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# TRStalker: an Efficient Heuristic for Finding NP-Complete Tandem Repeats 

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#### Abstract

Genomic sequences in higher eucaryotic organisms contain a substantial amount of (almost) repeated sequences. Tandem Repeats (TRs) constitute a large class of repetitive sequences that are originated via phenomena such as replication slippage, are characterized by close spatial contiguity, and play an important role in several molecular regulatory mechanisms. Certain types of tandem repeats are highly polymorphic and constitute a fingerprint feature of individuals. Abnormal TRs are known to be linked to several diseases. Researchers in bio-informatics in the last 20 years have proposed many formal definitions for the rather loose notion of a Tandem Repeat and have proposed exact or heuristic algorithms to detect TRs in genomic sequences. The general trend has been to use formal (implicit or explicit) definitions of TR for which verification of the solution was easy (with complexity linear, or polynomial in the TR's length and substitution+indel rates) while the effort was directed towards identifying efficiently the sub-strings of the input to submit to the verification phase (either implicitly or explicitly). In this paper we take a step forward: we use a definition of TR for which also the verification step is difficult (in effect, NP-complete) and we develop new filtering techniques for coping with high error levels. The resulting heuristic algorithm, christened TRStalker, is approximate since it cannot guarantee that all NP-Complete Tandem Repeats satisfying the target definition in the input string will be found. However, in synthetic experiments with $30 \%$ of errors allowed, TRStalker has demonstrated a very high recall (ranging from $100 \%$ to $60 \%$, depending on motif length and repetition number) for the NP-complete TRs. TRStalker has consistently better performance than some state-of-the-art methods for a large range of parameters on the class of NP-complete Tandem Repeats. TRStalker aims at improving the capability of TR detection for classes of TRs for which existing methods do not perform well.


## 1 Introduction

Tandem Repeats (TRs) are multiple (two or more) duplications of substrings in the DNA that occur contiguously, and may involve some base mutations (such as substitutions, insertions, and deletions). Tandem Repeats of several forms (satellites, microsatellites, minisatellites, and others) have been studied extensively because of their role in several biological processes ${ }^{3}$. In fact, TRs are privileged targets in activities such as fingerprinting or tracing the evolution of populations [KTCM08,VKN $\left.{ }^{+} 06\right]$. Several diseases, disorders and addictive behaviors are linked to specific TRs loci [ $\mathrm{WCJC}^{+} 94$ ]. The role of TRs has been studied also within coding regions [OEPS05] and in relation to gene functions [LPPV07]. Large scale Tandem Repeats comparative studies of the human genome are described in $\left[\mathrm{DNT}^{+} 08, \mathrm{WHG}^{+} 08\right]$. Data Bases of repetitive elements such as RepBase $\left[\mathrm{JKP}^{+} 05\right]$ and TRDB [GRB07] are now available; and the detection of repetitive elements via library-based similarity matching, for example by using the tool Repeatmasker ${ }^{4}$, is a popular practice. However, tools for $a b$ initio detection of repetitive elements that are not based on prior knowledge accumulated in data bases, are still important in order to extend our comprehension of the role of TRs in biological mechanisms. Existing ab initio tools are successful when the TR exhibit a moderate amount of noise and when the TR is easily certified. However, there is an emerging need for new tools that are able to cope with higher levels of noise and/or TR computationally more difficult to detect. For example, Boeva et al. [BRPM06] study so called Fuzzy Tandem Repeats and their role in gene expression, but their technique is limited since they do not handle insertions/and deletions.

[^0]Some of the most successful ab initio tools, such as TRF [Ben99] and ATRHunter [WYKG05], are based on a filtering multi-stage approach (see also $\left[\mathrm{PSdL}^{+} 09\right]$ ). In the first stage the input sequence is analyzed so to detect via statistical criteria likely position and length of candidate subsequences. The final stage is the validation one in which a more expensive validation test is applied to candidate substrings passing the first stages, so to determine an output matching the implicit definition of TR and the user-defined filtering parameters.
Our contribution is a novel filtering multi-stage algorithm TRStalker that introduces new techniques in all stages. For the first stage, where over-represented distances between probes are sought, we employ gapped $q$-grams [BK03] in place of the standard $q$-grams in order to collect evidence on the candidate substrings. Gapped q-grams have been used before in the context of textual and biological database searching, but less so in the area of tandem repeats detection (with the exception of the system TEIRESIAS [SGFR99]). Because of errors due to insertion/deletions the period of a TR is subject to fluctuations, we employ a weighting scheme with exponential decay so to reinforce the signal even in presence of this smearing effect. Finally we use ranking instead of thresholds when deciding the substrings to pass to the next phases, in order to concentrate the computational effort on the zones with candidates with higher weight. For the final validation stage we employ an NP-complete definition of TR involving the concept of generalized median string under edit distance [dIHC00,SP03], together with an efficient heuristic for computing such median strings [JABC03].
By extensive experimental comparisons of TRStalker with two state-of-the-art tools, namely TRF and ATRHunter, we did find out that TRStalker has consistently better performance for a large range of error and length parameters for the class of NP-complete Tandem Repeats, with a recall ranging from $100 \%$ to $60 \%$. Thus TRStalker improves the capability of TR detection for classes of TRs for which existing methods do not perform well. Tests performed on standard TRs definitions (verifiable in polynomial time) also show recall performance close to $100 \%$. Incidentally, this result confirms of the power of the new techniques developed for the initial filtering phase.
The paper is organized as follows: in Section 2 we describe some basic definitions of TRs; in Section 3 we survey the state of the art; in Section 4 we describe in detail the algorithmic principles of TRStalker; in Section 5 we describe the outcome of the experimental validation. Preliminary experiments with biological data are under way.

## 2 Basic Definitions

A Tandem Repeat in a DNA sequence is the repetition of two or more contiguous exact or approximate copies of a substring (called the motif) of the tandem repeat.
Exact TR. Formally, given an alphabet $\Sigma$, and a set of strings $x_{i} \in \Sigma^{*}$, consider the concatenation $X=x_{1} x_{2} . . x_{t}$. The string $X$ is a exact tandem repeat (ETR) of period $k$ and repeat number $t$, when $\left|x_{i}\right|=k$ and $x_{i}=x_{1}$, for each $i \in[1, . . t]$. In general we may suppose there is a longer string $Y$ of which $X$ is a substring. The string $x_{1}$ that is repeated exactly is called the motif of the ETR. A TR $X$ is called maximal if it cannot be extended in $Y$ while still being a TR.
Exact tandem repeats are sometimes found in biological sequences, but they tell us only part of the story, thus several notions of an approximate tandem repeat have been developed. Denote with $D_{H}(a, b)$ the hamming distance of two strings with equal length. If the length of $a$ and $b$ is different we consider the smallest possible mismatch in an alignment of the two strings without gaps. Denote with $D_{E}(a, b)$ the edit distance of the two strings $a$ and $b$.
Approximate TR (ATR). Several different definitions of Approximate Tandem Repeats proposed in the literature differ by the choices made along a few dimensional axes. The most important are the following:

1. The distance function (or similarity function) used to compare pairs of strings (repeats or motifs). Classical distance functions are (weighted or unweighted) Hamming and edit distances, but also more complex functions with block-operations have been considered. Usually distance functions of choice are also metrics. Sequence pair alignment scoring functions are often used as similarity functions.
2. The choice of pairing policy (adjacent pairs, all pairs, motif-instance pairs, etc..) is the most critical one, in particular the use of motifs not present in the input string (Steiner motifs) may easily change the validation task from polynomial to np -complete.
3. Different functionals for the final error score (typically the sum and the maximum functionals are used).
4. Use of an absolute error threshold or relative to the TR length. More generally the error thresholds can be complex functions of the structure of the TR.
5. Role of the ambient string $Y$. In most definition a string $X$ is a TR regardless of the properties of the string $Y$ containing it. In a few definitions a statistical analysis of $Y$ is used to determine some of parameters thus the property of being a TR depends on both $Y$ and $X$

In [SBT07] it is used the following definition: $X$ is called a $k$-edit Approximate Tandem Repeat when $\sum_{i=1}^{t-1} D_{E}\left(x_{i}, x_{i+1}\right) \leq k$, where the last repeat $x_{t}$ might be incomplete so $D_{E}\left(x_{t-1}, x_{t}\right)$ is computed as the minimum edit distance of $x_{t}$ and the prefixes of $x_{t-1}$. This definition is inspired by the evolutionary model of TRs in which it is assumed TRs are generated by duplicating the last copy of a previous TR, possibly with duplication errors that truncates it. A k-edit repeat is maximal if it cannot be extended either to the left or to the right without violating its definition.
In [WYKG04] for a similarity function $\phi$ that measures the alignment score of two sequences, it is defined a $\eta$-Simple Approximate Tandem Repeat ( $\eta$-SATR) a string $X=x_{1} \ldots x_{t}$ such that: there exists a motif $\bar{x} \in \Sigma^{*}$ so that for every $i \in[1, . ., t], \phi\left(\bar{x}, x_{i}\right) \geq \eta$. In other words the TR consists of $t$ duplications of a single consensus string $\bar{x}$ with mutations. Such string $\bar{x}$ is also called a Steiner motif if $\bar{x}$ is not constrained to be equal to some repeat $x_{j}$. Often in practice $\bar{x}$ is chosen as the repeat $x_{j}$ that minimizes the error function, and is called a Pivot motif. The distinction is critical since, as mentioned before, Steiner motifs lead to NP-complete recognition problems, while Pivot motifs do not.
The $\eta$-Neighboring Approximate Tandem Repeat ( $\eta$-NATR) is a string $X$, so that for each $i \in[1, . ., t-1]$, $\phi\left(x_{i}, x_{i+1}\right) \geq \eta$ [WYKG04]. The Pairwise Approximate Tandem Repeat (PATR) is a string $X$, such that for every pair of indices $i, j \in[1, . . t]^{2}$ with $i \neq j$ we have $\phi\left(x_{i}, x_{j}\right) \geq \eta_{i j}$, where $\eta_{i j}$ is set to be a monotonically decreasing function of $|i-j|$, thus allowing more slackness when comparing distant copies of the basic motif. In [KT04] it is used a definition similar to that of the Neighboring Approximate Tandem Repeat, except that the Hamming distance is used and that the threshold is not absolute but relative to the length. A $\gamma$-HATR ( $\gamma$-Hamming Approximate Tandem Repeat) is a string $X$ such that: for each $i \in[1, . ., t-1]$, $D_{H}\left(x_{i}, x_{i+1}\right) \leq\left|x_{i}\right| \gamma$.
In [SGFR99] a more complex definition is given that takes into account the substring alignment score density function for pairs of random substrings of a given length. Here the definition of a TR $X$ depends on the properties of the longer string $Y$ into which $X$ is embedded. In particular a ( $\mu, p$ )-TR must comply to two conditions: 1) $\sum_{i=0}^{t-1} \phi\left(x_{i}, x_{i+1}\right) \geq(t-1) \mu$, that imposes an average high similarity score for adjacent repeats, and 2) define $\alpha(p, k)$ as the value of similarity such that there is probability $p$ that two random substrings of length $k$ in $Y$ have similarity above $\alpha(p, k)$. There must be an index $q \in[1, . ., t]$ such that $\phi\left(x_{q}, x_{j}\right) \geq \alpha(p, k)$ for all $j \in[1, . . t]$. Note that this condition limits the dispersion of the similarity with respect to one of the copies (called the pivot).
TRF [Ben99] uses as final validation algorithm the Wraparound Dynamic Programming technique (WDP) that tests efficiently the alignments of a given candidate motif with the surrounding portions of the input sequence, so to determine the maximum number of adjacent repetitions within a user-defined score bound. This implies a notion of TR akin to that of simple approximate tandem repeats (SATR) with Pivot motif. Classical results on string alignments [Gus97, page 351] ensures that, for the metric score given by the sum of motif-repeats distances, the solution found using the optimal Pivot motif has a score within a factor $(2-1 / t)$ of the score induced by the optimal Stainer motif. For low levels of errors one could use a PivotSATR definition doubling the error threshold to capture a Steiner-SATR, however for higher error levels (say, above $25 \%$ ), doubling the error threshold forces the existing systems to work in a range of values (say, above $50 \%$ ) where most methods do not perform well.
Precision and Recall. For all of the above definitions once the tandem repeat and its motif are found it is possible to validate them according to the chosen definition. Thus we can assume that in principle any Approximate TR finder algorithm will report only legitimate TRs (thus precision is always $100 \%$ ) and the only important output quality measure is recall (that is how many of the implanted ATR's present in the input string have been reported as such by the algorithm).

## 3 State of the Art

We will briefly survey the state of the art in finding tandem repeats. First we will describe methods that for a given definition of TR are able to find all maximal substrings in the input that match the definition (exhaustive algorithms). Often exhaustive algorithms may not be available, or when available are too slow in practice, thus several heuristic algorithm have been developed which are shown experimentally to be able
to detect a large fraction of TRs efficiently. Note that the time/precision trade-off is severely influenced by the allowed error thresholds. Performance often degrades quickly with increasing error levels.
Exhaustive algorithms. When we allow no error, it is possible to find all maximal exact TRs in a string of length $n$ in time $O(n)$ [KK99,GS04]. When we allow two consecutive repeats to differ by an amount at most $k$ (either in Hamming or in edit distance) Landau, Schmidt and Sokol [LSS01] give exhaustive algorithms running in time $O(n k \log (n / k))$ for Hamming distance, and $O(n k \log k \log (n / k))$ for edit distance. A simpler algorithm with the same asymptotic complexity for the edit distance is proposed by Sokol, Benson and Tojeira [SBT07]. Kolpakov and Kucherov [KK03] improved the bound for the Hamming distance to $O(n k \log k+s)$ where $s$ is the number of TR found. For the Hamming distance, Krishnan and Tang [KT04] give an exhaustive method running sequentially in time $O\left(n^{3}\right)$, that can be easily implemented onto a parallel architecture, since every possible pattern length is searched independently.
Heuristic algorithms. The algorithmic techniques in [KK99,KK03] has been extended in the tool mreps [KBK03] so to be able to handle approximate TRs under edit distance, with some additional heuristic filtering steps.
The tool TRF (Tandem Repeat Finder) developed by Benson [Ben98,Ben99], based on statistical filtering of zones of DNA likely to contain TRs, is currently one of the standard heuristic methods. ATRHunter [WYKG04] by Wexler et al. is also based on a statistical filtering approach, placing greater emphasis in techniques for designing thresholds for the quantities of interest. Other proposed heuristics for finding TRs are REPuter [KS99, $\mathrm{KCO}^{+} 01$ ], STRING [PFAP03], TEIRESIAS [SGFR99], TandemSWAN [BRPM06]. A class of papers (see e.g. [SIRR04], [Bro07],[BJ03], [GSMS07]) tackle the problem of finding tandem repeats as a problem in signal processing theory and usually map the input string into a time-signal in a suitable numerical domain for which several spectral techniques can be used, such as the Periodicity Transform or the Fourier Transform. Other methods use data compression techniques to detect repetitive elements [RDD $\left.{ }^{+} 97\right]$.
The methods cited above are rather general since they aim at treating efficiently TRs in a wide range of length values. There is also a large class of methods that are aimed at handling particular or special classes of TRs such as: microsatellites (e.g. IMEx [MN07]), palindromic repeats (e.g. CRISPFinder [GVP07]), Variable Length Tandem Repeats (VLTR) and Multi-period Tandem Repeats (MPTR) [HJ02], Variable Number Tandem Repeats (VNTR) [SS06]. Since the focus of our research on TRs at present is on the more classical forms of TRs we do not dwell longer on them. However, we just note that often methods for MPTR, VNTR, VLTR use standard TR finding as a subroutine, thus our proposed algorithm can increase also our ability to detect such higher order structures. Other methods need as input additional parameters, such as the target period of the TR [BW94].
Systematic comparison among TR finding tools and algorithms operating "ab initio", that is without support of specific biological data bases has been tackled in recent years [LRJ07,SBMP08]. A survey of problems on Tandem Repeats in the context of evolutionary mechanisms, such as the construction of TR Evolutionary Trees, is proposed in [Riv04] (see also [EG02]).

## 4 Our algorithm: TRStalker

Our definitions of TR. We used two different definitions of TRs:

- Neighboring TR (NTR): it is a string $X$, so that for each $i \in[1, . ., t-1], D_{E}\left(x_{i}, x_{i+1}\right) \leq \mu\left|x_{i}\right|$, for a user defined parameter $0 \leq \mu \leq 1$
- Stainer-STR with sum: it is a string $X=x_{1} x_{2} \ldots x_{t}$ for which there exists a Steiner string $\bar{x} \in \Sigma^{*}$ so that $\sum_{i \in[1, . ., t]} D_{E}\left(\bar{x}, x_{i}\right) \leq \mu|X|$, for a user defined parameter $0 \leq \mu \leq 1$. In other words the TR consists of $t$ duplications of a single Steiner consensus string $\bar{x}$ with mutations.

The initial filtering phase is the same in either case, while the verification phase will be different for the two possible definitions.

An example To focus on the main ideas, let us consider the very simple case of Exact TR. Given a string $X=x_{1} x_{2} . . x_{t}$, embedded in a random string $Y$, where $x_{i}=x_{1}$ for all $i$ and $\left|x_{1}\right|=k$. Consider the class of ungapped $q$-grams, and the distance between any two occurrences of a given $q$-gram (also called probes) in $Y$. For q-grams in $X$, the period $k$ will appear at least $(k-q+1)(t-1)$ times as the distance between homologous probes. More generally the distance $h k$, an integer multiple of $k$, will appear at least $(k-q+1)(t-h)$ times for each value $h=1, . ., t-1$. For gapped $q$-grams similar formulae hold. For values of $k$ and $t$ large enough,
the period $k$ and its integer multiples will occur more frequently than the expected number of occurrences in a random string, thus the empirical number of occurrences of the value $k$ and its multiple will tend to be in the higher part of a ranking by frequency. This observation holds true as long as the length of the superstring $Y$ is sufficiently limited so that the frequencies generated by the random portion of $Y$ do not overrun the frequencies generated by $X$. An exact characterization of such a distribution in terms of the parameters $k, t$, $q$ and $|Y|$ is complex since it can be characterized as the sum of non-independent random variables each with a negative binomial distribution. However we avoid the issue of characterizing exactly such a distribution (1) by splitting the input string into blocks of predefined length and limiting the analysis to each block separately, with mechanisms to deal with TR stranded across the block boundaries; (2) by ranking the periods by weighted frequency and exploring only the top $L$ positions (for $L=50$ in our experiments). Note that in most cases the top ranking periods not corresponding to tandem repeats will be discarded quickly when the positional density is considered, thus we can be very slack in choosing $L$ without incurring in a computational cost. The choice of block-length could be more critical, but experiments show that blocks of length within factor 10-20 of the length of the TR sought do work well, so here too we do not need to define a sharp threshold.

Gapped q-grams The presence of substitutions/insert/deletions has the effect that many instances of qgrams will be affected by error and a match will be missed, thus reducing the frequency counts for the period $k$. To cope with this effect we use gapped $q$-grams $[\mathrm{BK} 03, \mathrm{BK} 02]$ that are more resilient to the presence of substitutions/insert/deletions. As suggested by experiments in [BK02] just few gaps are sufficient to be effective, thus we will use the patterns all gapped q-grams with at most 2 gaps and 3 letters.

Period detection. Our overall strategy for period detection is as follows. We split the input string into blocks of predefined length and we scan the block $Y$ and record for each gapped q-gram $P$ in $Y$ the distance of each occurrence of $P$ to the 5 preceding and following occurrences (10 in total). If a distance and its integer multiples has been detected we form a composite period by summing the occurrences of the single simple periods. The candidate periods (simple and composite) are then weighted with the anti-smearing procedure described below, sorted by weight. The candidate periods are processed in order from the highest weight.

Anti-smearing weighting. If $q_{1}$ and $q_{2}$ are occurrences of the same probe at distance $k$, before the implant of mutations, the effect of insertion and deletions on the positions between $q_{1}$ and $q_{2}$ is to alter their distance so that a different period $k^{\prime}$ is detected. The difference $k-k^{\prime}$ is equal to the algebraic sum of number of insertions and deletions in the positions between $q_{1}$ and $q_{2}$. Assuming that any such position can be an insert or a deletion independently with the same probability, the random variable $k-k^{\prime}$ is distributed as a sum of independent r.v. of value $+1 /-1$ with the same probability [MR95,Mul93], thus, by a Chernoff argument, its tail distribution decays exponentially, for any fixed number of terms in the summation ${ }^{5}$. Inspired by the above observation we devise a weighting scheme that increments the total weight of period $k$ if another period of value $k^{\prime}$ is discovered in a near-by position, with weights that decay exponentially with $\left|k-k^{\prime}\right|$. More formally, at position $i$ of the input string we have pattern $P(i)=P$, let $P\left(j_{1}\right), . . P\left(j_{5}\right)$ be the next 5 occurrence of $P$ and $j_{w}-i=x_{w}$ the detected distances for $w=1, . .5$. We give to the period $x_{w}$ a weight $1+\sum_{y \in Q} 2^{-\left|x_{w}-y\right|}$, that favors the presence of nearby distances with similar values, and this weight is added to the cumulative array indexed by period value. Afterwards we enqueue all values $x_{w}$ in the queue position buffer $Q$. The queue is of a fixed length $|Q|=20$ and the last elements (older) are dequeued and discarded ${ }^{6}$. The weighting scheme can be easily adapted to the case in which the insert and delete probabilities are different.

Positional density. The second property that TRs exhibit is that of having the same period (simple or composite) detected by probes in near-by positions, so that we can define a notion of $k$-density, that is the density of probes that contribute to the counter for the candidate period $k$. We search for position of high $k$-density as candidates for the starting point of a TR. More formally, let $p$ be the simple period under investigation. Consider the set $Q_{p}$ of probes (i.e. substrings of $Y$ that contribute to the popularity of $p$ (taking of the two matching probes only the first one) compute the union $\bigcup Q_{p}$, thus if a position is shared by several probes it will be counted only once. Let $f: N \rightarrow\{0,1\}$ the characteristic function that

[^1]for each position in $Y$ denote the membership of that position to $\bigcup Q_{p}$. Consider the $p$-window smoothing of $f: F(i)=\sum_{j=i}^{i+p} f(j)$ that computes the $p$-density of the function $f$. Finally we define a threshold $t(p)$ proportional to the average $p$-density by a user-defined constant, and we consider as a candidate position set $C P(Y, p)=\{i \in N \mid F(i) \geq t(p)\}$. The output of the second phase is a sequence of pairs $(p, i)$ where $p$ is a candidate period and $i$ a candidate position.

Validation. In the third phase we take each candidate pair $(p, i)$ and we test explicitly whether there is a tandem repeat of period $p$ starting in position $i$ according to the definitions above. When using the definition STR we use the incremental method for computing generalized media strings proposed in [JABC03]. When using the definition NTR we use a standard wraparound dynamic programming technique (WDP) [FLSS93]. In this phase we discover the (maximum) repetition number of the TR eventually extracted. As a post processing we check for inclusion the TRs found and we filter out those TR completely enclosed in another one. For TR in the same position and length but different period we report all of them.

## 5 Experiments

### 5.1 Measurements of quality.

Let a TR be characterized by the triple: $(b, p, r)$, where $b$ is the initial position, $p$ the period, $r$ the repetition number. Thus the TR covers the positions in $Y$ from index $b$ to $b+r p-1$, we identify the TR with its segment $\operatorname{Seg}(T R)=[b, b+r p-1]$. Given two tandem repeats $T R_{1}$ and $T R_{2}$ their Jaccard coefficient $J C$ is:

$$
J C\left(T R_{1}, T R_{2}\right)=\frac{\left|S e g\left(T R_{1}\right) \cap \operatorname{Seg}\left(T R_{2}\right)\right|}{\left|\operatorname{Seg}\left(T R_{1}\right) \cup \operatorname{Seg}\left(T R_{2}\right)\right|}
$$

Given a TR $T R_{0}$ and a set of TRs: $T=\left\{T R_{1}, \ldots, T R_{s}\right\}$ we define the best-match $B M\left(T R_{0}, T\right)$ :

$$
B M\left(T R_{0}, T\right)=\arg \max _{t \in T} J C\left(T R_{0}, t\right)
$$

and the best-match-score BMS:

$$
B M S\left(T R_{0}, T\right)=\max _{t \in T} J C\left(T R_{0}, t\right)
$$

In our controlled experiments we will have the ground truth $T R_{0}$, and we will score the set $T$ of output TRs by the best match score. For a series of experiments we will take the average of the BMS. At first sight one might consider this metric as overly generous. However, consider two algorithms $A$ and $B$ having as input the same string $Y$ into which $T R_{0}$ has been embedded returning two candidate sets $T_{A}$ and $T_{B}$. Because of the above considerations, if $A$ and $B$ use the same definition of TR with the same parameters, the two sets are both composed of valid TRs (no false positives). We cannot rule out the existence of other TRs in $Y$ besides the embedded ones, so we cannot penalize the presence in $T_{A}$ and $T_{B}$ of elements different from $T R_{0}$ provided they are valid. The BMS score measures how well the best choice in $T_{A}$ and $T_{B}$ approximates $T R_{0}$.
Modified Jaccard Coefficient. Even if $X$ is a TR according to the definition, when we embed $X$ in a string $Y$, it is well possible that $X$ is not maximal in $Y$, thus if an algorithm reports $X^{\prime} \supset X$ there will be a slight penalization in the JC measure. This problem arises a number of times, thus we decided to use a customized version of JC where the denominator is changed as $\max \left(\left|\operatorname{Seg}\left(T R_{1}\right)\right|,\left|\operatorname{Seg}\left(T R_{2}\right)\right|\right)$.

### 5.2 Experimental Setup

TRStalker has been compared with respect to TRF and ATRHunter. We used the on-line version of the ATRHunter program that is available at its website ${ }^{7}$. The operation of the ATRHunter program can be customized according to the following parameters:

- match, mismatch, gap, terminal gap: scores for match, mismatch, indel (insertion or deletion) and terminal indels;

[^2]- maximum motif length: only repeats with motif length shorter than or equal to this value are reported;
- definition of approximate tandem repeat: it is possible to select the definition of TRs among the following ones:
- similarity between adjacent copies;
- average similarity between adjacent copies;
- minimum alignement score with a repeating copy.

The operation of the TRF program can be customized through the following parameters:

- match, mismatch, indel score: weights used to compute the alignement score;
- match and indel probability: the expected value of matches and indels between two motif repetitions;
- minimum score: the program reports only the TRs that obtain an alignement score higher than this one;
- maximum period: the program looks for TRs where the period size is no larger than this value.

Since the parameters of operation of the TRF web version cannot be completely customized, we used both the web-based tool and the downloadable binary (version 4.00).

### 5.3 Synthetic Data

We carried out a first set of experiments by using synthetic data. This allows a fine grained control on the amount of mutations introduced within the regions covered by the TRs. The sequences we gave as input to the programs have been built according to the following steps:

1. the background sequence is generated by selecting the four bases $\mathrm{A}, \mathrm{C}, \mathrm{G}$, and T with equal probability;
2. a perfect TR is embedded within the previous sequence, the TR is generated as $r$ repetitions of a motif with length $l$;
3. the region covered by the TR is mutated according to substitution, insertion and deletion probabilities ( $p_{s}, p_{i}$, and $p_{d}$ ); the number of substitutions, insertions and deletion for every repetition of the motif is exactly equal to $l p_{s}, l p_{i}$, and $l p_{d}$;
4. if the TR is a Steiner-STR mutations are introduced in every repeat with respect to the consensus motif, if the TR is a Neighboring-TR mutations are introduced with respect to the previous repeat.

The experiments have been carried out running ATRHunter with these values: match, mismatch, gap and terminal gap score equal to $10-10$ (the most permissive setting on the website); maximum motif length equal to 500 bp . In order to select the definition of TRs, we performed a preliminary set of experiments: the definition that gave the best results was the third one (minimum alignement score). In this case, ATRHunter reports only the TRs that have a score higher than a given threshold. The value of the threshold has been set to 30 .
For the web-based version of TRF all the experiments have been carried out with these parameters: match, mismatch, and indel score equal to 2 , 3 , and 5 respectively; maximum period equal to 500 ; minimum score equal to 30 . For the binary version we used the following ones: match, mismatch, and indel score equal to 2,3 , and 3 respectively; match and indel probability equal to 0.75 and 0.20 ; maximum period equal to 500 ; minimum score equal to 30 . The parameters of the experiments have been set so to make sure that the minimum allowed score for all the tools tested is attained on the input data.

Results For the experiments on Neighboring-TR (Figure 1) we tested TR with motifs of length from 60 to 300 , and a number of repeats from 2 to 8 . TRStalker has recall always above $95 \%$. TRF (binary) has a recall always above $80 \%$ except for TR with repeat number 2 for which the recall drops to $60 \%$. ATRHunter has recall of about $60 \%$. These experiments confirm the effectiveness of the new techniques for the initial filtering steps.
Results on Steiner Simple TR with motifs of length from 60 to 300 , and a number of repeats from 2 to 8 are shown in figure (Figure 2). Here we notice that all methods have degraded performance for longer motifs (above 200 bases) while TRStalker still manages to have recall above $60 \%$. For shorter motifs (of less than 100 bases) TRF (binary) is able to match TRStalker only when the repeat number is above 6 . Thus for a large range of values TRStalker attains the best performance in recall, or a matching one, always above $80 \%$.
The time performance of TRStalker has not been yet optimized. At the moment it is within an order of magnitude of TRF and ATRHunter.


Fig. 1. The total length of the input sequence is 10000 bp ; the type of TR is Neighboring-TR (NTR); the amount of substitutions, insertions, and deletions are equal to $10 \%$ of the motif length each (thus with total error allowed of $30 \%$ ). Every point is the average of 30 measurements and the $95 \%$ confidence intervals are shown.

### 5.4 Biological data

A very preliminary testing of TRStalker on biological sequences has confirmed the potential of our method for finding very fuzzy TRs not detected by TRF and ATRHunter. We tested the following sequences:

1. L3609 Homo sapiens germline T-cell receptor beta chain, complete gene - 684,973 bp long.
2. U43748 Homo sapiens frataxin (FRDA) gene, promoter region and exon - 2,465 bp long.
3. Saccharomyces cerevisiae Chromosome I - 230,208 bp long.

The three algorithms have been run with the setting used in the synthetic experiments on Steiner Simple TRs (thus with a very permissive acceptance policy). In general, none of the three algorithms generates all TRs found by the two others. For our purposes we concentrate on those found by TRStalker only. In Table 1 we report some very long TRs that were detected by TRStalker but missed by the other two methods. We check the motif/repeat alignments using the tool jaligner ${ }^{8}$ using the BLOSUM62, that confirms the good quality of the motifs found (see table 2, figure 3).

[^3]

Fig. 2. Total length of the input sequence is 10000 bp ; the type of TR is a Steiner Simple TR (Steiner-STR); the amount of substitutions, insertions, and deletions are equal to $10 \%$ of the motif length each (thus with total error allowed of $30 \%$ ). Every point is the average of 30 measurements and the $95 \%$ confidence intervals are shown.

## 6 Conclusions

TRStalker is a novel efficient heuristic algorithm for finding NP-complete tandem repeats in biological sequences. TRStalker aims at improving the capability of TR detection for a class of TRs and error range for which existing methods do not perform well. Initial testing on synthetic data are encouraging and we plan extensive testing on biological data as future work.

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(c) Third period (score: 399.00).

Fig. 3. TRs found by TRStalker in the Yeast Chromosome I sequence as reported in Table 1. Alignment with the consensus string of the first (a), second (b), and third repeat found (c). The score reported here has been computed with a BLOSUM62 score matrix.

| $\mathbf{N}$. | Sequence | Sequence Length | TR Start | TR End | TR Length | Repetitions |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Human Beta T-cell receptor (L36092) | 684,973 | 411,000 | 413,128 | 2,128 | 2.00 |
| 2 | Human Beta T-cell receptor (L36092) | 684,973 | 448,000 | 449,684 | 1,684 | 2.00 |
| 3 | Human Beta T-cell receptor (L36092) | 684,973 | 636,116 | 638,622 | 2,506 | 2.00 |
| 4 | Yeast Chromosome I | 230,208 | 43 | 314 | 271 | 3.01 |
| 5 | Homo Sapiens Frataxin (U43748) | 2,465 | 2,036 | 2,414 | 377 | 2.01 |

Table 1. Examples of TRs found by TRStalker and missed by TRF and ATRHunter.

| N. | Repeat | Length | Identity | Gaps | Score |
| :---: | :---: | :---: | ---: | ---: | ---: |
| 1 | 1 | 1107 | $805 / 1107(72.72 \%)$ | $91 / 1107(8.22 \%)$ | 3657.00 |
| 1 | 2 | 1093 | $895 / 1093(81.88 \%)$ | $70 / 1093(6.40 \%)$ | 4291.00 |
| 2 | 1 | 878 | $638 / 878(72.67 \%)$ | $85 / 878(9.68 \%)$ | 3045.50 |
| 2 | 2 | 866 | $716 / 866(82.68 \%)$ | $52 / 866(6.00 \%)$ | 3568.00 |
| 3 | 1 | 1300 | $1000 / 1300(76.92 \%)$ | $94 / 1300(7.23 \%)$ | 5206.00 |
| 3 | 2 | 1313 | $1004 / 1313(76.47 \%)$ | $120 / 1313(9.14 \%)$ | 5176.50 |
| 4 | 1 | 96 | $75 / 96(78.12 \%)$ | $12 / 96(12.50 \%)$ | 415.50 |
| 4 | 2 | 96 | $65 / 96(67.71 \%)$ | $12 / 96(12.50 \%)$ | 384.50 |
| 4 | 3 | 92 | $69 / 92(75.00 \%)$ | $10 / 92(10.87 \%)$ | 399.00 |
| 5 | 1 | 193 | $149 / 193(77.20 \%)$ | $10 / 193(5.18 \%)$ | 723.50 |
| 5 | 2 | 191 | $146 / 191(76.44 \%)$ | $5 / 191(2.62 \%)$ | 765.00 |

Table 2. Motif/repeats alignment scores computed by jaligner using the BLOSUM62 score matrix with gap open penalty set to 10.0 and gap extend penalty set to 0.5 .
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[^0]:    ${ }^{3}$ As of today, in the PubMed bibliographic database 39214 articles mention the expression "tandem repeat" in their title or abstract (1270 in their title).
    ${ }^{4}$ http://www.repeatmasker.org/

[^1]:    ${ }^{5}$ Technically, in our case, also the number of terms in the summation is a random variable distributed as a binomial distribution, thus a full formal proof would need to take this into account.
    ${ }^{6}$ Although the value of the queue length of $|Q|$ could be made dependent of other parameters, we noticed that good performance could be attained even with this simple choice.

[^2]:    ${ }^{7}$ http://bioinfo.cs.technion.ac.il/atrhunter/

[^3]:    ${ }^{8}$ http://jaligner.sourceforge.net/

