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Analysis of a Wireless Sensors Dropping Problem in Environmental Monitoring

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Abstract

In this paper we study the following problem: we are given a certain region R to monitor and a requirement on the degree of coverage (DoC) of R to meet by a network of deployed sensors. The latter will be dropped by a moving vehicle, which can release sensors at arbitrary points within R. The node spatial distribution when sensors are dropped at a certain point is modeled by a certain probability density function $\mathcal{F}$. The network designer is allowed to choose an arbitrary set of drop points, and to release an arbitrary number of sensors at each point. Given this setting, we consider the problem of determining the optimal deployment strategy, i.e., the drop strategy such that the DoC requirement is fulfilled and the total number of deployed nodes n is minimum. We study this problem both analytically and through simulation, under the assumption that $\mathcal{F}$ is the two-dimensional Normal distribution of parameter σ centered at the drop point. We show that, for a given value of σ and DoC requirement, optimal deployment strategies can be easily identified.

The sensor dropping problem studied in this paper is relevant whenever manual node deployment is impossible or overly expensive, and partially controlled deployment (the network designer can choose the drop points, but the final node deployment is random) is the only feasible choice.

1 Introduction

Wireless Sensor Networks (WSNs) are expected to revolutionize the way natural phenomena and human activities are monitored in the next few years. By connecting tiny, smart sensor nodes by means of wireless transceivers, large WSNs can be formed and used in several application scenarios: smart home environments, intrusion detection, tracking of animal movements/behaviors, large-scale environmental monitoring, and so on [4, 9, 12, 15, 16].

Environmental monitoring is one of the scenarios in which WSNs are expected to bring a breakthrough with respect to current technologies. WSNs can be used to monitor large-scale natural phenomena (e.g., ocean currents) as well as medium to small-scale phenomena (e.g., movement of snow masses, gas leaks in volcanic areas, and so on), providing a much better observation accuracy than that achieved nowadays. In turn, this increased observation accuracy will result in more accurate prediction of natural phenomena (weather forecast, avalanche prediction, and so on), and the benefits to the community will be considerable [16].

Typically, a WSN used for environmental monitoring is designed to provide a certain level of QoS, which essentially measures the degree of spatio/temporal accuracy required by the particular application[1]. In this paper, we focus on one such spatial measure, termed the *degree of coverage*, which evaluates the percentage of a given region sensed by a set of network-connected sensors. We will formally define this concept in Section 3. While the degree of coverage is just one of the QoS measures that one would like to consider (others are, e.g., delay, throughput and packet loss rate), it is our opinion that the approach taken in this paper can be suitably adopted to investigate other spatial and/or temporal parameters, such as the exposure (see Section 7).

The degree of coverage (hereafter, DoC), as well as other QoS measures provided by a sensor network, depends heavily on the infrastructure, i.e., on the number and positions of the sensors used to monitor the area. Thus, a fundamental step in the network planning stage is to define the node deployment strategy. The strategy used to deploy WSNs depends on the application considered: when the environment is sufficiently known and/or not hazardous, sensors can be placed manually in pre-determined positions. This strategy provides full control on the node placement, allowing a careful choice of the optimal network topology at the design stage. However, manual sensor placement is impossible in many situations. In fact, the cost of sending human or robot operators in the geographical region to be monitored can be enormous. In other cases, it can be physically impossible

[1] In many application scenarios, the WSN should be able to provide several levels of QoS depending on the events currently going on in the monitored region. However, at the network planning stage the WSN must be designed as if the most stringent QoS requirement were to be met.

to send the operator in the monitored area due to a highly hostile environment.

In situations where manual deployment is not feasible, a typical alternative is to drop sensors from a moving vehicle, such as an airplane or a helicopter. This deployment strategy results in a somewhat random node distribution, in which the human operator can only control the sensor drop point(s). Since the network designer has only a partial control on sensor positions, estimating the expected DoC provided by the deployed network becomes a fundamental and challenging task.

In this paper, we consider the following network planning problem. We are given a certain region R to be monitored and a certain DoC requirement to be fulfilled. Sensors are assumed to be dropped by a moving vehicle. The spatial distribution of the sensors released at a certain drop point is accurately modeled by a certain probability density function $\mathcal{F}$. The network designer is allowed to choose an arbitrary set of drop points within R, and to release an arbitrary number of sensors at each point. Given this setting, we consider the problem of determining the *optimal drop strategy*, i.e. the strategy such that the DoC requirement is met, and the total number of deployed nodes n is minimized. Since the problem in the most general form is intractable, we consider the sub-problem in which $\mathcal{F}$ is the Normal distribution of parameter σ and the drop points are arranged in grids of arbitrary side, and we investigate both analytically and through simulation the conditions under which a certain deployment strategy is optimal.

The rest of this paper is organized as follows. In Section 2, we review related work on deployment strategies for WSNs, and we describe the original contributions of our paper with respect to the existing literature. In Section 3, we introduce the system model, and in Section 4 we define the sensor dropping problem studied in the remainder of the paper. In Section 5, we formally analyze two restricted versions of the problem defined in Section 4, while in Section 6 we present a thorough experimental analysis of the problem in its more general form. Section 7 concludes.

2 Related work and contribution

Sensor networks deployment strategies have been investigated in several recent papers. A great deal of work has been devoted to study strategies for WSNs used for intrusion detection. In [10], Meguerdichian et al. introduce the concept of exposure of a WSN, which is a measure of how well an object, moving on an arbitrary path within the monitored area, can be observed by the sensor network over a period of time. The higher the exposure, the more likely it is to detect a target moving in the sensor field. In the paper, the authors analyze the exposure of several regular and random node deployments. The problem of computing the exposure provided by an arbitrary WSN has also been addressed [11, 18]. The problem of optimal sensor placement for target location in a grid-like sensor field has been investigated in [2]. The authors solve the problem using integer linear programming. Unfortunately, computing the optimal sensor placement incurs a huge computational overhead, which becomes prohibitive for networks composed by few tenths of nodes.

Several authors have also investigated multi-step deployment strategies, which are based on the idea of deploying groups of sensors incrementally, possibly using the information provided by the sensors already on the field to direct the deployment of the next group of nodes. This idea has been exploited in particular in the context of multi-robot exploration. In this scenario, sensors are mounted on mobile robots, and the goal of the robot team is to form a network with certain characteristics. For instance, in [6] Howard et al. propose an incremental deployment algorithm in which sensors (robots) are placed one at a time, and the location of the next sensor is calculated based on the information provided by the robots already on the field. Other proposals along this line of research are [7, 19]. The drawback of this approach is that node deployment is a very expensive and time consuming process. Furthermore, this strategy can be applied only in restricted application scenarios.

Another approach to multi-step node deployment suitable for clusterized WSNs has been proposed in [21]. In this approach, sensors are initially randomly deployed. After initial deployment, the sensors communicate their location and ID to the cluster-head. Given this information, the cluster-head executes an algorithm based on "virtual forces", which computes a new position for every node in the cluster. New positions are chosen in such a way that network coverage is maximized and node movements are minimized. Once the new positions have been computed and communicated to the nodes, a one-time movement is carried out, and the sensors are re-deployed to these positions. Although interesting, this approach is feasible only in a restricted set of application scenarios, since it is based on very demanding assumptions (location-awareness and autonomous sensor mobility).

A multi-step random deployment strategy is proposed also in [3]. The goal of the designer is to deploy a WSN that guarantees a certain level of exposure while minimizing the cost, which in turn depends on the number of deployed nodes and on the number of sensor deployments. A maximum of M sensors can be positioned on the field, but node positioning is a totally random process. The basic idea is simple: only part of the available sensors are deployed first, and the sensors report their position to the base station. Given node positions, the base station computes the network exposure, and verifies whether the desired level of exposure is achieved. If not, additional sensors are deployed, and the process is repeated until either the level of coverage is satisfactory, or all the sensors have been deployed. In the paper, the authors analyze the infrastructure cost for varying number of sensors deployed in each step.

Contrary to the approaches discussed so far, the emphasis in [17] is on routing protocols for sensor networks. In particular, the authors investigate the effect of infrastructure decisions (number of sensors and deployment strategy) on the performance of routing protocols such as DSR, DSDV and AODV. They consider different deployment strategies (grid-like and random), and several performance metrics. The main finding of the paper is that by simply deploying more sensors, network performance can be harmed. Thus, the network infrastructure must be carefully optimized to fulfill the application requirements.

The study presented in this paper differ from previous work in several ways. With respect to other studies, our model is less demanding on the sensors: in particular, we do not assume location-awareness nor autonomous mobility. Thus, our results can be applied in a wider range of scenarios. More importantly, ours is the first paper that addresses *partially controlled* network deployment: the network designer can choose the drop points but, once the sensors are dropped, their exact location cannot be controlled (random placement). This is in sharp contrast with the current literature, which considers only totally controlled or completely uncontrolled deployment strategies. While totally controlled deployment is feasible in some application scenarios, we believe that completely uncontrolled deployment is somewhat unrealistic and overly pessimistic. Our feeling is motivated by the fact that current investigations on random deployments are based on the assumption of uniform node distribution [3, 17, 21], which seems to be scarcely representative of real world scenarios. In our approach, we assume that nodes are concentrated around the drop points according to a two-dimensional Normal distribution, where the amount of this concentration is modeled by the parameter σ of the distribution (which we call the *dispersion factor*). The dispersion factor is very important, since it provides a handle that can be used to account for different environmental conditions. In turn, these conditions determine the degree of control the network designer has on the final node deployment: the higher the dispersion factor, the less control the designer has on the final node positions. The main finding of this paper is that, given a value of σ and a certain DoC requirement, optimal drop strategies can be easily identified.

Admittedly, assuming that nodes are Normally distributed around the drop points might be not realistic in some situations. However, in lack of a better model, the Normal distribution is a reasonable choice, especially if compared with previous approaches based on the assumption of uniform node distribution.

This paper also contributes a clean theoretical analysis of the infrastructure/DoC tradeoff for restricted cases of our sensor dropping problem. First of all, given a region R and a division of R into square cells, we consider dropping k sensors at the center of each cell vs dropping $4k$ sensors at the unique common point of four cells, and we investigate the probability that any of the four cells receives at least one sensor as a function of σ. We prove that there is a value $\bar{\sigma}$ such that, for $\sigma < \bar{\sigma}$ the 4 drops approach is better, while for $\sigma > \bar{\sigma}$ the 1 drop solution is to be preferred. We also characterize, under both scenarios, the expected number of sensors that must be dropped in order to have at least one sensor *in each cell*, finding a matching tradeoff result.

As a second contribution of our theoretical analysis, given a cell-division of R, we devise an upper bound on the number of sensors that must be dropped at each cell in order to have a given fraction of the cells covered with given target probability (i.e., to meet a certain DoC requirement), again as a function of the dispersion factor σ. The bound turns out to be tight in case σ is relatively small. Interestingly, as the experimental analysis reported in Section 6 has shown, this is exactly the case in which the strategy S turns out to be optimal amongst all possible strategies. Thus, our bound can be used in practice to determine the minimum number of sensors to be deployed to meet the DoC requirement when σ is relatively small.

Finally, the results presented in this paper give a clear understanding of the infrastructure/DoC/environmental conditions tradeoff. In particular, by extensive simulation we verify that the effect of an increased DoC on the infrastructure depends heavily on the environmental conditions: for small values of σ, node deployment can be controlled quite carefully, and relatively few sensors are sufficient to meet even very stringent DoC requirements. On the contrary, when the dispersion factor is quite high, a relatively modest increase in the DoC requirement results in a dramatic increase of the number of deployed nodes.

3 System model

We assume an application scenario in which sensors are dropped from a moving vehicle, e.g. an airplane or helicopter. The target application is environmental monitoring. The deployed network must satisfy a DoC requirement, which specifies the accuracy of the monitoring provided by the network. In particular, we define DoC as follows.

Let R be the geographical region to be monitored. R is divided into a number of non-overlapping sub-regions (also called *sensing cells*, or *cells* for short), which are used to determine the requirement on the sensing coverage provided by the network. In particular, we consider a cell to be *covered* if at least one sensor resides in the cell. A typical DoC requirement used in our analysis is: "at least q% of the cells are covered with a target probability p%".

Although our ideas can be applied to any deployment region R and for any subdivision of R into non-overlapping cells, in the following we will make two simplifying assumptions:

- R is a square region of side 1 Km;
- R is divided into equally-sized square cells of side s, for some parameter s.

We remark that these assumptions are made only to simplify the presentation of our results. Once properly modified, the techniques presented in this paper can be applied to arbitrary shapes of the deployment region and of the cells.

In our model, the value of s (and consequently, the total number of cells in R) determines the *sensing granularity* required by the particular application. For instance, if sensors are used to monitor temperature, a smaller value of s results in a finer temperature field generated by the network. Thus, parameter s can be seen as an alternative to the concept of sensing range (i.e., a sensor "covers" an area within a certain range around it), which is commonly used in the literature. We believe that our definition of "sensing granularity" captures better than the sensing range the DoC requirements that typically arise in WSNs used for environmental monitoring.

Besides network coverage, we are also interested in ensuring network *connectivity*, i.e. the ability of a node to communicate (possibly using multi-hop paths) with any other node in the network. This property is fundamental in order to guarantee that *any* sensor node can be used to acquire a global view of the monitored area. This way, a mobile user connected to any of the sensor nodes can obtain a complete view of R. In case the information generated by the WSN must be sent to a fixed base station, we must also guarantee connectivity between the base station and some of the network nodes. If the base station is within R, the problem of connecting the WSN to the base station can be reduced to the problem of ensuring connectivity in the original WSN with one additional node. This node, which lies in a fixed position, mimics the base station, and it could have a larger transmitting range with respect to sensor nodes. If the base station lies on the boundary of R, the problem of connecting the WSN to the base station can be easily solved by dropping few additional sensors in the sub-region of R which is closest to the base station. For these reasons, and for clarity of presentation, in this paper we focus our attention on ensuring connectivity of the WSN only.

The communication graph used to check for network connectivity is generated according to the following simple (but widely accepted) rule: there exists a bi-directional wireless link between sensors u and v if and only the sensors are at distance at most r, where r is the nodes' transmitting range. We assume that all the nodes have the same transmitting range.

Summarizing, our typical DoC requirement can be re-formulated as follows: "Given a certain sensing granularity s, require that at least q% of the cells are covered with a target probability p%, and that all the sensors in these cells form a unique connected component". For brevity, we denote this specific requirement with $DoC(s, q, p)$.

4 A sensor dropping problem

In this paper, we analyze the following sensor dropping problem:

Definition 1 (Sensor Dropping Problem (SDP)). *We are given a square geographical region $R = [0, 1]^2$ and a DoC requirement $DoC(s, q, p)$. Sensor nodes are dropped from a moving vehicle. We assume that the node spatial distribution generated when sensors are dropped at a certain point (x, y), with $0 \leq x, y \leq 1$, is accurately approximated by a probability density function (pdf) $\mathcal{F}$. Sensors can be dropped at arbitrary locations within R. Given R, $DoC(s, q, p)$ and $\mathcal{F}$, which is the optimal drop strategy, i.e., the strategy such that the DoC requirement is met and the total number of deployed sensors is minimum?*

Solving this problem in the most general formulation is virtually impossible, mainly because of the

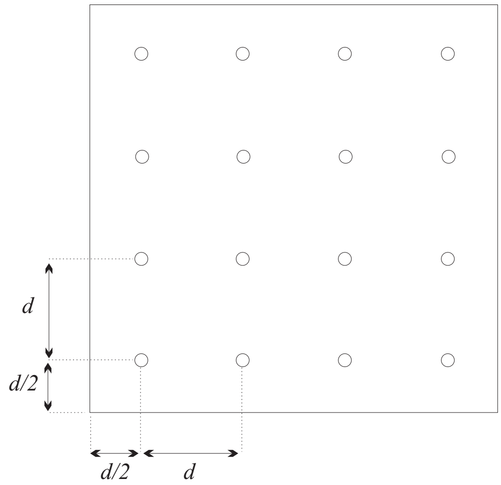

Figure 1: Grid strategy of parameter $d = 0.25$. Drop points are represented by circles. There are $4 \times 4 = 16$ drop points in total.

uncountable number of different drop strategies to consider. In order to render the problem manageable, we restrict the class of possible drop strategies to *grid strategies*, which we now define.

Definition 2 (Grid drop strategy). *The grid drop strategy of parameter d, with* $0 < d \leq 1$, *corresponds to releasing sensors at coordinates* $(\frac{d}{2} + i \cdot d, \frac{d}{2} + j \cdot d)$, *for* $i, j = 0, \ldots, \lfloor \frac{1}{d} \rfloor - 1$. *The total number of drop points in the grid strategy of parameter d is* $\lfloor \frac{1}{d} \rfloor^2$.

The grid strategy of parameter $d = 0.25$ is reported in Figure 1. Figure 2 shows the subdivision of R into cells when $s = 0.1$, and four different grid strategies.

In our analysis, we assume that $\mathcal{F}$ is the two-dimensional Normal distribution of parameter σ centered at the drop point. More formally, the pdf of the x-coordinate of a sensor dropped at point $(\bar{x}, \bar{y})$ is accurately approximated by $\mathcal{N}(\bar{x}, \sigma)$, while the y-coordinate is accurately approximated by $\mathcal{N}(\bar{y}, \sigma)$.

Parameter σ models the expected geographical dispersion of sensor nodes when dropped at a certain point. For this reason, σ is called the *dispersion factor*. For the sake of simplicity, we assume that σ does not depend on the particular drop point. In other words, we assume that the pdf that models the node spatial distribution is the same at each drop point. Finally, we assume that the dispersion along the x-axis is the same as the dispersion along the y-axis. Again, the goal of this assumption is only making the presentation of our results clearer. Our techniques can be easily extended to account for different dispersions along the two axes.

We are now ready to define the constrained version of SDP which will be analyzed in the reminder of the paper.

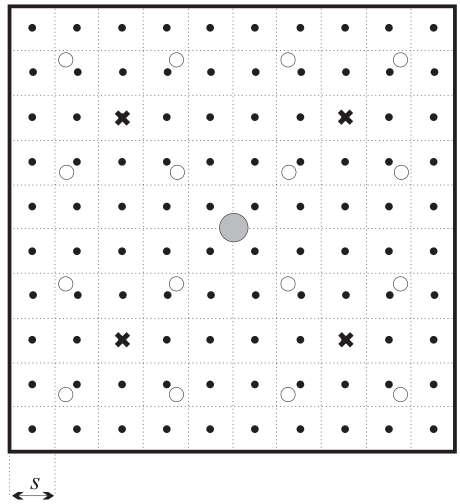

Figure 2: Subdivision of R into square cells when $s = 0.1$. Grid strategies of parameter $d = 0.1$ (black circle), $d = 0.25$ (white circle), $d = 0.5$ (cross) and $d = 1$ (shaded circle) are also shown.

Definition 3 (Grid Sensor Dropping Problem (GSDP)). *We are given a square geographical region* $R = [0, 1]^2$ *and a DoC requirement* $DoC(s, q, p)$. *Sensor nodes are dropped from a moving vehicle. We assume that the node spatial distribution generated when sensors are dropped at a certain point* (x, y), *with* $0 \leq x, y \leq 1$, *is accurately approximated by the two-dimensional Normal distribution of parameter* σ *centered at* (x, y). *Given R,* $DoC(s, q, p)$ *and* σ, *which is the optimal grid drop strategy, i.e., the grid strategy such that the DoC requirement is met and the total number of deployed sensors is minimum?*

5 Theoretical analysis

At first sight, the restricted version of the SDP introduced in the previous section could be investigated using occupancy theory, an important branch of Probability Theory (see, e.g., [8]). The classical occupancy model of randomly throwing n balls into N boxes has a number of well-known applications, from the classification of accidents according to the weekdays to the classification of the descendant of an individual according to the genotypes [5]. Occupancy theory has been used also in ad hoc networks, to investigate properties such as connectivity [14], and the performance of cell-based energy-conservation strategies [1] and of clustering protocols [20]. In our case, we can quite naturally view the sensing cells as the boxes and the sensors dropped as the balls thrown

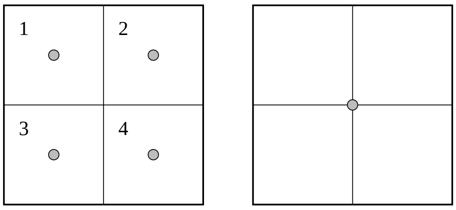

Figure 3: One vs four launches: the small circles indicates the (projection of the) point from which sensors are dropped.

into them[2]. However, there are some difficulties that prevent us from using the most studied occupancy models. Consider the launch of a sensor and suppose that p_i is the probability that it lands in cell i. Occupancy theory gives us models for the case of equiprobable allocations (which means that the probability of a ball going into box i is the same for all i, i.e., $p_i = \frac{1}{N}$, where N is the number of boxes) as well as nonequiprobable allocations (i.e., possibly $p_i \neq p_j$, for $i \neq j$). However, in our case the balls (sensors) are thrown from different places; this makes the elementary probabilities p_i also dependent on a further variable, say t, that can be interpreted as a time variable. In other words, we must replace p_i with $p_i(t), t = 1, 2, \ldots$, where $p_i(t)$ is the probability that the t-th sensor dropped goes into cell i. Moreover, the $p_i(t)$ are not given for free; in fact, each $p_i(t)$ has to be computed by integrating, over the area represented by cell i, the probability density function that governs the t-th launch. At the very least, for a large number of cells and many drop points, the computation of the basic probabilities $p_i(t)$ can be time consuming.

For the above mentioned reasons, we do not further pursue this line of attack; rather, we devise, using “special purpose” methods, analytical results that apply to two extremal instances of GSDP.

In the first scenario, we are faced with relatively large values of σ with respect to the sensing granularity. Under this circumstance, a simple yet interesting question that turns out to be analytically tractable is the following: is there any advantage from dropping sensors at the intersection of four cells with respect to dropping (a less number of) sensors over each of the four cells (see Figure 3)? In the second scenario we assume relatively small values of σ. In this case it is reasonable (as confirmed by the simulation results reported in Section 6) to drop a small number m of sensors over each cell. The problem we study here is to analytically bound m in order to have, say, 75% of the cells covered with probability greater than 0.95. We obtain this result by considering the sequence of launches as a sequence of independent trials (also known as *Bernoulli trials*), each of which has only two possible outcomes, success or failure, and bounding the tail probability of the number of empty cells by using one of the well-known Chernoff inequalities.

[2] The surface outside the deployment region R can be regarded as an additional box collecting the probability of a sensor not landing into R.

5.1 Four launches vs one launch

Consider a square region R of side $4u$ and suppose it is divided in four square cells of side $2u$, for some $u \geq 0$. We consider the following two experiments: (a) four sensors are dropped, one at the center of each cell, and (b) four sensors are dropped at the center of R (Figure 3). We compute the probability that cell 1 receives at least one sensor under (a) and (b); because of an easy symmetry argument, such probability is the same for all the cells. In all cases we assume that the pdf governing the dispersal of the sensors is Normal with mean centered at the drop point (which is always assumed to be at $(0, 0)$ without loss of generality) and standard deviation σ along both axes, denoted $N_{O,\sigma}$. We recall that, in polar coordinates,

$$N_{O,\sigma}(\rho,\theta) = \frac{\rho}{2\pi} \mathrm{e}^{-\frac{\rho^2}{2\sigma^2}}. \qquad (1)$$

Case (a). Assume the four cells are numbered 1 through 4 as in Figure 3. Let p_{ij} denote the probability that a sensor dropped at the center of cell i lands somewhere in cell j, $i, j = 1, \ldots, 4$, and let $q_{ij} = 1 - p_{ij}$. The probability p_1 that cell 1 receives at least one sensor is $p_1 = 1 - q_{11}q_{21}q_{31}q_{41}$. By symmetry, p_{ij} only depends on the relative positions of cells i and j. Hence, for instance, $p_{21} = p_{31} = p_{12}$ and $p_{41} = p_{14}$, and we are simply lead to compute p_{11}, p_{12}, and p_{14}. To this end, for $\theta \in \left[0, \frac{\pi}{4}\right]$ and $\rho \geq 0$, we define the following two functions:

$$C(\rho,\theta) \triangleq \int_0^{\theta} \int_0^{\frac{\rho}{\cos\theta}} N(\rho',\theta') \mathrm{d}\rho' \mathrm{d}\theta' = = \frac{\theta}{2\pi} - \frac{1}{2\pi} \int_0^{\theta} \mathrm{e}^{-\frac{\rho^2}{2\sigma^2 \cos^2\theta'}} \mathrm{d}\theta', \qquad (2)$$

and

$$S(\rho,\theta) \triangleq \int_{\theta}^{\frac{\pi}{4}} \int_0^{\frac{\rho}{\sin\theta}} N(\rho',\theta') \mathrm{d}\rho' \mathrm{d}\theta' = = \frac{1}{8} - \frac{\theta}{2\pi} - \frac{1}{2\pi} \int_{\theta}^{\frac{\pi}{4}} \mathrm{e}^{-\frac{\rho^2}{2\sigma^2 \sin^2\theta'}} \mathrm{d}\theta'. \qquad (3)$$

By definition, $C(\rho,\theta)$ (respectively, $S(\rho,\theta)$) is the probability that a sensor dropped at $(0, 0)$ lands somewhere in the triangle $(0,0),(\rho,0),(\rho,\rho\text{tg}\,\theta)$ (respectively, $(0,0),(\rho,\rho),(\rho\text{ctg}\,\theta,\rho)$), see Figure 4.

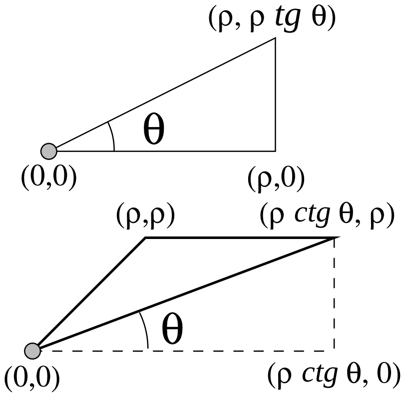

Figure 4: Integration domains within the definition of $C(\rho,\theta)$ and $S(\rho,\theta)$.

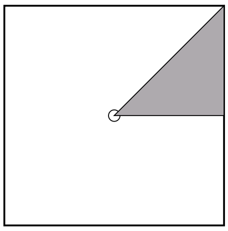

Figure 5: Area that receives $C(u,\frac{\pi}{4})$ of the total probability. Sides have length $2u$.

By combining $C(\rho,\theta)$ and $S(\rho,\theta)$, we can compute the probabilities p_{11}, p_{12}, and p_{14} as follows.

- $p_{11} = 8C(u,\frac{\pi}{4})$, where $C(u,\frac{\pi}{4})$ accounts for the probability over the area depicted in Figure 5.
- $\frac{1}{2}p_{12} = 2\left(C(3u,\bar{\theta}) + S(u,\bar{\theta}) - C(u,\frac{\pi}{4})\right)$, where $\bar{\theta} = \text{arcos}\left(\sqrt{10}/3\right)$, by the area decomposition suggested in Figure 6.
- $p_{14} = 2\left(C(3u,\frac{\pi}{4}) - C(u,\frac{\pi}{4})) - p_{12}\right)$. In fact, p_{14} can be computed by following the area decomposition suggested in Figure 7.

Substituting the single pieces into $p_1 = 1 - q_{11}(q_{21})^2 q_{41}$ and doing the algebra, we get:

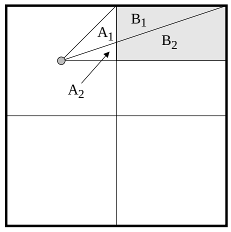

Figure 6: The probability over the light gray shaded area (B_1, B_2) is computed as the probability over (A_1, B_1) $(S(u,\bar{\theta}))$ plus the probability over (A_2, B_2) $(C(3u,\bar{\theta}))$ minus the probability over (A_1, A_2) $(C(u,\frac{\pi}{4}))$.

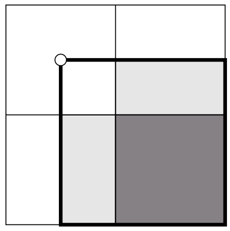

Figure 7: The probability over the dark gray shaded area is computed as the probability over the square with thick sides $(2C(3u,\frac{\pi}{4}))$ minus the probability over the light gray shaded areas (p_{12}) minus the probability over the inner white square $(2C(u,\frac{\pi}{4}))$.

$$p_1 = 1 - \left(\frac{4}{\pi}\int_0^{\frac{\pi}{4}} e^{-\frac{u^2}{2\sigma^2\cos^2\theta}}\,d\theta\right)\cdot \left(1 - \frac{1}{\pi}\left(\int_0^{\frac{\pi}{4}} e^{-\frac{u^2}{2\sigma^2\cos^2\theta}}\,d\theta - \int_0^{\bar{\theta}} e^{-\frac{9u^2}{2\sigma^2\cos^2\theta}}\,d\theta + - \int_{\bar{\theta}}^{\frac{\pi}{4}} e^{-\frac{u^2}{2\sigma^2\sin^2\theta}}\,d\theta\right)\right)^2 \cdot \left(1 - \frac{1}{\pi}\left(\int_{\bar{\theta}}^{\frac{\pi}{4}} e^{-\frac{u^2}{2\sigma^2\sin^2\theta}}\,d\theta - \int_{\bar{\theta}}^{\frac{\pi}{4}} e^{-\frac{9u^2}{2\sigma^2\cos^2\theta}}\,d\theta\right)\right)$$

Case (b). Suppose we drop four sensors at the center of R. The probability the any of these goes into cell 1 is $\hat{p}_1 = 1 - (\hat{q}_1)^4$, where $\hat{q}_1$ is the probability that any given sensor does not land in cell 1. In turn, $\hat{q}_1 = 1 - 2C(2u,\frac{\pi}{4})$, so that (after simple algebra) we

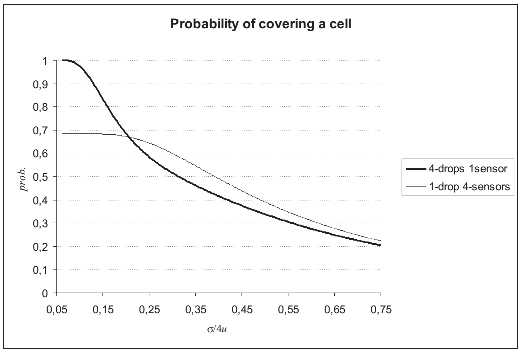

Figure 8: Probability of having at least one sensor in cell 1 under cases (a) and (b) as a function of $\frac{\sigma}{4u}$.

obtain

$$\hat{p}_1 = 1 - \left(\frac{3}{4} + \frac{1}{\pi} \int_0^{\frac{\pi}{4}} \mathrm{e}^{-\frac{u^2}{2\sigma^2 \cos^2 \theta}} \mathrm{d}\theta \right)^4 .$$

Comparison. In Figure 8 we report the probability of cell 1 receiving at least one sensor in the cases (a) and (b) discussed above. The probability is plotted against the ratio $\frac{\sigma}{4u}$, i.e., the dispersion factor is normalized with respect to the side of the deployment region. Clearly, for symmetry reasons, this is also the probability that any other cell receive at least one sensor. The plot confirms the intuition that there must be a value of $\frac{\sigma}{4u}$ (approximately 0.206) beyond which the probability of having at least a sensor in a given cell with the single launch option is higher.

Let us now consider the expected number of sensors that must be dropped in cases (a) and (b) in order to have at least one sensor *in each cell.* We recall that, for independent trials (i.e., launches), each one having the same success probability p, the number of attempts that must be made before the first success occurs is a geometric random variable, i.e., the first success occurs at trial i with probability $p(1-p)^{i-1}$. The expectation of a geometric random variable of parameter p is $\frac{1}{p}$.

Let us start with scenario (a). In this scenario, the random experiment is the launch of four sensors, one for each of the four drop points. Let X_i, for $i = 1, \ldots, 4$, be the random variable that denotes the minimum number of experiments (each dropping four sensors) after which cell i contains at least one sensor. It is immediate that X_i has geometric distribution of parameter p_1, where p_1 is defined as above. Note that the X_is are not independent, since the fact that a certain cell is occupied (or empty) has the effect of decreasing (or increasing) the probability that another cell is occupied. Nevertheless, in the following we derive a formula for the expected number of sensors to drop under the assumption that the X_is are independent, and we show by simulation that the approximation committed is negligible.

Let C be the random variable that denotes the minimum number of experiments after which *all* the cells are covered with at least one sensor. It is immediate that $C = \max_{i=1,\ldots,4} X_i$. We derive the cumulative distribution function of C as follows:

$$Prob(C < k) = 1 - Prob(C \geq k) =$$

$$1 - Prob(\exists i : X_i \geq k) = 1 - (1 - Prob(\forall i : X_i < k))$$

Assuming independence of the X_is, we can write:

$$Prob(C < k) \approx \prod_{i=1}^{4} Prob(X_i < k) .$$

Since the X_is are geometric random variables of parameter p_1, we have:

$$Prob(C < k) \approx \left(\sum_{j=1}^{k-1} p_1 (1-p_1)^{j-1} \right)^4 . \quad (4)$$

Using the cumulative distribution function, we can compute the probability that C equals k as follows:

$$Prob(C = k) = Prob(C < k+1) - Prob(C < k) .$$

It follows that the expectation of the random variable C is:

$$\mathrm{E}[C] \approx \sum_{k=1}^{+\infty} k \cdot Prob(C = k) .$$

Plugging equation (4) into the expression above and simplifying the resulting formula (we have used Mathematica™), we obtain:

$$\mathrm{E}[C] \approx \frac{-25 + 69p_1 - 85p_1^2 + 58p_1^3 - 22p_1^4 + 4p_1^5}{p_1(-2+p_1)(3-3p_1+p_1^2)(2-2p_1+p_1^2)} . \quad (5)$$

Note that the value above must be multiplied by four to obtain the number of expected sensors to drop, since each experiment in this scenario consists of dropping four sensors. So, $\mathrm{E}[X_a] = 4 \cdot \mathrm{E}[C]$, where X_a is the random variable that denotes the number of sensors to be dropped in scenario (a) to have every cell covered.

To verify the quality of the approximation committed in formula (5), we have simulated scenario (a), generating a large number of data sets. As seen from Figure 9, our formula is a very good approximation of $\mathrm{E}[C]$.

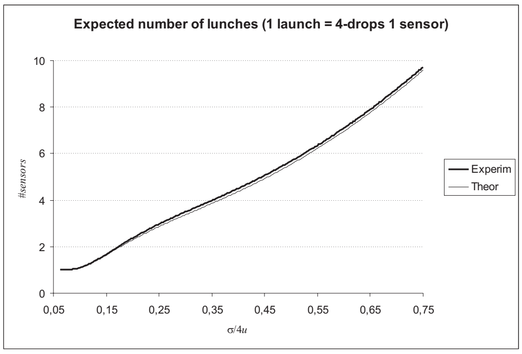

Figure 9: Expected number of launches to have one sensor in each cell under scenario (a) with our formula (Theor) and as resulting from simulations (Exper), as a function of $\frac{\sigma}{4u}$.

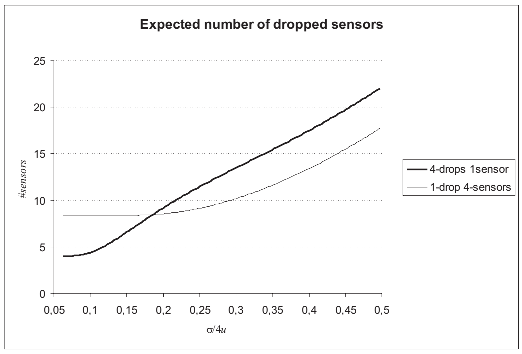

Figure 10: Expected number of dropped sensors to have one sensor in each cell under cases (a) and (b) as a function of $\frac{\sigma}{4u}$.

Turning to scenario (b), we observe that it is not necessary always to drop $4k$ sensors (for some $k > 0$), and a finer analysis is required. The problem can be viewed as a coupon collecting task (see, e.g., [13]): there are four coupons (the four cells) that we want to collect, called *valid* coupons, and one which we are not interested in, corresponding to the region outside R. Actually, we may assume that the latter is a fifth coupon that we have already collected. The number of attempts that we have to make before collecting all the valid coupons is the sum of the expected values of four geometric variables, X_1 through X_4, characterized by decreasing probability of success. More precisely, let p the probability of collecting any of the four valid coupons during one attempt. Note that in our case $4p < 1$, since the coupon we are not interested in has nonzero probability of occurrence. For the first attempt we are happy if we get any of the four valid coupons, and this happens with probability $4p$. For the second attempt, we look for one of only three valid coupons, with corresponding total probability $3p$. By iterating the reasoning it is not difficult to see that the expected number of attempts before collecting all the valid coupons under scenario (b) is

$$\begin{aligned} \mathrm{E}[X_b] &= \mathrm{E}[X_1 + X_2 + X_3 + X_4] = \sum_{i=1}^{4} \mathrm{E}[X_i] \\ &= \sum_{i=1}^{4} \frac{1}{ip} = \frac{25}{12p}, \end{aligned} \tag{6}$$

where $p = 1 - \hat{q}_1$.

Figure 10 shows the plot of the expected values $\mathrm{E}[X_a]$ and $\mathrm{E}[X_b]$. Since $\mathrm{E}[X_a]$ and $\mathrm{E}[X_b]$ are not integers, in general, we also report the rounded values in Table 1, whose entries are computed from the expected values as follows: under scenario (b) the nearest largest integer; under scenario (a) the nearest largest integer divisible by 4. If you consider two consecutive rows of Table 1, say rows i and $i+1$, then row $i+1$ gives the number of sensors to be dropped when $\frac{\sigma}{4u}$ is strictly greater than the value in row i and not greater than the value in row $i+1$.

5.2 An upper bound on the minimum number of deployed sensors

In this section, we assume that a launch is done over the center of every cell (i.e., $d = s$), and that σ is small with respect to the side of the deployment region R. Under these hypotheses, we provide an upper bound on the minimum number of nodes that must be deployed in order to meet a certain DoC requirement. As shown by the simulation results reported in the next section, when σ is relatively small the deployment strategy which drops sensors at the center of each cell is optimal amongst all possible grid drop strategies. Thus, if the dispersion factor is relatively small the probabilistic bound presented in this section can be used in practice to estimate the minimum number of nodes that must be dropped with *any* grid drop strategy in order to meet a certain DoC requirement.

Under the stated assumptions, the probability that a sensor dropped over cell C_i lands somewhere in C_i is high. Also, it is reasonable to expect that the contributions of "mislanded" sensors (i.e., sensors dropped over C_j that lands in C_i, for $i \neq j$) to the probability that a cell is not empty be small. Ignoring such contributions leads us to regard the N launches (one for each cell) as a sequence of independent Bernoulli trials; as a consequence, we can use known techniques to bound the tail probability of the sum of Bernoulli variables to bound the number of empty cells. Note that, ignoring the effects of the mislanded sensors may only lead to a less tight bound; however, the

$\sigma/4u$	4-drops	1-drop
< 0.175	8	9
0.24	12	10
0.2625	12	10
0.29	16	10
0.3275	16	11
0.36	16	12
0.3625	16	12
0.3875	20	13
0.415	20	14
0.4375	20	15
0.455	20	15
0.46	24	16
0.48	24	17
0.4975	24	18

Table 1: Expected number of sensors to be dropped for having one sensor in each cell under (a) and (b).

result is perfectly valid (in the probabilistic sense).

Assume then that $m \geq 1$ sensors are dropped over any cell, and define

$$X_i = \begin{cases} 1 & \text{if no sensor lands in } C_i \\ 0 & \text{if at least one sensor lands in } C_i \end{cases} \quad (7)$$

where the sensors considered in (7) are only those dropped over C_i. Let $p = p(m) = \text{prob}[X_i = 1]$. Using equation (2) in Section 5.1, we can write

$$p = \left(1 - 8C\left(u, \frac{\pi}{4}\right)\right)^m = \left(\frac{4}{\pi}\int_0^{\frac{\pi}{4}} \mathrm{e}^{-\frac{u^2}{2\sigma^2\cos^2\theta}}\mathrm{d}\theta\right)^m.$$

Let $X = \sum_{i=1}^N X_i$, where N is the number of cells, and let $\mu = E[X]$. Clearly, $E[X_i] = p$ and, by linearity of expectation, $E[X] = \sum_{i=1}^N E[X_i] = Np$. We recall that our goal is to devise a probabilistic "guarantee" on the number of empty cells. For definiteness, suppose that we want at most 25% of the cells uncovered with probability at least 0.95. We can use the following Chernoff's bound (see, e.g., [13]):

$$\text{prob}[X > (1+\delta)\mu] < \left[\frac{\mathrm{e}^\delta}{(1+\delta)^{1+\delta}}\right]^\mu, \quad (8)$$

for any $\delta > 0$. Equating $(1+\delta)\mu = 0.25N$, we get $\delta = \frac{1}{4p} - 1$. Substituting the values of δ and μ into (8) and bounding the right hand side with 0.05, we get

$$\left[\frac{\mathrm{e}^{\frac{1}{4p}-1}}{\left(\frac{1}{4p}\right)^{\frac{1}{4p}}}\right]^{Np} < 0.05,$$

σ	p_{64}	p_{100}	m_{64}	m_{100}
0.01	8e-10	1e-06	0.1229	0.1832
0.015	6e-05	0.0017	0.2655	0.3935
0.02	0.0036	0.0247	0.4563	0.677
0.025	0.0247	0.0889	0.6953	1.0356
0.03	0.0731	0.182	0.9836	1.4711
0.035	0.1428	0.2828	1.3222	1.9843
0.04	0.2224	0.378	1.7118	2.5757
0.045	0.3026	0.462	2.1527	3.2455
0.05	0.378	0.5339	2.645	3.9939
0.055	0.4462	0.5946	3.1888	4.821
0.06	0.5066	0.6456	3.7843	5.7266
0.065	0.5595	0.6884	4.4314	6.711
0.07	0.6055	0.7244	5.1302	7.774
0.075	0.6456	0.755	5.8806	8.9157
0.08	0.6804	0.7809	6.6828	10.136
0.085	0.7107	0.8032	7.5367	11.435
0.09	0.7372	0.8224	8.4424	12.813
0.095	0.7605	0.8389	9.3997	14.27
0.1	0.7809	0.8534	10.409	15.805

Table 2: Upper bound on the number of sensors to be dropped over any cell to have at least 75% of the cells covered with probability at least 0.95, in case of 8×8 and 10×10 grids.

and, after simple algebra,

$$p + \frac{1}{4}\ln\frac{1}{p} > \frac{\ln 20}{N} + \frac{1}{4}(1 + \ln 4). \quad (9)$$

For small p, we can ignore the term p in (9). Finally, substituting $p = \left(1 - 8C\left(u, \frac{\pi}{4}\right)\right)^m$ into (9) and solving for m, we get

$$m > \frac{1}{-\ln\left(1 - 8C(u, \frac{\pi}{4})\right)}\left(\frac{4}{N}\ln 20 + 1 + \ln 4\right). \quad (10)$$

In Table 2 we show some computed bounds for the cases of a 8×8 and a 10×10 grid. The first column gives the values of the standard deviation σ (as in the previous section, we are interested in the value of σ relative to the side of the region R; however, the latter is assumed to be 1 here). Columns 2 and 3 contain $p = p(1) = prob[X_i = 1]$ for the two cases of 8×8 and 10×10 cells. Finally, the last two columns report the right-hand side of (10) for the two grids considered.

We have verified the accuracy of our bound through simulation, computing the minimum number of nodes to be deployed in order to meet the DoC requirement (i.e., at least 75% of cells covered with probability at least 95%) for different values of σ. The results of our simulations, which are shown in Figure 11, show that the bound reported in equation (10), once properly rounded to the least larger integer, is a very

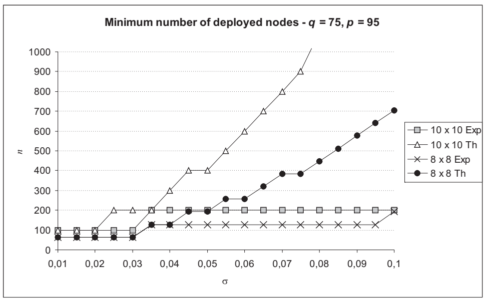

Figure 11: Minimum number of deployed sensors in order to cover at least 75% of the cells with probability at least 95% for different values of σ and of the sensing cell side. The graphic reports both the experimental values (Exp) and the values obtained using equation (10) (Th).

accurate estimation of the number of requested nodes for values of σ up to 0.04–0.05. For larger values of the dispersion factor, the approximations used in the computation of the bound (i.e., disregarding the contribution of "mislanded" sensors, and the use of the Chernoff's formula) become too large.

6 Experimental analysis

In this section we report the results of the extensive simulations performed to investigate GSDP.

The simulator that we have designed has the following input parameters:

- the required sensing granularity s, with $0 < s \leq 1$;
- the parameter d of the grid drop strategy, with $0 < d \leq 1$;
- the transmitting range r;
- the dispersion factor σ;
- the number of sensors n_d to be dropped at the single drop point. The total number of deployed sensors is then $n = n_d \cdot \lfloor \frac{1}{d} \rfloor^2$.

In a single experiment, n_d sensors are dropped at each drop point according to the Normal distribution of parameter σ. Once all the n sensors have been deployed, the communication graph G is generated by connecting with an edge all the pairs of nodes that are within distance r. Then, the largest connected component LCC_G of G is calculated. Finally, the fraction of sensing cells covered by at least one node in LCC_G is computed and recorded in the output file.

Note that, since node positions are generated according to a pdf with unbounded support, sensors may lay outside R. These "out of bound" nodes are useless for the purpose of coverage, but can be used as bridges to increase network connectivity.

Parameter	Settings
s	1/8, 1/9, 1/10
d	1, 1/2, 1/3,...,1/10
r	0.15, 0.25
σ	from 0.005 to 0.05 in steps of 0.0025
	from 0.06 to 0.2 in steps of 0.01
	from 0.225 to 0.5 in steps of 0.025
n_d	from 1 to 1000

Table 3: Settings of the input parameters used in our experiments. The values of s, d, r and σ are expressed in Kilometers, i.e. they are normalized with respect to the side of the deployment region R.

In our simulations, we have considered about 49000 different settings of the input parameters and, for each setting, we have performed a number of experiments varying from 250 to 1000. The settings of the various parameters used in our simulations are shown in Table 3.

An observation concerning n_d is in order. In order to keep the simulation time reasonable, we have imposed an upper bound of 1000 to the total number n of deployed nodes. Thus, depending on the value of d, the maximum value of n_d considered is much smaller than 1000. For instance, when $d = 0.1$ the maximum value of n_d considered in our experiments is 10.

6.1 Optimal drop grid size

In the first set of experiments, we have evaluated the optimal grid size as the dispersion factor σ increases. Given the value of σ, of r, and a certain DoC requirement $DoC(s, q, p)$, we have calculated the value of d corresponding to the optimal drop strategy, i.e. the value of d such that $DoC(s, q, p)$ is satisfied and n is minimum. This calculation has been performed by post-processing the huge amount of output data generated by our experiments.

The simulation results for $r = 0.15$, $s = 0.1$, $p = 95$, and different values of q are reported in Figure 12. Figure 13 reports the same results when $s = \frac{1}{8} = 0.125$. In the figures, 75-95 (and the other plots) must be interpreted as follows: in at least 95% of the cases, at least 75% of the cells is covered by at least one sensor in LCC_G.

From figures 12 and 13 it is seen that as the dispersion factor σ increases, the size of the optimal grid strategy increases as well. When σ is around 0.3 or above, the optimal drop strategy is to drop all the sensor at the center of the deployment region R, independently of the DoC requirement. Note that in

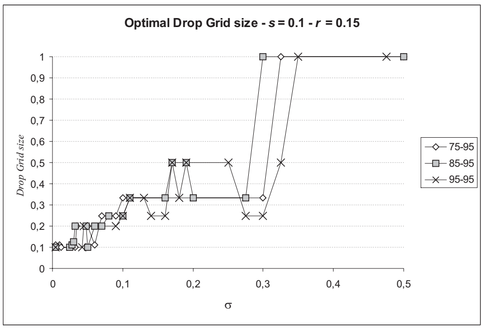

Figure 12: Optimal grid size for increasing values of σ. The sensing granularity is $s = 0.1$, and the transmitting range is $r = 0.15$.

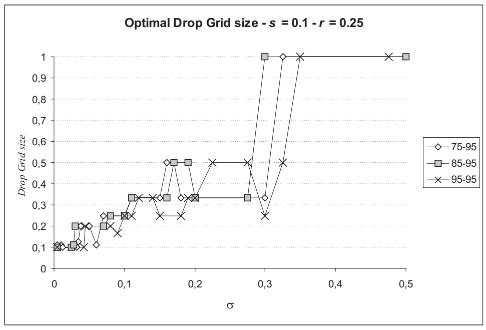

Figure 14: Optimal grid size for increasing values of σ. The sensing granularity is $s = 0.1$, and the transmitting range is $r = 0.25$.

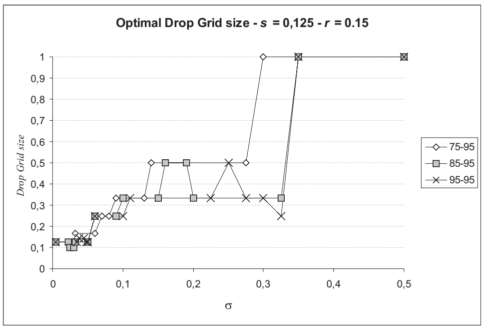

Figure 13: Optimal grid size for increasing values of σ. The sensing granularity is $s = 0.125$, and the transmitting range is $r = 0.15$.

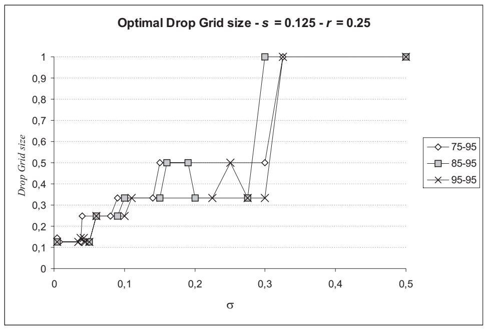

Figure 15: Optimal grid size for increasing values of σ. The sensing granularity is $s = 0.125$, and the transmitting range is $r = 0.25$.

this case it is likely that the uncovered cells reside on the boundary of R. On the other hand, when the expected dispersion is very small ($\sigma = 0.03$ and below), the grid strategy of minimum size 0.1 is always optimal. The effect of the DoC requirement on the optimal grid size is only marginal.

In order to evaluate the effect of the transmitting range on the optimal grid size, we have repeated the same set of experiments setting r to 0.25. The results are shown in figures 14 and 15. As it is seen from the figures, the effect of the larger transmitting range on the simulation results is negligible. This means that, with the DoC requirements used in our experiments, a transmitting range of 0.15 is already sufficient to provide connectivity on the average.

6.2 Number of deployed nodes

In the second set of experiments, we have evaluated how the total number of deployed sensors varies as a function of the dispersion factor σ. We have considered four different grid drop strategies ($d = s$, $d = \frac{1}{6} = 0.166$, $d = \frac{1}{3} = 0.333$ and $d = 1$), with various DoC requirements. The transmitting range is set to 0.15. We have repeated the experiments setting r to 0.25, obtaining virtually indistinguishable results. For this reason, we only report the results obtained with $r = 0.15$.

The results of our experiments for different *DoC* requirements are reported in figures 16–19. From the graphics it is seen that:

- for any of the DoC requirements considered, the behavior of n as σ increases depends on the drop strategy: if the drop grid is very fine ($d = s$), we have a linear increase of n with σ (up to the discretization induced on n by the fact that n_d is an integer). For relatively coarser grids, n decreases with σ quite sharply initially; then, it increases with a relatively modest slope. When the dispersion factor is large enough (σ around 0.35 and above), n increases with σ independently of the grid size. This fact is quite intuitive: as the dispersion factor increases, the node deployment becomes more and more difficult to control, and the total number of sensors to be deployed to meet the DoC constraint increases.
- for any of the DoC requirements considered, we

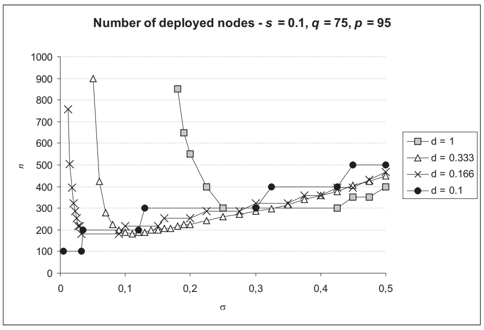

Figure 16: Total number of deployed sensors with increasing values of σ and different grid strategies. The DoC requirement to be met is $DoC(0.1, 75, 95)$.

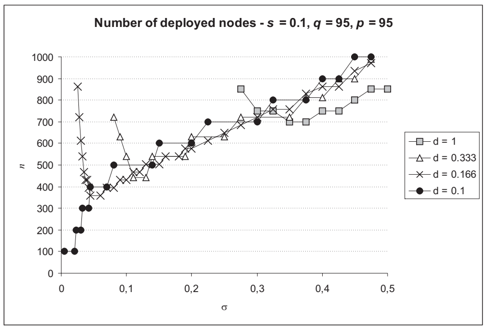

Figure 17: Total number of deployed sensors with increasing values of σ and different grid strategies. The DoC requirement to be met is $DoC(0.1, 95, 95)$.

can distinguish three regimes in the plots: for small values of σ (σ around 0.1 or below), dropping the sensors using the finest grid ($d = s$) is a good choice, that often minimizes n. For large values of σ (σ around 0.3 and above), the best option is always to deploy sensors using a unique drop point located at the center of R. For intermediate values of σ, setting $d = 0.333$ seems to be the best choice.

– the DoC requirement has a dramatic effect on n. For instance, when $\sigma = 0.1$, less than 200 nodes are sufficient to satisfy $DoC(0.1, 75, 95)$, while more than 400 nodes are necessary to satisfy $DoC(0.1, 95, 95)$. In words, increasing the required percentage of covered cells by 20% results in a more than 100% increase in the number of deployed sensors. Changing the constraint on s has a less significant impact on n: when $\sigma = 0.1$ and the requirement is $DoC(0.125, 75, 95)$, the number of nodes to be deployed is around 125. Thus, a 25% increase in s results in a decrease in the order of 37% of the number of deployed sensors (we recall that a larger value of s results in a less stringent DoC requirement). Note that the situation is very different when σ is very small (σ below 0.025); in this case, up to 200 nodes are sufficient to satisfy both $DoC(0.1, 75, 95)$ and $DoC(0.1, 95, 95)$. This is due to the fact that with these values of σ node deployment can be accurately controlled, and dropping 1 or 2 sensors at the center of every sensing cell is sufficient to meet basically all the DoC requirements.

Before concluding this section, we present sample sensor deployments that help us clarifying why the apparently trivial strategy of dropping all the sensors at the center of R is optimal for large values of σ.

We consider the two extreme drop strategies: having a drop point in the center of every sensing cell ($d = s$), and having a unique drop point at the center of R. Figure 20 shows sample deployments obtained with the two dropping strategies when $\sigma = 0.04$ with $n = 200$ nodes. Figure 21 shows similar samples when $\sigma = 0.4$ and $n = 300$. In case of small σ, the finer drop strategy results in a far better coverage: 94% of the cells are covered, as compared to only 10% covered cells when $d = 1$. The situation is reversed when $\sigma = 0.4$: in this case, the cell coverage is 70% with $d = s$ and 82% with $d = 1$. The better coverage of the coarser drop strategy is due to the fact that when all the sensor are dropped at the center of deployment region, less sensors are expected to fall outside the boundaries of R: the number of "out of bound" sensors is 56.7% when $d = s$, and only 35% when $d = 1$.

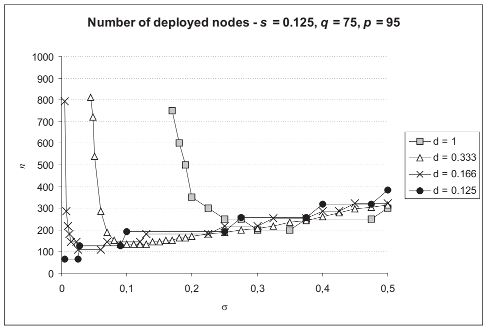

Figure 18: Total number of deployed sensors with increasing values of σ and different grid strategies. The DoC requirement to be met is $DoC(0.125, 75, 95)$.

7 Final remarks

In this paper we have investigated in detail a wireless sensors dropping problem that can arise when the WSN is used for environmental monitoring. By introducing the concept of sensing granularity, we have

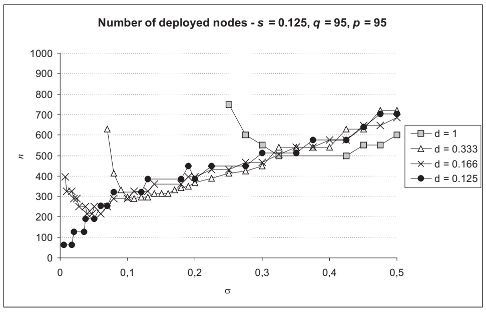

Figure 19: Total number of deployed sensors with increasing values of σ and different grid strategies. The DoC requirement to be met is $DoC(0.125, 95, 95)$.

considered a target spatial DoC requirement, and studied both theoretically and experimentally the relation between the environmental conditions that influence node dispersal (the dispersion factor) and the optimal deployment strategy.

We believe that one of the main contributions of this paper is the definition of a simple yet powerful methodology to tackle the problem of optimal WSN deployment: 1) define a quantity that can be used to measure QoS (in our case, sensing coverage) 2) define a pdf that models the node dispersal at the single drop point (the Normal distribution in this paper) as a function of the environmental conditions (modeled by the dispersion factor in our study) 3) define a reasonable set of drop strategies (grid drop strategies in our case) 4) once the above parameters are set, the optimal deployment strategy can be identified.

The methodology described above can be used to study node deployment strategies in other application scenarios, such as target detection. For instance, it is clear that sensing granularity and exposure (see [18]) are related measures : the finer the granularity, the easier it is for the WSN to detect a target moving within the monitored region. An in-depth investigation of the relation between these two important QoS measures is the subject of ongoing studies.

Another way to extend the work presented in this paper is to analyze sensor deployment problems in which the QoS requirement accounts also for *temporal* accuracy, such as the latency with which the events are reported to the base station. We are currently working on this topic, and on the related topic of deploying a network with a minimal guarantee on the operational lifetime. We believe that a viable approach to tackle these more complex problems is to combine our methodology with techniques used in the analysis of topology control, clustering and energy-conserving protocols.

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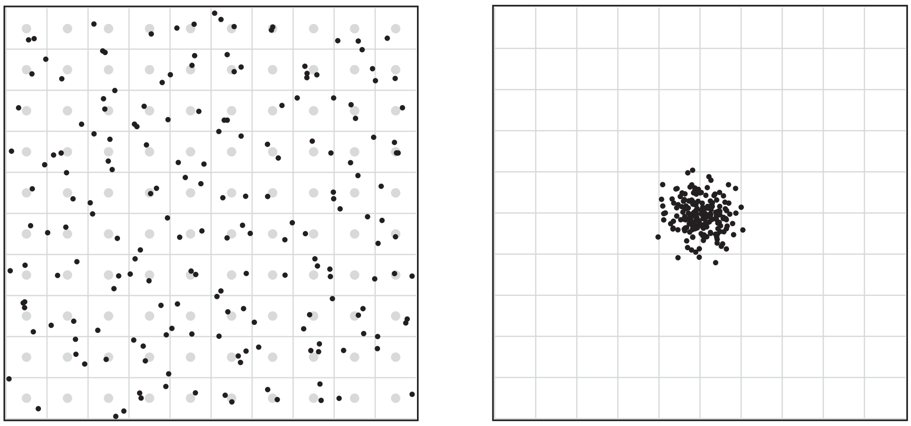

Figure 20: Sample sensor deployments when $\sigma = 0.04$ with different dropping strategies: $d = s$ (left) and $d = 1$ (right). Black circles represent sensors, and shaded circles represent the drop points. Only sensors that lay within R are shown.

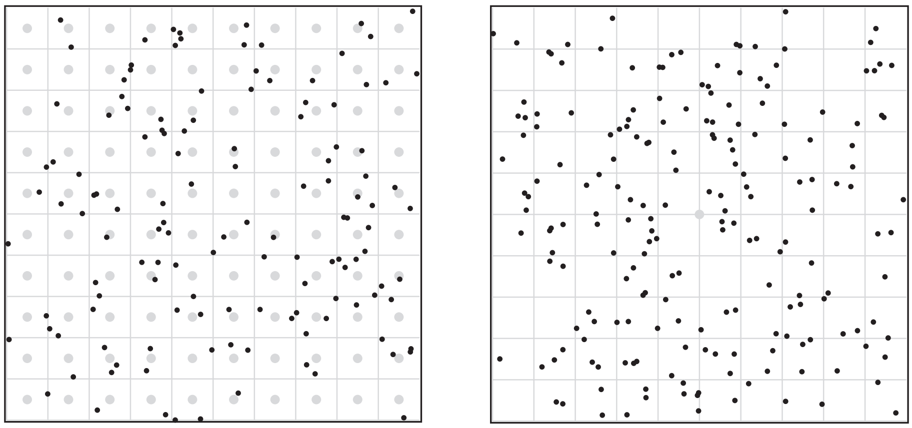

Figure 21: Sample sensor deployments when $\sigma = 0.4$ with different dropping strategies: $d = s$ (left) and $d = 1$ (right). Black circles represent sensors, and shaded circles represent the drop points. Only sensors that lay within R are shown.

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