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Abstract— **In this paper, we consider the accuracy of bounded physical interference models for use in multi-hop wireless networks. In these models, physical interference is accounted for but only for a subset of nodes around each receiver, and interference from farther transmitters is ignored. These models are very often used, both in theoretical analyses and simulations, with an "interference range" that defines the distance from a receiver beyond which interference is ignored. In this paper, we prove that, if the interference range is chosen as any unbounded increasing function of the number of nodes in the network, the total ignored interference converges to zero as the number of nodes approaches infinity. This result is proven under both constant node density and uniform random node distribution assumptions. We also prove that, if the interference range is considered to be a constant, e.g. a multiple of the transmission range, the total ignored interference does not converge to zero and, in fact, can be several orders of magnitude greater than the noise for networks of moderate size. The theoretical results are enhanced by simulations, which evaluate the bounded models relative to the true physical interference model and demonstrate, empirically, that slowly increasing interference ranges are necessary and sufficient to achieve good accuracy. Our results also demonstrate that a scheduling algorithm that considers a fixed interference range will produce schedules with a very high percentage of failing transmissions, which would have substantial negative impacts on performance and fairness in such networks.**

I. INTRODUCTION

Multi-hop wireless networks are now used in many different contexts, from mobile ad hoc networks to wireless sensor networks to wireless mesh networks. In such networks, spatial reuse of a wireless communication channel, in which two links that are sufficiently far apart are used concurrently in the same channel, is possible. Maximizing spatial reuse is highly desirable in order to try to maximize achievable throughput in the network. This is particularly the case, for example, in wireless mesh networks where typical applications demand high performance and throughput optimization is critical. Unfortunately, the amount of spatial reuse that can be exploited is strictly limited by interference that is generated when multiple simultaneous transmissions occur in the same channel.

Wireless interference has been traditionally modeled in terms of the communication graph, where nodes within 1 or 2 hops from a communicating link interfere with its transmission, and thus must be silent. Later, a somewhat more realistic model of interference, called the *protocol model* [7], has been considered. Here, a geometric notion of *interference range* is defined — the receiver of a link must not be within the interference range of another concurrent transmitter. These models suffer from the limitations that (i) interference is considered in a 'binary' sense (interference either totally eliminates the ability to communicate or is non-existent), and (ii) interference is considered only between *pairs* of nodes or links. In reality, wireless interference is much more complex. Whether a communication is successful depends on whether signal power exceeds the sum of the interference powers plus noise by a threshold that is a property of the physical layer radio design. This SINR (signal to interference plus noise ratio)-based model is known as the *physical interference model* [7]. The complexity here is that interference is neither binary nor pairwise; aggregated interference from all communicating nodes must be considered to decide whether a communication is successful. Theory aside, recent performance studies with 802.11-based mesh networks also demonstrate that multiple interferers must be considered to evaluate interference limited capacity of a link [4].

While physical interference models that account for all possible transmissions throughout the network are the most accurate, such models are very complex. This complexity manifests itself in several ways, e.g. NP-hardness of many important problems when considering physical interference [18] and use of approximated physical interference models in wireless network simulations [13], [16] to prevent simulation times from being slowed down by large factors. It is precisely the type of approximated physical interference model that is used in packet-level wireless network simulators that is the focus of our study in this paper. The models used in the simulators can be considered a hybrid between physical and protocol interference models, where physical interference is considered but only within a bounded range of a receiver (similar to the interference range assumption in the protocol interference model).

In what follows, we formalize bounded physical interference models and do a thorough study of their benefits and limitations with respect to true physical interference modeling. To be more specific, we consider models where physical interference from nodes within a circular region surrounding a receiver is accounted for, but interference outside this region is ignored. The models are parameterized by the size of this "close-in" region, which we refer to as the close-in radius. We prove that, if the close-in radius is set to be a constant (independent of the number of nodes n in the network), the neglected interference remains large enough to cause significant errors in the accuracy of the model. This corresponds to the standard assumption in the protocol interference model and in all network simulations of which we are aware. We also prove that, if the close-in radius is an arbitrary unbounded increasing function of n, that the neglected interference vanishes as $n \to \infty$, meaning that the approximate interference model approaches the accuracy of the true interference model asymptotically. We end with simulation results that validate the theoretical results and indicate a strong potential for practical application of our results when random transmission sets are used (representative of CSMA/CA networks, we believe) and point out some challenges with application in STDMA networks, where densely-packed schedules of node transmissions are constructed.

II. Related work

Since interference is a major factor limiting performance in wireless networks, the problem of interference-aware protocol design has been deeply investigated in the literature. In particular, scheduling transmissions in a wireless network has been subject of intensive research, due to its central role in understanding the transport capacity limits of the network. A seminal work in this area is [7], in which Gupta and Kumar study the transport capacity of wireless networks under two different interference models, the physical and the protocol interference model. Contrary to the physical model, the protocol interference model is a bounded model, since decision on whether a certain communication is successful depends only on the presence of concurrent transmitters within a bounded area centered at the receiver. Another bounded interference model commonly used in the literature is the graph-based interference model, in which a certain communication graph representing communication links is assumed, and only links whose endpoints are up to a certain hop distance h on the communication graph from link (u, v) can interfere with (u, v).

Due to their simplicity, and to the fact that they somehow resemble the behavior of 802.11 MAC layer, bounded interference models have been mostly used in the literature to design interference-aware protocols. This is the case, for instance, of the protocols presented in [1], [11], [14], [15]. Given the complexity of dealing with (unbounded) physical interference, only a few protocols based on this model have been proposed so far. In particular, [5], [6], [9] consider the problem of scheduling, but they provide solutions which are computationally infeasible even for small size network. Recently, some of the authors of this paper have presented the first computationally efficient algorithm for scheduling transmissions under the physical interference model [3]. A notable feature of the GreedyPhysical scheduling algorithm of [3] is that it has a proven approximation bound on performance with respect to the optimal scheduler.

Another branch of research which is closely related to the study reported in this paper is concerned with investigating complexity of scheduling problems under different interference models. Just to cite two recent works, [17] studies the complexity of optimally scheduling transmissions under graph-based interference models, while [18] is concerned with the complexity of both scheduling and one-shot scheduling under the physical interference model. We also want to mention a recent work by Inaltekin and Wicker [8], in which the total interference at a certain node under the physical interference model is estimated. However, differently from our work, the analysis of [8] is focused on evaluating the effect of the 'singularity at 0'[1] on the interference level. As a consequence of that, the study reported in [8] is based on a different network model.

In this paper, we consider a bounded physical interference model, in which the SINR values at the nodes is considered to determine whether a transmission is successful, but only interference caused by 'close-in' transmitters in accounted for when computing the SINR. In particular, the focus of this paper is in investigating the effect of ignoring interference beyond a certain distance on accuracy of the interference model. To the best of our knowledge, the only paper implicitly using the notion of bounded physical interference model is [3], in which this notion is used to derive the approximation bound on GreedyPhysical's performance. However, the bound on the minimum value of the close-in range necessary to 'safely' ignore far-away interference presented therein is much looser than the ones presented in this paper, and holds only for random uniform node deployment.

It is worth observing that a bounded physical interference model is typically used by network simulators (e.g., ns2 [13] and GTNeTS [16]) to approximate physical layer behavior without slowing down too much the simulation running time. Hence, the results presented in this paper give insights on the impact of using a bounded physical interference model on accuracy of simulation results.

III. A Bounded Physical Interference Model

In this section, we formally define a bounded physical interference model and compare it against the 'true' physical interference model. In the true physical interference model [7], the successful reception of a message sent by node u and destined to node v depends on the SINR at v. To be specific, denoting by $P_v(x)$ the received power at v of the signal transmitted by node x, a packet along link (u, v) is correctly received if and only if:

$$\frac{P_v(u)}{N + \sum_{w \in V' - \{u\}} P_v(w)} \geq \beta \ , \qquad (1)$$

where N is the background noise, V' is the subset of nodes in V that are transmitting simultaneously, and β is a constant threshold (often called the *SINR threshold* or *packet capture threshold*) that depends on the desired data rate, the modulation scheme, etc. In the true physical interference model, every concurrent transmission in the network must

[1] 'Singularity at 0' arises in those asymptotical studies in which an increasing number of nodes is distributed in a fixed region. In this situation, distance between sender and receiver can become arbitrarily close to zero, leading to an inconsistence in the physical interference model. Note that 'singularity at 0' does not arise in the study reported in this paper.

be explicitly considered when evaluating whether any single given transmission is succesful.

In the bounded physical interference model considered in this paper, henceforth referred to as the BPI model, we explicitly consider all concurrent transmissions within a given region enclosing the receiver of a particular transmission. We refer to the region considered by the BPI model for a particular receiver as its *close-in region*. In the BPI model, concurrent transmissions outside of a receiver's close-in region are ignored.

More formally, the BPI model is defined as follows:

Definition 1: In the BPI model with close-in region CR, a packet along link (u, v) is correctly received if and only if

$$\frac{P_v(u)}{N + \sum_{w \in V' \cap CR - \{u\}} P_v(w)} \geq \beta \ ,$$

We are concerned with deriving conditions on the close-in regions such that an assumption that total interference outside these regions can be ignored is accurate. To be specific, we would like to derive necessary and sufficient conditions such that total interference from outside any given close-in region approaches zero as the number of nodes in the network approaches infinity.[2] We also refer to the total interference from outside the close-in region as *far-away interference*.

As is common in the literature, we consider circular close-in regions, which can be characterized by a single parameter, referred to as the *close-in radius* (equivalent to the concept of *interference range* used in many papers). Note that our model definition allows non-circular regions, but we focus on analysis under the common assumption of circular regions in the remainder of the paper. Given close-in regions that are approximately circular, one can easily construct slightly larger circular regions enclosing them in order to apply the results derived herein. Circular regions can also be used with propagation models such as log-normal shadowing by constructing regions that are circular with respect to virtual distance. We recall that in the log-normal shadowing model, the attenuation of the received signal (in dB) at a certain distance d from the transmitter is modeled as the sum of a deterministic quantity which obeys log-distance path loss with a certain path loss exponent α, and of a random component that is modeled as a random variable with Normal distribution, zero mean, and variance σ. We can interpret this model as converting the actual distance between transmitter and receiver into a 'virtual distance', which is computed as the distance between the sender and the receiver if attenuation of the signal (computed according to the log-normal shadowing model) was governed only by log-distance path loss. Note that the 'virtual distance' can be either smaller than (when the random component of the attenuation is positive) or larger than (when the random component of the attenuation is negative) the actual distance.

[2]Total interference in this context refers to the sum of the received powers in the left hand side of Inequality 1 coming from nodes outside the close-in region.

Given our consideration of circular close-in regions, our analysis focuses on necessary and sufficient conditions on the close-in radius such that far-away interference can be ignored. The BPI model is parameterized by the close-in radius, which we consider to be a function of the number of nodes in the network for the purposes of asymptotic analyses. We denote the BPI model with a close-in radius of $s(n)$ by $\mathrm{B}PI_{s(n)}$.

Note that the BPI model can be termed "optimistic" with respect to the true physical interference model in that it strictly underestimates the total interference at a particular receiver. This could have negative impacts, for example, when such a model is used to derive schedules for wireless networks employing spatial-reuse TDMA (STDMA) [12]. In STDMA, optimistic assumptions on interference could lead to construction of an infeasible transmission schedule, i.e. a schedule in which some transmissions will fail due to underestimation of interference. We evaluate the impact of the BPI model with different assumptions about close-in radius both for randomly chosen transmission sets and for "well-scheduled" transmission sets, i.e. transmission sets that are carefully chosen by a provably good scheduling algorithm designed to work with physical interference [3].

An interesting problem in this area is to develop a good "conservative" interference model, which is guaranteed to overestimate far-away interference but with a very small error. Such a model would be "safe" for scheduling but the interference upper bound must be very tight in order to not degrade performance substantially due to reductions in spatial reuse. Since a major problem in deriving tight upper bounds on far-away interference is to accurately upper bound the number of concurrent transmitters in different parts of the network, this problem appears to be closely related to the problem of one-shot scheduling[3], which was recently proved to be NP-hard under physical interference [18].

IV. Analysis

A. Preliminaries

We are given a communication graph $G = (V, E)$, where V is the set of nodes in the multi-hop wireless network and E is the set of intended communication links, i.e. we assume that all traffic in the network will use links in the given set E even if other potentially viable links are present. A *feasible transmission set* T is a set of transmitter, receiver pairs such that: $i)$ every node appears at most once as a transmitter or receiver in T; $ii)$ $\forall (t, r) \in T$, $(t, r) \in E$; and $iii)$ $\forall (t, r) \in T$, the SINR inequality (for whichever physical interference model is under consideration) is satisfied at r when all transmitters in T are transmitting simultaneously and no other nodes are transmitting.

In the analysis, we assume the following concerning radio signal propagation: $a1$. Radio signal propagation obeys the log-distance path loss model, with path loss exponent $\alpha > 2$; and, $a2$. All nodes use the same transmit power P, which

[3]In the one-shot scheduling problem, one is interested in scheduling as many transmissions as possible in a single slot.

is such that the resulting transmission range r (in absence of interference) is at least 1. The last part of Assumption a2 amounts to normalizing the unit of distance in the network to be equal to the transmission range.

We consider the two following network deployment scenarios:

– *Constant density scenario*: A number n of nodes is deployed in a region R, with the only condition that node density is assumed to be upper bounded by a constant ρ. More specifically, for each subregion $R' \subseteq R$ such that the area $a(R')$ of R' is at least π, there are at most $\rho a(R')$ nodes in R'.
– *Random uniform scenario*: A number $n = (8+\varepsilon)C\ln C$ of nodes is deployed uniformly at random in a square area R of side $l = \sqrt{C}$, where ε is an arbitrary positive constant.[4]

B. Constant density scenario

We start with proving the following upper bound on the interference generated by nodes outside a close in radius $s \geq 2r$ centered at the intended receiver of a communication.

Theorem 1: Assume the constant density scenario, and let u be an arbitrary node in the network which is at the receiver end of a communication link; the interference generated by nodes located at distance $d > s$ from u, where $s \geq 2r$ is the close-in radius, is upper bounded by

$$C(\alpha) = \frac{\pi\rho P}{s^{\alpha-2}} \cdot \frac{1}{4 - 5\left(\frac{4}{5}\right)^{\frac{\alpha}{2}}} .$$

Proof: Consider an infinite hierarchy $\mathcal{C} = \{\mathcal{C}_0, \mathcal{C}_1, \dots\}$ of circles centered at u, where the radius of $\mathcal{C}_0$ is s, and the radius of $\mathcal{C}_{\mathrm{k}}$ is sv^k, for some constant $v > 1$. It is easy to see that hierarchy $\mathcal{C}$ covers all the plane, i.e., all the network nodes. Let us now bound the contribution to interference from transmitters located in $\Delta_{k+1} = \mathcal{C}_{\mathrm{k+1}} - \mathcal{C}_{\mathrm{k}}$. The area of Δ_{k+1} is $\pi s^2 v^{2k}(v^2-1)$, and the number of transmitters in Δ_{k+1} is at most $\rho\pi s^2 v^{2k}(v^2-1)$ (this comes from the constant density assumption). The contribution to the interference level at node u due to each one of these transmitters (call it w) is upper bounded by $\frac{P}{s^\alpha v^{k\alpha}}$, since we are assuming a decay of power inversely proportional to $d(u,w)^\alpha$, and $d(u,w) > sv^k$. Hence, the total contribution to interference due to transmitters in Δ_{k+1} is upper bounded by $\frac{P}{s^\alpha v^{k\alpha}} \cdot \rho\pi s^2 v^{2k}(v^2-1) = \frac{\pi\rho P}{s^{\alpha-2}} \cdot \frac{v^2-1}{v^{k(\alpha-2)}}$. The upper bound to the total interference level at u due to transmitters at distance larger than s from u can be obtained by evaluating

$$\sum_{k=0}^{+\infty} \frac{\pi\rho P}{s^{\alpha-2}} \cdot \frac{v^2-1}{v^{k(\alpha-2)}} ,$$

which converges to $\frac{\pi\rho P}{s^{\alpha-2}} \cdot \frac{v^\alpha(v^2-1)}{v^\alpha - v^2}$.
The bound above depends on the parameter v chosen to build the hierarchy of circles. We observe that a lower value of v results in a better bound, given the finer hierarchy that is used in the construction. However, the hierarchy cannot be made 'too fine', since otherwise the upper bound on the density of nodes might be violated: in fact, the upper bound on the density of nodes holds only if the area of the region of interest is $\Omega(1)$. To ensure this property, we set v in such a way that the area of Δ_1 (and, consequently, those of Δ_k, with $k > 1$) is at least equal to the area of a circle of radius $r = 1$; by observing that $s \geq 2r$, we get $v = \frac{\sqrt{5}}{2}$. The upper bound on interference then becomes $C(\alpha) = \frac{\pi\rho P}{s^{\alpha-2}} \cdot \frac{1}{4-5\left(\frac{4}{5}\right)^{\frac{\alpha}{2}}}$. ■

The following corollary shows that setting the close-in radius to any unbounded increasing function of n is a sufficient condition for asymptotic accuracy of the BPI model.

Corollary 1: If the close-in radius s is chosen in such a way that $s = f(n)$, where $f(n)$ is an arbitrary unbounded increasing function of n, then the total interference at an arbitrary receiver node u due to nodes located at distance greater than s from u converges to 0 as $n \to \infty$.

The next lemma shows that if the close-in radius s is chosen to be a constant, it is not asymptotically safe to ignore interference beyond distance s from the receiver, i.e. an unbounded increasing function of n is a necessary condition on the close-in radius for asymptotic accuracy of the BPI model. To prove this result, we extend the constant density scenario to also include a lower bound on node density.

– *Extended constant density scenario:* For each subregion $R' \subseteq R$ such that the area $a(R')$ of R' is at least π, there are at least $\tau a(R')$ nodes in R', where $\tau \leq \rho$.

Theorem 2: Assume the extended constant density scenario, and assume s is set to an arbitrary constant $h > 1$. Then, there exists a node deployment and a transmission set T for that deployment such that: $i)$ T is a feasible set under the $\mathrm{B}PI_s$ model; and $ii)$ the total interference at some receiver u due to nodes located at distance greater than s from u converges to $C'(\alpha) > 0$ as $n \to \infty$, where

$$C'(\alpha) = \frac{\pi P h^{2-\alpha}}{(\lfloor h \rfloor + 2)^2} \cdot \frac{1}{2^{\frac{\alpha}{2}} - 2} .$$

Proof: Assume node are placed on a square grid of side $\sqrt{n}$, and adjacent nodes in the grid are placed exactly at distance $r = 1$ (the transmission range). It is immediate to see that grid node placement satisfies to condition for extended constant density scenario. The close-in radius s is set to an arbitrary constant $h > 1$. Consider the following transmission set T: transmitters are located $\lfloor h \rfloor + 2$ hops away from each other in the grid along the horizontal and vertical direction (see Figure 1); for each transmitter node u in T, one of the neighbors of u in the grid is selected as the intended receiver. It is easy to see that transmission set T is feasible under the OPI with close in radius h: in fact, for every intended receiver node v in T, the closest interferer is at distance $> h$ from v; hence, all interferers are ignored when computing the SINR at every receiver node in T, and the transmission set is feasible. Let us now lower bound the interference at a certain receiver node v due to nodes outside the close-in range under the true physical model. By using a hierarchy similar to the one used

[4]With some minor technicalities, the proof presented in the following can be adapted to the case where the deployment area is a disk of area C.

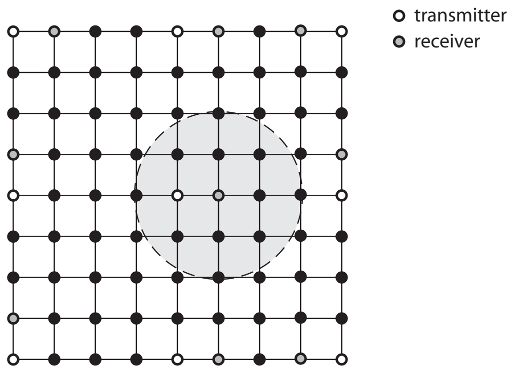

Fig. 1. Transmission set used in the proof of Theorem 2. The figure refers to the case $s = 2$.

in the proof of Theorem 1, by observing that the density of transmitter nodes per unit of area in transmission set T is $\frac{1}{(\lfloor h \rfloor + 2)^2}$, and that each of these transmitters generates an interference level at least $\frac{P}{(hv^{k+1})^\alpha}$ when considering area Δ_{k+1}, we can lower bound the total interference at the node v in the center of the grid as follows:

$$\frac{\pi P h^{2-\alpha}}{(\lfloor h \rfloor + 2)^2} \cdot \sum_{k=0}^{\bar{k}} \frac{v^2 - 1}{v^{k(\alpha-2)} v^\alpha} ,$$

where $\bar{k} = \log_v \left(\frac{\sqrt{n}}{2h} \right)$. For $n \to \infty$, the above lower bound converges to $\frac{\pi P h^{2-\alpha}}{(\lfloor h \rfloor + 2)^2} \cdot \frac{v^2-1}{v^\alpha - v^2}$. The lemma follows by observing that, by setting $v = \sqrt{2}$, we guarantee that the area of Δ_1 is at least as large as the area of a circle of radius h, and the lower bound on transmitter density is satisfied. ■

It is easy to see that, for realistic parameter values, $C'(\alpha)$ is very high. For instance, by setting $P = 100mW$, $\alpha = 3$, $h = 2r$ (corresponding to the standard assumption that the interference range is twice the transmission range), and $r = 250m$, we get $C'(\alpha) = -55.2148dBm$, which is at least two order of magnitude higher than the standard noise value. The above value is the limit of the total interference, which could actually be approached only for very large networks. But even if we consider relatively small networks and truncate the summation in the proof of Theorem 2 accordingly, we obtain a relatively high value of the total interference. For instance, with $n = 49$ and the same parameters as above, we obtain a lower bound to the interference at the node in the center of the grid equal to $-55.2166dBm$, which is very close to the asymptotic value of $C'(\alpha)$.

Corollary 1 and Theorem 2 imply that $s = f(n)$, where $f(n)$ is an arbitrary unbounded increasing function of n, is the minimum possible setting of the close-in radius that ensures an asymptotically negligible error of the BPI model with respect to the true physical interference model. Another implication of Corollary 1 and Theorem 2 is a good upper bound on the density of transmitter nodes in the constant density scenario. More specifically, Corollary 1 implies that a transmission set $T_{f(n)}$ in which intended receivers are separated from the closest interferers by at least distance $f(n)$, where $f(n)$ is an arbitrary increasing function of n, is asymptotically feasible, implying that a transmitter density of $O(\frac{n}{f(n)})$ is asymptotically feasible. On the other hand, by Theorem 2, a transmitter density of $\Omega(n)$ in asymptotically infeasible.

It is also worth observing that the speed of convergence towards the interference bounds stated in theorems 1 and 2 depends on α: higher values of α lead to a faster convergence to the asymptotic value of far-away interference. On the other hand, $\alpha > 2$ is a necessary condition for convergence of far-away interference towards a finite value. This observation is in accordance with the study reported in [8].

C. Random uniform scenario

Let us now consider the random uniform scenario. We will make use of the following concentration result, proved in [2]:

Lemma 1: Consider any partitioning of the deployment region R into C cells of area 1, and assume $n = (8+\varepsilon)C \ln C$ nodes are distributed uniformly at random into R. Then, the maximally occupied cell contains at most $f(C)$ nodes w.h.p.[5], where $f(C)$ is defined as follows:

$$f(C) = e \ln C - \ln \left(\sqrt{2\pi e \ln C} \right) + g(C) ,$$

where $g(C)$ is an arbitrary increasing function of C with $g(C) \in O(\log C)$.

We are now ready to prove an upper bound to the total interference due to nodes at distance at least $s \geq 2r$ from a certain receiver.

Theorem 3: Assume the random uniform scenario, and let u be an arbitrary node in the network which is at the receiver end of a communication link; the interference generated by nodes located at distance $d > s$ from u, where $s \geq 2r$ is the close-in radius, is upper bounded by

$$C_{rand}(\alpha) = \frac{6 f(C) P}{s^{\alpha-2}} \cdot \frac{2^{\frac{\alpha}{2}}}{2^{\frac{\alpha}{2}} - 2} ,$$

w.h.p.

Proof: Consider an infinite hierarchy $\mathcal{C} = \{\mathcal{C}_0, \mathcal{C}_1, \dots\}$ of circles centered at u, where the radius of $\mathcal{C}_0$ is s, and the radius of $\mathcal{C}_\mathrm{k}$ is sv^k, for some constant $v > 1$. It is easy to see that hierarchy $\mathcal{C}$ covers all the plane, hence the deployment region R. Let us now bound the contribution to interference from transmitters located in $\Delta_{k+1} = \mathcal{C}_{\mathrm{k}+1} - \mathcal{C}_\mathrm{k}$. The area of Δ_{k+1} is $\pi s^2 v^{2k}(v^2 - 1)$, and, if we consider a square cell partitioning of R with cells of unit side, it contains at most $2s^2 v^{2k}(2v^2 - 1)$ cells. This bound is obtained by subtracting the area of the square inscribed into circle $\mathcal{C}_\mathrm{k}$ from the area of the square circumscribed to circle $\mathcal{C}_{\mathrm{k}+1}$. From Lemma 1, it follows that the number of nodes in Δ_{k+1} is $f(C)\pi 2s^2 v^{2k}(2v^2 - 1)$ w.h.p. The contribution to the interference level at node u due to each one of these transmitters (call it w) is upper bounded by $\frac{P}{s^\alpha v^{k\alpha}}$, since we are assuming a decay of power inversely proportional to

[5] In this paper, w.h.p. means with probability that converges to 1 as $C \to \infty$.

$d(u,w)^\alpha$, and $d(u,w) > sv^k$. Hence, the total contribution to interference due to transmitters in Δ_{k+1} is upper bounded by $\frac{2f(C)P}{s^{\alpha-2}} \cdot \frac{(2v^2-1)}{v^{k(\alpha-2)}}$, w.h.p. The upper bound to the total interference level at u due to transmitters at distance larger than s from u can be obtained by evaluating

$$\sum_{k=0}^{+\infty} \frac{2f(C)P}{s^{\alpha-2}} \cdot \frac{(2v^2-1)}{v^{k(\alpha-2)}} ,$$

which converges to $\frac{2f(C)P}{s^{\alpha-2}} \cdot \frac{v^\alpha(2v^2-1)}{v^\alpha - v^2}$, w.h.p.
The upper bound of $C_{rand}(\alpha)$ is obtained by setting $v = \sqrt{2}$ in the above formula, which ensures that the area of Δ_1 (and, consequently, of all the Δ_ks) is at least 1, i.e., it contains at least one cell. ∎

Corollary 2: If the close-in radius s is chosen in such a way that $s = f(n)$, where $f(n)$ is an arbitrary function of n such that $\frac{\log n}{f(n)} \to 0$ as $n \to \infty$, then the total interference at an arbitrary node u due to nodes located at distance greater than s from u converges to 0 as $n \to \infty$, w.h.p.

Observe that, compared to the constant density scenario, we have a slightly stronger requirement on the close-in radius to have convergence to 0 of the total interference generated from nodes outside the close-in radius. Namely, $f(n)$ cannot be an arbitrary increasing function of n, but an increasing function of n which grows to infinity faster than $\log n$. This additional constraint is due to the higher node density in the random uniform scenario ($\Theta(\log C)$ nodes per unit of area, instead of $O(1)$ as in the constant density scenario).

Theorem 4: Assume the random uniform scenario, and assume s is set to an arbitrary constant $h > 1$. Then, there exists a transmission set T for such that: $i)$ T is a feasible set under the $\mathrm{B}PI_s$ model; and $ii)$ the total interference at some receiver u due to nodes located at distance greater than s from u converges to $C'_{rand}(\alpha) > 0$ as $n \to \infty$ w.h.p., where

$$C'_{rand}(\alpha) = \frac{4Ph^{2-\alpha}}{(\lfloor h \rfloor + 1)^2(2^\alpha - 4)} .$$

Proof: Due to space limitations, we only give a sketch of the proof. The idea is to subdivide the deployment region into C cells of side 1. By setting n as in the random uniform scenario, we have from Theorem 2, page 96, of [10] that the minimum number of nodes in a cell is $\Omega(\log C)$ w.h.p. We can then build a transmission set T as follows. We consider cells which are $\lfloor h \rfloor + 1$ away from each other, and we randomly pick a sender and a receiver from each one of these cells (we can do that, since the cell contains $\Omega(\log C)$ nodes w.h.p.). It is immediate to see that this transmission set is feasible under the BPI model with $s = h$, since the minimum distance between any receiver and the closest interferers is larger than s. The lower bound on the interference caused by nodes outside the close-in region on a specific receiver node can be obtained by using a construction and arguments similar to the one used in the proof of Theorem 2. ∎

Similarly to the constant density scenario, by using realistic parameters ($P = 100mW$, $\alpha = 3$, and $h = 2r$), we have $C'_{rand}(\alpha) \approx -61dBm$, which is at least two orders of magnitude larger than the typical noise value.

V. Simulation Results

In this section, we first evaluate the accuracy of the BPI model on randomly chosen transmission sets in order to demonstrate agreement with the theoretical analyses of the preceding section. We then focus on a more detailed evaluation of the impact of the BPI model on scheduling when the physical interference model is used to construct the schedule.

A. Simulations with Random Transmission Sets

In order to verify the theoretical analyses, we considered n-node networks configured as a $\sqrt{n} \times \sqrt{n}$ grid with an average separation between nodes of 250 meters (the nominal transmission range for 802.11b). In order to eliminate some discretization effects, once nodes were placed in a grid with exactly equal separation between each pair of neighboring nodes, we slightly perturbed the node positions. We did this by setting the final node location to be a point that was uniformly distributed within a circle of radius 50 meters around the initial node location. This produced a maximum separation between neighboring nodes of 350 meters. Transmit power was set to 100 milliwatts, noise was set to -90 dBm, and β was set to 10 dB, which corresponds to the value needed for a data rate of 11 Mbps with 802.11b. We considered values of α of 2.5 (considered to be a typical rural outdoor environment), 3.0 (a typical urban outdoor environment), and 3.5 (a cluttered urban outdoor environment).

In these initial simulations, for each value of n considered, we chose a mean transmission set size such that roughly half of transmission sets of that size chosen at random were feasible under the true physical interference model. We then considered random sets with sizes normally distributed around the mean. For each random transmission set considered, we determined whether the set was feasible under both the true physical interference model and the BPI model, and whenever there was a disagreement between the two models, this was counted as an error for the BPI model.[6]

With the setting as described, we first kept the close-in radius as a constant 500 meters. This corresponds to the rule-of-thumb widely used in analyses and simulations, where the interference range is set to twice the nominal transmission range. We then considered a slowly-growing close-in radius. To be specific, we made the close-in radius proportional to $n^{0.1}$, choosing the constant so that at the smallest value of n, the radius was equal to 500 meters (i.e. the same as for the constant close-in radius case). Figure 2 shows the percentage error for the BPI model resulting from these simulations. From the figure, we clearly see the trends predicted by our theoretical analyses. For constant close-in radius, the BPI model's error quickly converges to a constant, depending on α. The error is about 42% for $\alpha = 2.5$, 25% for $\alpha = 3.0$, and 15% for $\alpha = 3.5$. Clearly, errors of this size could cause significant problems with results produced under the

[6]In practice, since the BPI model is strictly optimistic, errors only occurred in one direction, i.e. if the set was infeasible under the true model, it was also infeasible under the BPI model.

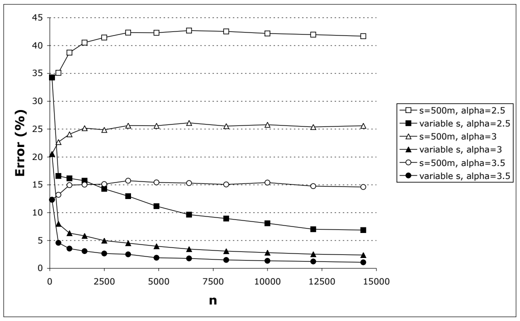

Fig. 2. Error in BPI Model vs. n for Varying α and Different Assumptions on the Close-in Radius

assumption of constant interference range. When the close-in radius is proportional to $n^{0.1}$, it is also clear that the error converges toward zero. However, the convergence is quite slow for $\alpha = 2.5$. These results show that considering interference range to be a slowly growing function of n is a promising approach to limiting complexity while maintaining accurate interference modeling, when α is not too small.

B. Simulations of Physical-Interference-Based Scheduling

In constructing a schedule for use within spatial-reuse TDMA (STDMA) [12], interference is the limiting factor. Recently, we proposed a scheduling algorithm for Mesh networks, referred to as Algorithm GreedyPhysical, that accounts for physical interference and has a proven approximation ratio with respect to the optimum scheduler [3]. In this section, we investigate the impact of the BPI model on scheduling by generating schedules with GreedyPhysical using the BPI model, and evaluating them against the true physical interference model. We note that scheduling can be considered to be a worst-case scenario for approximate physical interference models because scheduling algorithms try to pack the maximum possible number of transmissions into a single slot and, thus, they tend to maximize the total amount of interference in the network.

B.1 Simulation Set-up

Algorithm GreedyPhysical was designed to work with Mesh networks in which clients access the network via a set of wireless routers that are connected to each other to form a backbone wireless network. The backbone wireless network carries traffic between a designated set of routers, known as gateways, and the clients. Since all traffic goes between clients and gateways, routing is typically done via a forest, rooted at the gateway nodes. In these evaluations, we assume a simple shortest-path forest is used, where the set of shortest paths from each router (which aggregates traffic from/towards its registered clients) to its closest gateway node are merged into a forest. Given the routing forest and a set of traffic demands, one for each router, the total demand on a link is given by the sum of all router demands in the subtree below that link, i.e. router demands are aggregated as one moves up the tree. Given the link demands, GreedyPhysical constructs a schedule that includes each link in the forest a sufficient number of times in order to satisfy all of the demands. A single slot in the schedule consists of a transmission set, which is a feasible transmission set under whatever interference model is used by GreedyPhysical to calculate feasibility.

If we execute Algorithm GreedyPhysical under the BPI model, some of the schedules that it produces will contain infeasible transmission sets under the true physical interference model, due to the optimistic nature of the BPI model. In the results that follow, we evaluate the percentage of transmissions that fail in an average schedule produced by GreedyPhysical under the BPI model, when the transmission sets are evaluated under the true physical interference model. This is a measure of the impact of the BPI model's inaccuracy on the final schedule produced. If the BPI model is used with GreedyPhysical in a real network and the percentage of failed transmissions can be reduced to a small value, e.g. below 10%, then the original schedule produced by GreedyPhysical can be extended slightly and any transmissions that fail can be moved into the additional slots.

The setting used with GreedyPhysical for the remainder of this section is as follows. For a given n, nodes are distributed as a $\sqrt{n} \times \sqrt{n}$ grid to form the wireless backbone network. A randomly selected set of wireless routers (nodes), consisting of 10% of the total number of routers, is chosen to be the set of gateway nodes. Once the gateway nodes are determined, a shortest path forest is constructed connecting the (non-gateway) routers to their nearest gateway. Each router is then assigned a demand that is uniformly distributed between 1 and 10 units, representing the total demand from clients connected directly to that router. Demands are then aggregated, as described earlier, in order to produce the link demands. Simulations are done for various values of n, α, and the close-in radius. Both a simple log-distance path loss model and a lognormal shadowing model are used.

When considering lognormal shadowing, we consider the close-in radius to be defined in terms of virtual distance. This corresponds to selecting a set of transmitters with at least a certain minimum received power at a given receiver instead of the transmitters within a certain geometric distance. Since the intent of bounded interference models is to select the "most interfering" nodes and neglect the others, we believe this is a natural way to define the close-in region with shadowing. In an actual network, if received power is used to estimate distance, then the situation matches exactly our definition.

Other parameters were set as in the random transmission set simulations, i.e. transmit power was set to 100 mW, noise was assumed to be -90 dBm, and β was set to 10 dB.

B.2 Simulation Results for Scheduling

We begin by investigating the accuracy of the BPI model with respect to the close-in radius for different values of n. Figure 3a shows the percentage of failed transmissions in

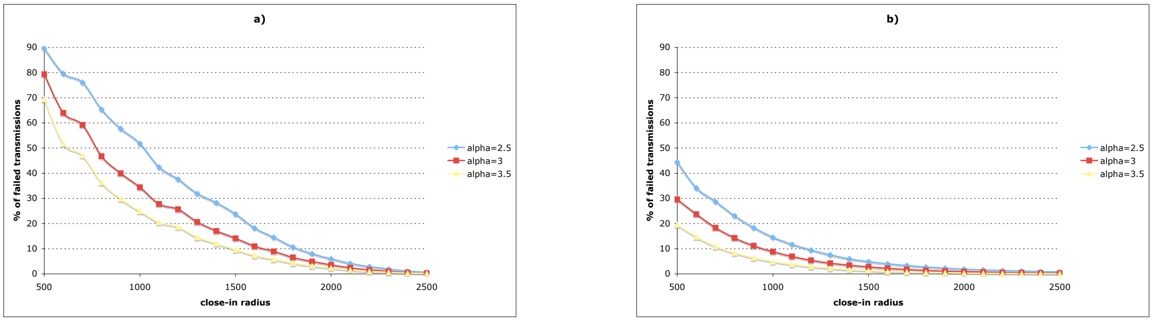

Fig. 3. Percentage of Failed Transmissions vs. Close-in Radius, $n = 100$; a) Without Log-Normal Shadowing; b) With Log-Normal Shadowing

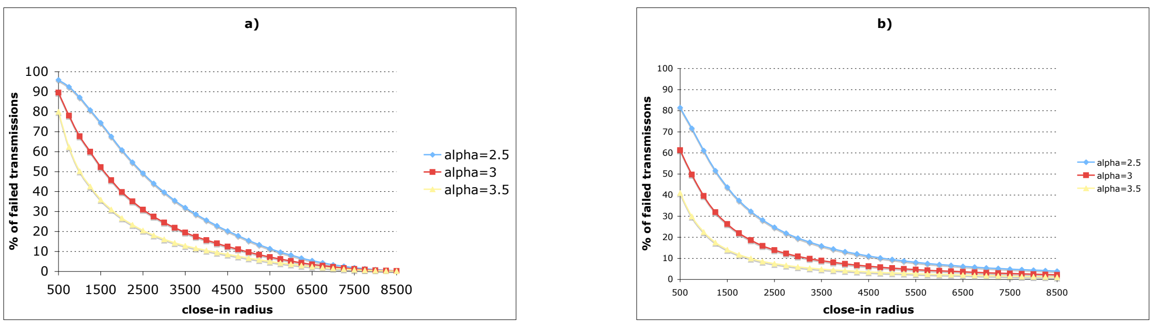

Fig. 4. Percentage of Failed Transmissions vs. Close-in Radius, $n = 900$; a) Without Log-Normal Shadowing; b) With Log-Normal Shadowing

the schedule generated by GreedyPhysical versus the close-in radius for $n = 100$ and three values of α. Figure 3b shows the same quantity except with log-normal shadowing.

In these figures, one can see that, as expected, the value of α has a significant impact on the error. It is also evident that log-normal shadowing actually reduces the error of the BPI model, in that a much lower percentage of transmissions fail at small to moderate values of the close-in radius. Equivalently, for a given desired percentage of failed transmissions, a much smaller close-in radius suffices with log-normal shadowing. We believe this is because, with log-normal shadowing and considering the virtual distance from any given receiver to other nodes, there is a higher density of nodes closer to the receiver. Thus, the situation when considering virtual distance is somewhat skewed away from the constant density scenario toward one in which there is some clustering around the receiver. This has the effect of bringing more nodes within the close-in radius, which then increases the accuracy of the BPI model. We have verified this clustering effect in the simulations we conducted.

Figure 4 shows the percentage of failed transmissions with the same parameters as above except that $n = 900$. The same dependence on α is clearly visible. While the difference with log-normal shadowing is somewhat smaller than in the previous case, there is still a substantial increase in accuracy when log-normal shadowing is used.

We now compare the accuracies of the BPI model with fixed close-in radius and with increasing close-in radius, as it applies to physical-interference-based scheduling using GreedyPhysical. Here, we considered the propagation model without log-normal shadowing, since this actually worsens the performance of the BPI model compared to with shadowing. Figure 5 shows the percentage of failed transmissions versus n for these two versions of the BPI model, $\alpha = 3.5$, and several different scaling functions for the close-in radius. We know from the earlier plots that close-in radius needs to be fairly high in this situation, so we set the baseline close-in radius to 1000 meters, which is four times the nominal transmission range. With the close-in radius fixed to this value, we see that the "error" converges to a constant, i.e. a transmission failure rate of about 58%. When the close-in radius increases with n, the failure rate decreases. However, a fairly fast increase of the close-in radius (proportional to $\sqrt{n}$) is required to decrease the percentage of failed transmissions below 10% for the values of n considered in these data sets.

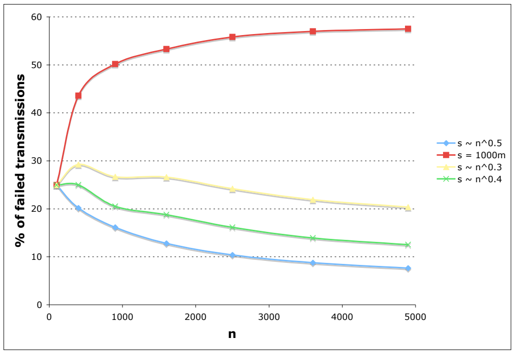

Fig. 5. Percentage of Failed Transmissions vs. n, $\alpha = 3.5$, log-distance path model, no shadowing

VI. Discussion and Future Work

We have seen very good agreement with the theoretical analysis when simulating random transmission sets. With fixed close-in radius, the error of the BPI model quickly converges to a constant, which can be quite large even for a fairly large α. In addition, error converges toward zero when the close-in radius is set to as slowly increasing a function as $n^{0.1}$, and convergence is quite fast except for the smallest value of α considered. We believe that the random transmission set scenario should be fairly representative of CSMA/CA protocols under relatively high load situations. Thus, our results indicate a potential for the BPI model (with increasing close-in radius) to be quite useful in evaluations of CSMA/CA protocols under more realistic interference assumptions. One example of this would be, in CSMA/CA simulations using packet-level simulators such as ns-2 and GTNetS, to scale the interference range slowly as network size is increased, rather than using the standard assumption of interference range that is a multiple of the fixed transmission range. However, more simulations should be done to study the random scenario since our results have considered only one typical setting in this case.

When considering a challenging environment for physical interference modeling, namely well-scheduled, i.e. densely packed, transmission sets, the picture is somewhat different for the BPI model. While the fixed close-in radius assumption is clearly even worse in this situation than with random transmission sets, there are some challenges in applying the theoretical results in practice. While we do see convergence toward zero error in the scheduling simulations, convergence is significantly slower than for random transmission sets, even for close-in radius growing as fast as $n^{0.5}$ and a high value of α. One promising result in this case is that log-normal shadowing actually helps the BPI model, due to its tendency to produce a higher concentration of nodes relatively close to a given receiver when considering virtual distance. Experimental validation of this effect in a real shadowing environment would be very helpful in determining the most realistic model to use in evaluating accuracy in this situation. The practical constraints imposed by well-scheduled transmission sets indicate that the approach to approximating physical interference via conservative models that provably overestimate the total far-away interference is one that should be considered. The challenge here is deriving good upper bounds on far-away interference, which will require some analysis of the densest possible packing of transmissions, which is related to the NP-hard problem of one-shot scheduling.

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