YG: a node u transmits with the minimum power $p_{u,\rho}$ such that there is at least one neighbor in every cone of angle ρ centered at u. The obtained communication graph is made symmetric by adding the reverse edge to every asymmetric link. The authors show that setting $\rho \leq 2\pi/3$ is a sufficient condition to ensure connectivity. A set of optimizations aimed at pruning energy-inefficient edges without impairing connectivity (and symmetry) is also presented. Further, the authors prove that if $\rho \leq \pi/2$, every node in the final communication graph has degree at most 6. A more detailed analysis of CBTC, along with an improved set of optimizations (which, however, rely on distance estimation), can be found in [Li et al. 2001]. The CBTC protocol has been extended to the case of nodes in the three-dimensional space in [Bahramgiri et al. 2002]. The authors of [Bahramgiri et al. 2002] also presented a modification of the protocol aimed at ensuring k-connectivity. In [Huang et al. 2002], the CBTC protocol is implemented using directional antennas[6].

In [Borbash and Jennings 2002], Borbash and Jennings introduced a distributed protocol which is also based on directional information. The goal of the protocol is to build the Relative Neighbor Graph of the network in a distributed fashion. The choice of the RNG as the target graph of the protocol is due to the fact that it guarantees connectivity and it shows good performance in terms of average transmitting range, node degree and hop diameter.

In [Li et al. 2003], Li et al. introduced LMST, a fully distributed and localized protocol aimed at building an MST-like topology. The authors show that: (1) the protocol generates a strongly connected communication graph; (2) the node degree of any node in the generated topology is at most 6; and (3) the topology can be made symmetric by removing asymmetric links without impairing connectivity. Furthermore, the authors show through simulation that LMST outperforms CBTC and the protocol of Rodoplu and Meng in terms of both average node degree and average node transmitting range. A drawback of LMST is that it requires location information, which can be provided only with a considerable hardware and/or message cost.

Another class of topology control protocols is based on the so called *k-neighbors graph* of a set of points N, which is obtained by connecting each node to its k closest neighbors in N. The MobileGrid protocol of [Liu and Li 2002] and the LINT protocol of [Ramanathan and Rosales-Hain 2000] try to keep the number of neighbors of a node within a low and high threshold centered around an optimal value. When the actual number of neighbors is below (above) the threshold, the transmitting range is increased (decreased), until the number of neighbors is in the proper range. However, for both protocols no characterization of the optimal value of the number of neighbors is given, and, consequently, no guarantee on the connectivity of the resulting communication graph is provided. Another problem of the MobileGrid and LINT protocols is that they estimate the number of neighbors by simply overhearing control and data messages at different layers. This approach has the advantage of requiring no overhead, but the accuracy of the resulting neighbor number estimate heavily depends on the traffic present in the network. In the extreme case, a node which remains silent is not detected by any of its actual neighbors. The k-NEIGH protocol of [Blough et al. 2003] is also based on the k-neighbors graph. The goal of k-NEIGH is the keep the number of neighbors of a node equal to, or slightly below, a given value k. The communication graph that results is made symmetric by removing asymmetric edges. The authors show, using a result of [Xue and Kumar], that the communication graph generated by k-NEIGH when $k \in \Theta(\log n)$ is connected with high probability. Furthermore, the authors analyze the time and message complexity of the protocol, and present simulation results that show that the topology generated by k-NEIGH is, on the average, 20% more energy efficient than that generated by CBTC.

[6]Directional antennas have the ability to propagate the radio signal only in specific directions.

4.3 Discussion of energy cost

The results of Sections 4.1 and 4.2 can be used to evaluate the potential benefit (in terms of energy cost) achieved by topology control protocols. In fact, the solution of the range assignment problem RA can be seen, at least to a certain extent, as the best possible result of the execution of a topology control protocol. On the other hand, the critical transmitting range for connectivity considered in Section 4.1 is representative of the scenario in which no topology control is feasible.

The following theorem is a straightforward consequence of the results presented in [Santi and Blough 2003].

THEOREM 4.6. *Let l be a positive real number sufficiently large, and let N be a set of n nodes positioned uniformly and independently at random in $R = [0, l]^d$, with $d = 1, 2, 3$. Assume the distance-power gradient α is 2, and denote by $c_{min}(N)$ the cost of the r-homogeneous range assignment such that r is minimum and the resulting communication graph is connected. Then, with high probability:*

$$c_{min}(N) = \begin{cases} O(\frac{l^2 \log^2 l}{n}) & \text{for } d=1 \\ O(l^2 \log l) & \text{for } d=2 \\ O(l^2 n^{1/3} \log^{2/3} l) & \text{for } d=3 \end{cases}$$

The bounds of Theorem 4.6 can be compared to similar bounds obtained in [Blough et al. 2002; Clementi et al. 1999; 2000; Clementi et al. 2000; Kirousis et al. 2000] for the range assignment problem. For one-dimensional networks, the following result on equally spaced instances of RA has been proved [Clementi et al. 2000; Kirousis et al. 2000]:

THEOREM 4.7. *Let N be a set of n collinear points equally spaced at distance $\delta > 0$. The energy cost of the solution of* RA *on input N is $\Theta(\delta^2 n)$.*

Assuming that the n nodes are placed along a line of length l, the bound of Theorem 4.7 can be restated as $\Theta\left(\frac{l^2}{n}\right)$. It is not difficult to show that equally spacing nodes is the most energy-efficient placement. It follows immediately that the energy cost of any instance (including a random one[7]) of RA is $\Omega\left(\frac{l^2}{n}\right)$ with high probability. Comparing this bound with the upper bound reported in Theorem 4.6 for $d = 1$, we have that the asymptotic gap between the energy cost of the optimal range assignment and that of the optimal homogeneous range assignment is at most $\log^2 l$. Hence, the asymptotic benefit of the adoption of a topology control mechanism in one-dimensional networks is at most a factor of $\log^2 l$.

Bounds on the energy cost of the solution of the random instance of RA in two and three dimensions have been obtained in [Blough et al. 2002], and are $\Theta(l^2)$ for $d = 2$ and $\Theta(l^2 n^{1/3})$ for $d = 3$. By Theorem 4.6, we can conclude that the asymptotic benefit of the adoption of a topology control mechanism is at most a factor of $\log l$ in two-dimensional networks, and at most a factor of $log^{2/3} l$ in three-dimensional networks.

[7]Here, with random instance we mean an instance of the problem in which node positions are chosen uniformly at random in the deployment region $R = [0, l]^d$.

The comparison of the bounds on the energy cost of the optimal solution of RA and CTR in one, two and three-dimensional networks indicates that the benefit, expressed in terms of energy cost, of the adoption of a topology control mechanism increases with the length l of the side of the deployment region, but becomes less significant for networks of higher dimension.

5. MOBILE NETWORKS

In Section 4, we have analyzed several problems related to energy-efficient communication in stationary wireless ad hoc networks. In this Section, we will discuss how does mobility affect topology control in general.

The impact of mobility on topology control is two-fold:

– *increased message overhead*: the implementation of any distributed topology control protocol causes a certain message overhead, which is due to the fact that nodes need to exchange messages in order to set the transmitting range to the appropriate value. In case of stationary networks, the topology control protocol is in general executed once at the beginning of the network operational time, and the "efficiency" of the protocol (expressed here in terms of message overhead) has relatively little importance. In presence of mobility, the topology control protocol must be executed periodically, in order to account for the new positions of the nodes. The frequency of this periodic protocol re-execution depends on the mobility pattern, which determines the expected topology change rate. Thus, reducing message overhead is fundamental when implementing topology control mechanisms in mobile networks, especially in the case of high mobility scenarios.

– *non-uniform node spatial distribution*: as it will be discussed in details later, some mobility patterns cause a non-uniform node spatial distribution. This fact should be carefully taken into account in setting important network parameters (e.g., the critical transmitting range) at the design stage.

From the discussion above, it is clear that the impact of mobility on the effectiveness of topology control techniques heavily depends on the mobility pattern. For this reason, we first present the mobility models which have been considered in the literature.

5.1 Mobility models

The most widely used mobility model in the ad hoc network community is the random waypoint model [Johnson and Maltz 1996]. In this model, every node chooses uniformly at random a destination in $[0, l]^d$, and moves towards it along a straight line with a velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. When it reaches the destination, it remains stationary for a predefined pause time t_{pause}, and then it starts moving again according to the same rule.

A similar model is the random direction model, in which nodes move with direction chosen uniformly in the interval $[0, 2\pi[$, and velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. After a randomly chosen time, taken usually from an exponential distribution, the node chooses a new direction. A similar procedure is used to change velocity, using an independent stochastic process.

Contrary to the case of the random waypoint and the random direction model which resemble, at least to some extent, intentional motion, the class of Brownian-like mobility models resembles non-intentional movement. For example, in the model used in [Blough et al. 2002], mobility is modeled using parameters p_{stat}, p_{move} and m. Parameter p_{stat} represents the probability that a node remains stationary during the entire simulation time. Hence, only $(1 - p_{stat})n$ nodes (on the average) will move. Introducing p_{stat} in the model accounts for those situations in which some nodes are not able to move. For example, this could be the case when sensors are spread from a moving vehicle, and some of them remain entangled, say, in a bush or tree. This can also model a situation where two types of nodes are used, one type that is stationary and another type that is mobile. Parameter p_{move} is the probability that a node moves at a given step. This parameter accounts for heterogeneous mobility patterns, in which nodes may move at different times. Intuitively, the smaller is the value of p_{move}, the more heterogeneous is the mobility pattern. However, values of p_{move} close to 0 result in an almost stationary network. If a node is moving at step i, its position in step $i+1$ is chosen uniformly at random in the square of side $2m$ centered at the current node location. Parameter m models, to a certain extent, the velocity of the nodes: the larger m is, the more likely it is that a node moves far away from its position in the previous step.

Observe that in case of random direction or Brownian-like motion, nodes may in principle move out of the deployment region. Since a standard approach in simulations is to keep the number of network nodes constant, we need a so called *border rule* [Bettstetter 2001], which defines what to do with nodes that are about to leave the deployment region. In this situation, a node can be:

- bounced back according to some rule;
- positioned at the point of intersection of the boundary with the line connecting the current and the desired next position;
- wrapped around to the other side of the region, which is considered as a torus;
- "deleted", and a new node is initialized according to the initial distribution;
- forced to choose another position, until the chosen position is inside the boundaries of the deployment region.

Depending on the choice of the border rule, non-uniformity in the node spatial distribution can be produced. For example, the second rule described above places nodes exactly on the boundary of the region with higher probability than at other points. In fact, the only two rules that do not appear to favor one part of the region over another are the torus rule and the one in which a node is eliminated when it would cross the boundary and a new node is created in its place. However, these rules appear quite unrealistic, and are used mainly to artificially generate a more uniform node spatial distribution.

For a more exhaustive survey of mobility models in wireless networks, the reader is referred to [Bettstetter 2001].

5.2 Critical transmitting range

If deriving analytical results for stationary networks is difficult, even more challenging is deriving theoretical results regarding *mobile* ad hoc networks, even in the simpler case of topology control, i.e., in case of homogeneous range assignment.

When the range assignment is homogeneous, the message overhead is not an issue, since the nodes' transmitting range is set in the design stage, and cannot be changed dynamically. However, the node spatial distribution generated by the mobility model could be an issue. For instance, it is known [Bettstetter 2001; Bettstetter and Krause 2001; Bettstetter et al. 2002; Blough et al. 2002] that the random waypoint model generates a node spatial distribution which is independent of the initial node positions, and in which nodes are concentrated in the center of the deployment region. This phenomenon, which is known as the *border effect*, is due to the fact that in the random waypoint model a node chooses a uniformly distributed destination point rather than a uniformly distributed angle. Therefore, nodes located at the border of the region are very likely to move back toward the middle of the region. The intensity of the border effect mainly depends on the pause time t_{pause}. In fact, a longer pause time tends to increase the percentage of nodes that are "resting" at any given time. Since the starting and destination points of a movement are chosen uniformly in $[0,l]^d$, this implies that a relatively long pause time generates a "more uniform" node spatial distribution.

An immediate consequence of the fact that the node spatial distribution in presence of mobility is in general non-uniform is that results concerning the critical transmitting range in stationary networks (which are based on the uniformity assumption) cannot be directly used. For this reason, the relationship between the critical transmitting range without and with mobility must be carefully investigated.

In [Sanchez et al. 1999], Sanchez et al. analyze the probability distribution of the critical transmitting range in presence of different mobility patterns (random waypoint, random direction, and Brownian-like) through simulation. The simulation results seem to indicate that the mobility pattern has little influence on the distribution of the critical transmitting range. Unfortunately, the significance of the findings of [Sanchez et al. 1999] is partly impaired by the fact that the toroidal border rule is used in simulations, and that the values of the mobility parameters used in the experiments (such as t_{pause} in the random waypoint model) are not reported.

In [Santi and Blough 2003; 2002], Santi and Blough investigate the relationship between the critical transmitting range in stationary and in mobile networks through extensive simulation. They consider random waypoint and Brownian-like motion, and analyze different "critical values" for the node transmitting range, which are representative of different requirements on network connectivity (for instance, connectivity during 100% and 90% of the simulation time). The simulation results show that a relatively modest increase of the transmitting range with respect to the critical value in the stationary case is sufficient to ensure network connectivity during 100% of the simulation time. The increase is about 21% in the random waypoint and about 25% in the Brownian-like model. Furthermore, the simulation results show that the transmitting range can be considerably reduced (in the order of 35–40%) if the requirement for connectivity is only on 90% of the simulation time.

Further insights on the relationship between the stationary and mobile critical transmitting range can be derived from the statistical analysis of the node spatial distribution of mobile networks reported in [Blough et al. 2002]. Again, the authors consider random waypoint and Brownian-like mobility, and perform several statistical tests on the node spatial distribution generated by these models. The results of these tests show that the distribution generated by Brownian-like motion is virtually indistinguishable from the uniform distribution, and confirm the occurrence of the border effect in random waypoint motion, whose intensity heavily depends on the value of t_{pause}. In the extreme case of $t_{pause} = 0$, the random waypoint model generates a node spatial distribution which is considerably different from the uniform. Overall, the analysis of [Blough et al. 2002] indicate that Brownian-like mobility should have little influence on the value of the critical transmitting range, while the effect of random waypoint mobility on the critical transmitting range should heavily depend on the settings of the mobility parameters.

The quality of the observation above is confirmed by the probabilistic analysis reported in [Santi 2002], which is, to the best of our knowledge, the only theoretical result concerning the critical transmitting range in presence of mobility reported in the literature so far. Denoting with r and r_m the critical transmitting range in case of stationary and random waypoint mobile networks, respectively, the author shows that $r_m - r \in O(1)$ if $t_{pause} \neq 0$, and that $r_m \gg r$ otherwise (asymptotically, as $n \to \infty$). The author validates this result through simulations, whose results show an interesting "threshold phenomenon": for small values of n ($n \leq 50$), r_m is less than r, while for larger value of n the situation is reversed. This phenomenon is caused by the border effect induced by random waypoint mobility, which tends to concentrate nodes in the center of the deployment region: when n is small, the probability of finding at least one node close to the border is very low, and the critical transmitting range is smaller than in the stationary case. However, when n is large enough, some of the nodes actually lie close to the border of the deployment region, forcing an higher value of r_m.

5.3 Non-homogeneous topology control

In case of non-homogeneous topology control, the more relevant effect of mobility is the message overhead needed to update nodes' transmitting range in response to node mobility. The amount of this overhead depends on the frequency with which the reconfiguration protocol used to restore the desired network topology is executed. In turn, this depends on several factors, such as the mobility pattern and the properties of the topology generated by the protocol. To clarify this point, let us consider two topology control protocols P_1 and P_2. Protocol P_1 builds the MST in a distributed fashion, and set the nodes' transmitting range accordingly, while protocol P_2 attempts to keep the number of neighbors of each node below a certain value k, as in the k-NEIGH protocol of [Blough et al. 2003]. Protocol P_1 is based on global and very precise information, since the MST can be built only if the exact position of every node in the network is known. In principle, P_1 should be reconfigured every time the relative position of any two nodes in the network changes, since this change could cause edge insertion/removal in the MST. On the other hand, P_2 can be easily computed in a localized fashion, and can be implemented using relatively imprecise information such as distance estimation. In this case, the protocol should be re-executed only when the "relative neighborhood" relation of some node changes. It is quite intuitive that this occurs less frequently than edge insertion/removal in the MST. It should also be observed that having a non up-to-date topology is much more critical in case of the MST than in case of the k-neighbors graph: in fact, a single edge removal in the MST is sufficient to disconnect the network, while several edges can in general be removed from the k-neighbors graph without impairing connectivity. Overall, we can reasonably state that P_1 should be re-executed much more frequently than P_2. Further, we observe that the reconfiguration procedure needed to maintain the MST is more complicated than that required by the k-neighbors graph, since it relies on global information. So, we can conclude that protocol P_1 is not suitable to be implemented in a mobile scenario; in other words, it is not *resilient to mobility*.

From the discussion above, it is clear that a mobility resilient topology control protocol should be based on a topology which can be computed locally, and which requires little maintenance in presence of mobility. Many of the topology control protocols presented in the literature meet this requirement. However, only some of them have been defined to explicitly deal with node mobility.

In [Li et al. 2001], an adaptation of the CBTC protocol to the case of mobile networks is discussed. It is shown that, if the topology ever stabilizes and the reconfiguration protocol is executed, then the network topology remains connected. The reconfiguration procedure is adapted to the case of k-connectivity in [Bahramgiri et al. 2002].

In [Rodoplu and Meng 1999], Rodoplu and Meng discuss how does their protocol can be adapted to the mobile scenario, and evaluate the protocol power consumption in presence of a mobility pattern which resembles the random direction model.

The MobileGrid [Liu and Li 2002] and LINT [Ramanathan and Rosales-Hain 2000] protocols, which are based on the k-neighbors graph, are explicitly designed to deal with node mobility. They are zero-overhead protocols, since the estimation of the number of neighbors is based on overhearing of data and control traffic. However, no explicit guarantee on network connectivity is given, and only simulation results are reported by the authors.

A more subtle effect of mobility on certain topology control protocols is due to the possibly non-uniform node spatial distribution generated by the mobility pattern. This fact should be considered in setting fundamental protocol parameters, such as the "desired number of neighbors" in k-neighbors graph based protocols [Blough et al. 2002; Liu and Li 2002; Ramanathan and Rosales-Hain 2000]. In other words, it could be the case that the number of neighbors k needed to obtain connectivity w.h.p. in presence of uniform node distribution is significantly different from the value k_m needed when the node distribution is non-uniform, such as in presence of random waypoint mobility. Clearly, if nodes are expected to move with random waypoint-like mobility, k_m must be used instead of k in the protocol implementation.

6. OPEN ISSUES

Topology control has received increasing attention in the wireless ad hoc networks community in these recent years, as witnessed by the considerable body of research on this field reported in this paper. However, several aspects related to topology control have not been carefully investigated yet. In this final Section, we outline some of them, which we hope will motivate researchers to undertake further studies on this field.

More realistic models. The point graph model used to derive most of the results presented in this paper is an idealized model of a real ad hoc network. Although point graphs have proved useful to derive "qualitative" results, they can hardly be used to obtain the accurate quantitative information needed by the network designer. So, the need for a more realistic network model is urgent.

There are several ways in which the point graph model can be modified in order to be more realistic. For instance, we could define the occurrence of links between nodes in probabilistic rather than deterministic terms. A possible model could be the following: given nodes u and v at distance $\delta_{u,v}$, we have a link between u and v with probability 1 if $\delta_{u,v} \leq \overline{\delta}$, where $\overline{\delta}$ is an arbitrary constant, and with probability $p(\delta_{u,v}) < 1$ otherwise, where $p(\delta_{u,v})$ is an arbitrary decreasing function of the distance with values in $[0,1]$. This characterization of the occurrence of a wireless link is far more realistic than the 1/0 characterization used in the point graph model. For example, there could exist nodes u, v, w with $\delta_{u,v} = \delta_{u,w} > \overline{\delta}$ such that link (u,v) exists and link (u,w) does not. Thus, the radio coverage area is in general not regular, as it is the case in real wireless networks. A similar characterization of the wireless link is used in the Random Vertex model recently proposed by Faragó [Faragó 2002], where the link between nodes u and v occurs with probability $p_l(u,v)$, where $p_l(u,v)$ is an arbitrary function (with values in $[0,1]$), with the property that $\delta_{u,v} \leq \delta_{u,w}$ implies $p_l(u,v) \geq p_l(u,w)$. Furthermore, the author introduces the concept of *node availability*, i.e., node u is available with a given probability $p_a(u)$. Faragò shows that the study of monotone properties[8] (e.g., connectivity) on certain Random Vertex graphs can be reduced to the study of the same property on a properly defined random graph. This result is potentially interesting, given the overwhelming literature on traditional random graphs. Whether the Random Vertex model could be used to derive an analytical characterization of fundamental network properties (e.g., the critical transmitting range) in a more realistic setting is an open question.

Another possibility to make the network model more realistic is to take into account interferences between nodes. For example, in [Dousse et al. 2003] a bidirectional link between nodes u and v exists if the signal to noise ratio at the receiver is larger than some threshold, where the noise is the sum of the contribution of interferences from all other nodes and of a background noise. The authors analyze the impact of such wireless link model on network connectivity. Further investigation in this direction is needed.

More realistic node distribution. A simplifying assumption commonly used in the analysis of ad hoc networks is that nodes are uniformly distributed in the deployment region. Although this assumption seems reasonable in some settings, it is quite unrealistic in many scenarios. For instance, as discussed above, this assumption does not hold when the nodes move according to the random waypoint model. Further, when nodes are dispersed from a moving vehicle, the assumption of uniform distribution is only a rough approximation of the actual node distri-

[8]A property P of a graph is *monotone* whenever $P(G) \Rightarrow P(G')$, where G' is a super-graph of graph G and $P(G)$ (respectively, $P(G')$) denotes that graph G (respectively, G') satisfies P.

bution. Thus, the analysis of network properties in presence of non-uniform node spatial distributions is another step forward in the direction of a more realistic characterization of ad hoc networks.

More accurate analysis of mobile networks. More work needs to be done to investigate the effect of mobility on topology control. In particular, the following issues shall be addressed:

– *is mobility beneficial or detrimental?* On one hand, we have seen that mobility causes an increased message overhead to restore the desired topology. On the other hand, mobility has the positive effect of balancing the node energy consumption: in stationary networks, if a node u has twice the transmitting range of node v, it is likely to deplete its battery much faster than node v. In presence of mobility, nodes change the transmitting range dynamically, and a more balanced energy consumption is likely to occur. Since the ultimate goal of topology control is to extend network lifetime, the overall effect of mobility on network lifetime should be carefully investigated.

– *determination of the optimal frequency for reconfiguration.* As outlined in Section 5, there is a trade off between the message overhead caused by a topology control protocol and the "quality" of the topology generated. In general, to have a "high quality" topology (e.g., a connected topology), we should execute the reconfiguration protocol frequently. On the other hand, each execution of the reconfiguration protocol causes a significant message overhead. The careful investigation of this trade off would help in answering the previous issue.

Group mobility. In most of the mobility models considered in the literature (such as the random waypoint, random direction, and Brownian-like model), nodes move independently one of each other. However, in many realistic scenarios network nodes move in groups. This could be the case, for instance, of sensors dispersed in the ocean to monitor water temperature, which are moved by ocean flows. Or, the case of cars on a freeway, which exchange messages with the purpose of rapidly propagating information about traffic conditions. Thus, the impact of group mobility on topology control should be carefully investigated.

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