Consiglio Nazionale delle Ricerche

Quality Scores in Sponsored Search

M. Budinich

IIT TR-14/2011

Technical report

Luglio 2011

Istituto di Informatica e Telematica

Quality Scores in Sponsored Search

Michele Budinich*
m.budinich@imtlucca.it

July 6, 2011

Abstract

In sponsored search auctions (SSA) advertisers bid on particular keywords for the opportunity to display their ad besides the search results for the corresponding query. The amount of information on the details of these auctions is limited. Most of the scientific literature has focused on a simplified model, that nonetheless appears to explain many of the observed behaviors of these markets.

However one fundamental feature is missing from these models: the ability of the search engine to directly affect the rankings of the ads. The main tool search engines have to do this are *quality scores* (QS), which have already been considered in the current models, albeit in a limited way.

In this paper we extend the model to include quality scores that are independent of other properties of the auction, and show how this modifies the theoretical properties of the market.

Finally we also consider a scarcely studied cooperative behavior, in which a group of bidders collude (forming a so called *ring*) to decrease their overall payment and share the profits. We show that, in the sponsored search setting, rings are not always profitable, and consider the effect of quality scores in this scenario.

1 Introduction

Sponsored search is a form of on-line advertising in which search engines sell space on their result pages to advertisers. One fundamental property that differentiates sponsored search from previous advertising models is that advertisers pay only when a user clicks on their ad: what is actually being sold by the search engine are thus clicks by users who searched for a particular term. It would be unfeasible for the search engines to devise a price for clicks on each different query, both for the extremely large number of queries and for the uncommon nature of the good being sold (i.e. clicks). To overcome this problem, search engines have adopted *auctions*, which are a classical tool used by economists to determine the price for goods of unknown value when there are many potential buyers.

Advertisers choose a set of keywords they are interested in, and submit a bid for each one of them. When a query is made, the search engine selects some of the advertisers that have bid on that keyword, and displays a set of ads that maximize potential revenue. When an ad receives a click the search engine must determine how much to charge the bidder. There are numerous mechanisms with different properties, however the most popular in this context are based on the generalized second price auction (GSP). The GSP works as follows:

- bidders are ranked by decreasing bid order,
- when a bidder receives a click, his payment will be equal to the bid of the bidder directly below him (i.e. the next lower bid).

*This research was supported by a grant from the IIT - CNR in Pisa, Italy, and carried out under the supervision of prof. Bruno Codenotti.

Although GSP looks similar to the classical Vickrey-Clarke-Groves (VCG) mechanism [11, 5, 7] (in fact VCG and GSP coincide when there is only one available advertising slot), its properties are very different. The main departure from VCG is in the fact that truth-telling is not an equilibrium in GSP [6, 10]. However GSP is simpler to describe and to illustrate even to uneducated users, and thus it found its way into the practice of sponsored search auctions. Over the past few years, the algorithmic community has taken a close look at sponsored search auctions, touching in different ways this paradigm of online advertising, see, e.g. [1, 2, 8].

It is well known that the auctions in use by search engines are actually more complicated, namely they include *quality scores* (QS) (see, for instance, Nisan et al. [9], Edelman et al. [6], Varian [10]). Quality scores can be easily incorporated into the GSP model:

- bidders are ranked by decreasing bid times quality score order,
- when a bidder receives a click, his payment will be equal to the minimum bid required to maintain his position.

Notice that, when all quality scores are equal, this extended GSP is exactly as the one previously described.

In most of the literature (again, see, for instance, Nisan et al. [9], Edelman et al. [6], Varian [10]), quality scores have been considered as tightly connected to clickthrough rates (i.e. the number of clicks an ad receives). This indeed captures many features of sponsored search auctions, however, we believe, it does not include a fundamental characteristic of these auctions: that search engines have a direct control over the ranking of advertisements.

In this paper we give a different definition of quality scores, that highlights the possibility that the search engines might use these values to arbitrarily affect the ranking. We review some of the known results for the GSP in this setting, and give some insight on how important this kind of behavior might be for the search engine.

We also consider a well known strategic behavior that affects a wide variety of auctions: *bidding rings*. A bidding ring is an agreement among a set of bidders prior to the auction itself. In its most classical setting, only one of the bidders will actually participate in the auction. Due to the decreased competition he is likely to pay a lower price; by splitting part of these savings among the other ring participants, the coalition can ensure that every member is better off by participating in the ring. We show that, in sponsored search, bidding rings are not always profitable, and show how quality scores affect these behaviors.

The paper is organized as follows: Section 2 introduces notation and the classical definition of quality scores; Section 3 defines our new model for quality scores, comparing it to the previous ones. In Section 4 we study the theoretical properties of equilibria in our model, while in Section 5 we give an overview of bidding rings in sponsored search auctions.

2 Previous Models

Position auctions are the auctions used by most search engines to sell advertising slots that appear along search results when users make a query. Each advertisement slot $i = 1, \dots K$ has a specific clickthrough rate (CTR) α_i, and we assume that the CTR's are decreasing, i.e. $\alpha_1 > \cdots > \alpha_K > \alpha_{K+1} = 0$. Advertisers $j = 1, \dots, n$ are characterized by a private valuation v_j, that can be interpreted as the maximum amount advertiser j is willing to pay for each click. Advertisers submit bids b_j to the search engine, which assigns them to the available slots. Each time a search engine user clicks on an advertisement, say advertiser j in slot i, the search engine makes j pay a certain price p_j, so that j's utility can be expressed as

$$u_j = \alpha_i(v_j - p_j). \tag{1}$$

This simplistic model does not take into account the quality of an ad: its performance is based only on the slot it is shown in.

More complex models for sponsored search include an ad-specific component in the clickthrough rates; intuitively, in any fixed position i, a "higher-quality" ad gets more clicks. A general way to formalize this is to assign a specific CTR to every possible slot/bidder combination, i.e. bidder i in slot j will receive $\mathsf{CTR}_{i,j}$ clicks. Although this general model has been studied (see Aggarwal et al. [1]), most theoretical literature has focused on the separable clickthrough rates model. This simplified model assumes that each advertiser j has a clickthrough rate β_j so that, when he appears in position i the overall CTR will be

$$\mathsf{CTR}_{i,j} = \alpha_i \beta_j.$$

In this setting ads are ranked according to the product $\beta_j b_j$. Without loss of generality we assume that agents are indexed by their rank, i.e $\beta_1 b_1 > \cdots > \beta_n b_n$, so that bidder i is assigned to slot i. The GSP auction, as previously described, charges each bidder the minimum bid required to retain his position, i.e. bidder in position i will pay

$$p_i = \frac{\beta_{i+1} b_{i+1}}{\beta_i}, \tag{2}$$

And bidder i's utility is

$$\begin{aligned} u_i &= \alpha_i \beta_i \left(v_i - p_i\right) \\ &= \alpha_i \beta_i \left(v_i - \frac{\beta_{i+1} b_{i+1}}{\beta_i}\right). \end{aligned} \tag{3}$$

Although the introduction of the advertiser specific clickthrough rate components β appears to yield a more complex model, it is actually a special case of the simpler model in which all $\beta_j = 1$. To see why, consider the utility for a GSP (as in (1), with $p_j = b_{j+1}$) in which we scale bids and values of bidder i by β_i for all i:

$$u_i = \alpha_i(\beta_i v_i - \beta_{i+1} b_{i+1}),$$

which is exactly (3). Thus it is sufficient to study the GSP model without quality scores, since their introductions is equivalent to a scaling of bids and valuations.

3 A Different Take on Quality Scores

In the actual position auctions used by search engines quality scores assigned to bidders also have a slightly different meaning than the one previously described. Although they do reflect the ad-specific clickthrough rate, there is also a component to these values that has nothing to do with the number of clicks received[1]. These components could be used by the search engine to have some control over which ads are displayed, since the overall reputation of the sponsored search market is of paramount importance. Consider for example an ad that has a very misleading text (such as a well crafted spam message) and receives lots of clicks. Once the search engine has verified the illicit or dubious nature of the advertisement, it might wish to drive it away from the results page before cheated users stop clicking on ads.

We wish to study the effects of these particular components of the quality scores. For simplicity we will not consider the CTR components of the quality scores β, since even in our model, their introduction will be simply equivalent to a rescaling of bids and valuations.

In what follows, unless otherwise noted, with the term "quality scores" we intend just the non-CTR based components, which we call δ_j for advertiser j. Advertisers are sorted by decreasing δb. Again, without loss of generality, we assume that the advertisers are named according to this

[1] See, for instance, http://goo.gl/nRfxi.

order, so that advertiser i gets slot i. As before, payments are given by (2). However utilities are now different, since the quality scores δ do not affect the CTR:

$$\begin{aligned} u_i &= \alpha_i \left(v_i - p_i\right) \\ &= \alpha_i \left(v_i - \frac{\delta_{i+1} b_{i+1}}{\delta_i}\right). \end{aligned} \tag{4}$$

This utility cannot be considered just as a scaled version of (1), and, to the best of our knowledge, it has not been studied in the scientific literature.

4 Equilibrium

We follow Varian [10] and give some properties of the full information Nash equilibria of this auction. The results are similar to the ones presented by Varian for the next-price auction, however the quality scores δ play a central role.

As in the usual next-price auction, the equilibrium conditions are different if we consider the player's utility when moving to higher or lower slots. Namely, when a player moves to a slot above him, he has to overbid the agent currently in that slot. If bidder i increases his bid to move up to slot $j < i$ his payment will be

$$p_{i,j} = \frac{\delta_j b_j}{\delta_i}.$$

However, when he moves to a lower slot, he can bid much less that the current occupier's bid: he needs to bid higher than that agent's current price. So, if i lowers his bid to end in slot $k > i$, he will end up paying

$$p_{i,k} = \frac{\delta_{k+1} b_{k+1}}{\delta_i}.$$

Definition 1 (Nash Equilibrium)**.** *A Nash equilibrium (NE) set of bids satisfies, for all i,*

$$\alpha_i \left(v_i - \frac{\delta_{i+1} b_{i+1}}{\delta_i}\right) \geq \alpha_j \left(v_i - \frac{\delta_j b_j}{\delta_i}\right) \text{ for all } j < i, \tag{5}$$

$$\alpha_i \left(v_i - \frac{\delta_{i+1} b_{i+1}}{\delta_i}\right) \geq \alpha_j \left(v_i - \frac{\delta_{j+1} b_{j+1}}{\delta_i}\right) \text{ for all } j > i. \tag{6}$$

Note that we can rewrite (5) and (6), respectively, as

$$\alpha_i \left(v_i - p_i\right) \geq \alpha_j \left(v_i - p_{j-1} \frac{\delta_{j-1}}{\delta_i}\right) \text{ for all } j < i, \tag{7}$$

$$\alpha_i \left(v_i - p_i\right) \geq \alpha_j \left(v_i - p_j \frac{\delta_j}{\delta_i}\right) \text{ for all } j > i. \tag{8}$$

As for the next-price auction setting we consider a particular subset of these equilibria.

Definition 2 (Symmetric Nash Equilibrium)**.** *A symmetric Nash equilibrium (SNE) set of bids satisfies, for all i,*

$$\alpha_i \left(v_i - \frac{\delta_{i+1} b_{i+1}}{\delta_i}\right) \geq \alpha_j \left(v_i - \frac{\delta_{j+1} b_{j+1}}{\delta_i}\right) \text{ for all } j. \tag{9}$$

Again, (9) can be stated in terms of prices as

$$\alpha_i \left(v_i - p_i\right) \geq \alpha_j \left(v_i - \frac{\delta_j}{\delta_i} p_j\right) \text{ for all } j. \tag{10}$$

4.1 Basic Properties of SNE

As in the classical case, each agent has a non-negative surplus in a SNE.

Fact 3. *In a SNE, $v_i \geq p_i$ for all i.*

Proof. Consider the first non-visible slot, $K+1$, for which we know that $\alpha_{K+1} = 0$. By (10)

$$\alpha_i (v_i - p_i) \geq \alpha_{K+1} \left(v_i - \frac{\delta_{K+1}}{\delta_i} p_{K+1} \right) = 0,$$

and since $\alpha_i \geq 0$ this implies $v_i \geq p_i$. □

Now we consider the equilibrium prices, and show that, when scaled by δ, they are monotonically decreasing.

Fact 4. *In a SNE, for all i, $\alpha_{i-1}\delta_{i-1}p_{i-1} \geq \alpha_i \delta_i p_i$. Furthermore $\delta_{i-1}p_{i-1} \geq \delta_i p_i$, and if $v_{i-1} > v_i$ then $\delta_{i-1}p_{i-1} > \alpha_i \delta_i p_i$.*

Proof. Applying (10) to slots i and $i-1$ we get

$$\alpha_i (v_i - p_i) \geq \alpha_{i-1} \left(v_i - \frac{\delta_{i-1}}{\delta_i} p_{i-1} \right),$$

which can be rewritten as

$$\begin{aligned} \alpha_{i-1}\delta_{i-1}p_{i-1} &\geq \delta_i \left(\alpha_i p_i + v_i(\alpha_{i-1} - \alpha_i) \right) \\ &\geq \alpha_i \delta_i p_i, \end{aligned} \tag{11}$$

where the second inequality follows from the fact that $v_i(\alpha_{i-1} - \alpha_i) \geq 0$.

From Fact 3 we know that $v_i \geq p_i$, so that (11) implies

$$\begin{aligned} \alpha_{i-1}\delta_{i-1}p_{i-1} &\geq \delta_i \left(\alpha_i p_i + p_i(\alpha_{i-1} - \alpha_i) \right) \\ &\geq \alpha_i \delta_i p_i + \delta_i p_i (\alpha_{i-1} - \alpha_i) \\ &= \alpha_{i-1} \delta_i p_i, \end{aligned}$$

which completes the proof. □

We can now verify that, indeed, every SNE is a NE.

Fact 5 ($SNE \subset NE$)**.** *A SNE set of prices is also a NE set of prices.*

Proof. Since, by Fact 4, $\delta_{j-1}p_{j-1} \geq \delta_j p_j$,

$$\alpha_j \left(v_i - \frac{\delta_j}{\delta_i} p_j \right) \geq \alpha_j \left(v_i - \frac{\delta_{j-1}}{\delta_i} p_{j-1} \right).$$

Substituting p_{j-1} with (2) the right hand side term becomes

$$\alpha_j \left(v_i - \frac{\delta_j b_j}{\delta_i} \right).$$

By the SNE condition and the above inequalities we get that, in SNE,

$$\alpha_i \left(v_i - \frac{\delta_{i+1} b_{i+1}}{\delta_i} \right) \geq \alpha_j \left(v_i - \frac{\delta_{j+1} b_{j+1}}{\delta_i} \right) = \alpha_j \left(v_i - \frac{\delta_j}{\delta_i} p_j \right) \geq \alpha_j \left(v_i - \frac{\delta_j b_j}{\delta_i} \right),$$

which gives exactly (5). □

Next we show that, in SNE, the values $\delta \times v$ are in decreasing order.

Fact 6. *In SNE, $\delta_i v_i \geq \delta_j v_j$ if and only if $i < j$.*

Proof. Consider the SNE conditions (10) for agent i moving to slot j and agent j moving to slot i:

$$\begin{array}{ll} i \text{ to } j & \alpha_i(v_i - p_i) \geq \alpha_j(v_i - \frac{\delta_j}{\delta_i} p_j) \\ j \text{ to } i & \alpha_j(v_j - p_j) \geq \alpha_i(v_j - \frac{\delta_i}{\delta_j} p_i). \end{array}$$

They can be rewritten as

$$\begin{array}{ll} i \text{ to } j & \delta_i v_i(\alpha_i - \alpha_j) \geq \alpha_i \delta_i p_i - \alpha_j \delta_j p_j \\ j \text{ to } i & \delta_j v_j(\alpha_j - \alpha_i) \geq \alpha_j \delta_j p_j - \alpha_i \delta_i p_i. \end{array}$$

Summing these two we get

$$(\alpha_i - \alpha_j)(\delta_i v_i - \delta_j v_j) \geq 0,$$

which shows that $\delta_i v_i$ and $\delta_j v_j$ must be sorted in the same way as α_i and α_j. By our assumptions on CTRs this completes the proof. □

Finally we show that an important property of the classical setting holds here as well: it is sufficient to verify the SNE conditions for one slot above and one below to ensure they hold for all slots. For ease of notation we show this in a setting with 3 slots; we verify that if the conditions are met for slots 1 and 2 and slots 2 and 3, then they are also met for slots 1 and 3.

Fact 7. *If a set of bids satisfies* (10) *for $i+1$ and $i-1$, then it satisfies them for all j.*

Proof. The SNE conditions for slots 1 and 2 and slots 2 and 3 are:

$$\begin{array}{ll} 1 \text{ to } 2 & \alpha_1(v_1 - p_1) \geq \alpha_2(v_1 - \frac{\delta_2}{\delta_1} p_2) \\ 2 \text{ to } 3 & \alpha_2(v_2 - p_2) \geq \alpha_3(v_2 - \frac{\delta_3}{\delta_2} p_3), \end{array}$$

which we can rewrite as

$$1 \text{ to } 2 \quad \delta_1 v_1(\alpha_1 - \alpha_2) \geq \alpha_1 \delta_1 p_1 - \alpha_2 \delta_2 p_2 \tag{12}$$

$$2 \text{ to } 3 \quad \delta_2 v_2(\alpha_2 - \alpha_3) \geq \alpha_2 \delta_2 p_2 - \alpha_3 \delta_3 p_3. \tag{13}$$

By Fact 6 and our assumptions on CTRs we know that $\delta_1 v_1 \geq \delta_2 v_2$, so that (13) implies

$$\delta_1 v_1(\alpha_2 - \alpha_3) \geq \alpha_2 \delta_2 p_2 - \alpha_3 \delta_3 p_3. \tag{14}$$

Summing (12) and (14) we get

$$\alpha_1 (v_1 - p_1) \geq \alpha_3 \left(v_1 - \frac{\delta_3}{\delta_1} p_3 \right),$$

which is exactly the SNE condition for slot 1 and 3. The other direction is similar. □

Using these facts it is possible to give a characterization of SNE bids. Since, in SNE, agent in position i does not want to move to slot $i+1$

$$\alpha_i(v_i - p_i) \geq \alpha_{i+1} \left(v_i - p_{i+1} \frac{\delta_{i+1}}{\delta_i} \right).$$

Similarly agent in slot $i+1$ wouldn't prefer slot i, so

$$\alpha_{i+1}(v_{i+1} - p_{i+1}) \geq \alpha_i \left(v_{i+1} - p_i \frac{\delta_i}{\delta_{i+1}} \right).$$

By combining these two inequalities, we obtain that

$$\delta_i v_i (\alpha_i - \alpha_{i+1}) + \alpha_{i+1}\delta_{i+1}p_{i+1} \geq \alpha_i \delta_i p_i \geq \delta_{i+1}v_{i+1} (\alpha_i - \alpha_{i+1}) + \alpha_{i+1}\delta_{i+1}p_{i+1}.$$

Switching from prices to bids, and considering slots $i-1$ and i, the above becomes

$$\delta_{i-1}v_{i-1} (\alpha_{i-1} - \alpha_i) + \alpha_i \delta_{i+1} b_{i+1} \geq \alpha_{i-1}\delta_i b_i \geq \delta_i v_i (\alpha_{i-1} - \alpha_i) + \alpha_i \delta_{i+1} p_{i+1}. \tag{15}$$

Given that $\alpha_{K+1} = 0$ we can solve for the lower bound recursion, and see that the minimum equilibrium bids are

$$b_i = \frac{1}{\alpha_{i-1}\delta_i} \sum_{j \geq i}^{K+1} \delta_j v_j (\alpha_{j-1} - \alpha_j). \tag{16}$$

4.2 Efficiency

One of the important theoretical characteristics of GSP auctions is that, although it is easy to see that they are not truthful, they always admit a full information Nash equilibrium in which the payments are the same as in the VCG auction. We formally define this characteristic, as in Babaioff and Roughgarden [4].

Definition 8 (Efficient Mechanisms)**.** *A position auction mechanism is efficient if, for every valuation profile v, there exist a full information Nash equilibrium profile of bids b such that*

1. *slots are assigned in order of decreasing valuations (i.e. the efficient allocation in this setting) and*
2. *the equilibrium prices are the VCG prices.*

Notice that, even with utilities as defined in (4), the VCG mechanism will still assign higher slots to bidders with higher values, and the payments will be the same as in the classical setting. In other words, since δ values do not affect bidders' valuations, but just their bids, the VCG mechanism is unaffected by their introduction.

Fact 9. *If utilities are of the form (4), then, if for all i $\delta_i = 1$, the mechanism is efficient.*

Proof. If all $\delta_i = 1$, then we are in the classical GSP setting, for which the existence of efficient equilibria is well known (see Varian [10] or Edelman et al. [6]). □

Unfortunately a necessary condition for the mechanism to be efficient is not as clean due to some technicalities.

Fact 10. *Let the bidders be named so that $v_1 > v_2 > \ldots v_n$. Then, if the mechanism is efficient in SNE, it must be the case that, for $i = 2, \ldots n-1$, $\delta_i = 1$.*

Proof. Assume, by way of contradiction, that the mechanism is efficient but not all δ_i's (for $i \in [2, n-1]$) are set to 1. For ease of exposition assume that $\delta_2 \neq 1$ while all other $\delta_i = 1$. Consider a set of valuations $v_1 > v_2 > v_3 \cdots > v_n$. If $\delta_2 < 1$, then just set v_3 so that $\delta_2 v_2 < v_3 < v_2$. On the other hand, if $\delta_2 > 1$, just set v_1 so that $\delta_2 v_2 > v_1 > v_2$. By Fact 6 we know that, even if the mechanism is sorting bidders by $\delta_i b_i$, in SNE $\delta_i v_i \geq \delta_j v_j$ only if $i < j$. Thus, in both cases, our mechanism will not attain an efficient allocation. □

Notice that we exclude the δ_1 and δ_n from the statement. This is because, with all δ's to 1 and $\delta_1 > 1$ the bidders are still sorted according to valuation (and similarly for $\delta_n < 1$).

i	v_i	δ_i	$\delta_i v_i$
1	4	1	4
2	1	3	3
3	400	0.02	2
4	2	0.5	1

Table 1: Values used in our example.

slot	α_i	b_i	p_i
1	500	> 0.76	2.3
2	250	0.76	0.53
3	100	320	200
4	0	2	0

Table 2: Equilibrium bids and payments when using quality scores δ.

4.3 An Example

To give an idea of how the search engine might use δ's to affect the ranking consider the following example, in which there is a bidder (which we call 3) that has an very high value, i.e. is willing to make large bids. However, for the sake of this example, we assume he is a spammer, and that the search engine has assigned him a low quality score. Table 1 describes the scenario (note that the bidders are named in decreasing δv product). We can compute the minimum SNE bids (using (16)), having fixed some CTR values. Table 2 shows the bids and CTRs used. The expected revenue to the search engine in this case is 21283.3.

Given the same bidders and valuations, let's consider now a setting in which all δ's are set to 1, i.e. the classical next-price sponsored search auction. We also compute the SNE lower bound bids for this case (using the same CTRs). Values are reported in Table 3. Computing the revenue

i	v_i	b_i	p_i
3	400	> 2.8	2.8
1	4	2.8	1.6
4	2	1.6	1
2	1	1	

Table 3: Equilibrium bids and payments for the same bidders in a classical GSP setting.

in this case gives 1900.

Finally we can analyze how the revenue changes as the quality score for the spammer, δ_3, changes. The plot in Figure 1 compares the revenue as δ_3 increases, keeping all other parameters in the model fixed. As comparison we also show the GSP revenue and total value in the system (i.e. upper bound to the maximum revenue attainable).

The "jumps" in the revenue as δ_3 increases are due to the fact that bidder 3 moves to upper slots as this happens. It is interesting to note that, in Figure 1, the maximum revenue is obtained when δ_3 is just enough to put 3 in the first slot. Since all other values are fixed we know that, among other agents, 2 has the highest ranking. We can thus compute the value of δ_3 that maximizes the revenue as the value that solves this equation:

$$\delta_3 b_3 = \delta_2 b_2.$$

Note that, in this case, bidder 3 will pay exactly his bid.

Notice that, if we fix bids, the maximum revenue the search engine can obtain is by setting δ's so that $\delta_i b_i = \delta_j b_j \forall i, j$. This implies that every bidder is going to pay his bid (which is the

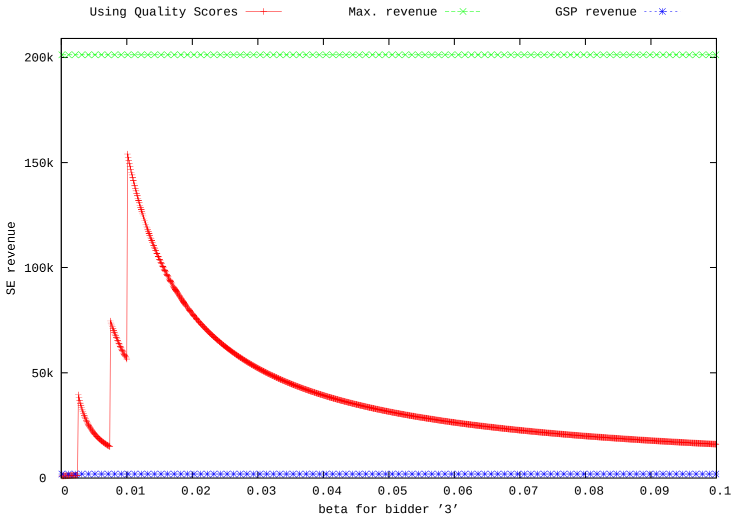

Figure 1: SE revenue as the quality score for bidder 3 increases.

maximum possible payment, since the auction rules guarantee that a bidder will never be charged more than his bid).

However this requires that the bids are fixed, or that the quality scores are dynamically computed once the bids arrive. If we assume this is the case, then, to the bidders, this type of auction will be indistinguishable from a first price auction (assuming the search engine breaks ties by giving higher slots to higher bids).

5 Rings

We now describe the effects of bidding rings in position auctions. The concept behind bidding rings is that two or more bidders collude and decide to bid as a single entity. This is a very well known behavior in auctions, and in most cases it gives a higher payoff to all bidders participating in the ring.

In sponsored search auctions it is conceivable that bidding rings might be very easy to form: consider for instance a company that is in charge of advertising for many firms (Ashlagi et al. [3] consider such possibility, albeit without side payments), or an advertiser that ensures prominent advertising space on his landing page for the other ring participants.

For a ring to be profitable it must be the case that the increased utility for the bidder that will actually make the bid (call him a) must be more than the loss by the bidders that do not participate anymore in the auction. When this happens, a will be able to pay the other ring participants.

For simplicity, throughout this section, we consider a 3-slot 4-bidders setting.

5.1 Classical Setting

We first consider the classical setting, in which all quality scores are set to 1. Assume bidder 1 wants to form a ring with bidder 2. Currently bidder 2's utility is

$$\begin{aligned} u_2 &= \alpha_2(v_2 - p_2) \\ &= \alpha_2(v_2 - b_3) \\ &= \alpha_2\left(v_2 - \frac{v_3(\alpha_2 - \alpha_3) + v_4\alpha_3}{\alpha_2}\right), \end{aligned} \tag{17}$$

where the last inequality follows from (16) with $\delta_i = 1$. Similarly, bidder 1's utility is

$$u_1 = \alpha_1 \left(v_1 - \frac{v_2(\alpha_1 - \alpha_2) + v_3(\alpha_2 - \alpha_3) + v_4\alpha_3}{\alpha_1} \right). \tag{18}$$

After the ring has formed, bidder 2 will get paid not to participate, and the new scenario is described in Table 4. Since 2 is not participating his utility from the auction will be 0, while we

slot	CTR	bidder i	b_i
1	α_1	1	b'_1
2	α_2	3	b'_3
3	α_3	4	b'_4
4	$\alpha_4 = 0$	-	-

Table 4: The auction after bidders 1 and 2 formed a ring. Notice that bidder 2 is absent.

can compute the new SNE and see that 1's utility will now be

$$\begin{aligned} u'_1 &= \alpha_1(v_1 - p'_2) \\ &= \alpha_1(v_1 - b'_3) \\ &= \alpha_1 \left(v_1 - \frac{v_3(\alpha_1 - \alpha_2) + v_4(\alpha_2 - \alpha_3)}{\alpha_1} \right). \end{aligned} \tag{19}$$

Since $v_2 \geq v_3 \geq v_4$ and $\alpha_3 v_4 \geq 0$, 1 will profit from the non-participation of 2, i.e. $u'_1 \geq u_1$. However, is this increased profit enough to pay 2? Formally, is $u'_1 - u_1 \geq u_2$? By substituting and simplifying, we can rewrite $u'_1 - u_1 - u_2$ as

$$\alpha_1(v_2 - v_3) - \alpha_2(2v_2 - 3v_3 + v_4) - \alpha_3(2v_3 + 3v_4).$$

First notice that this quantity might be negative, i.e. the ring might not be profitable. Fix v_2 and v_1, and let $v_3 \simeq v_4 \simeq v_2$, so that the above becomes

$$\simeq 0 - 0 - 5v_2\alpha_3 \leq 0.$$

To see that, in some cases, the ring might be profitable, just set $\alpha_1 = 4, \alpha_2 = 2, \alpha_3 = 1$, so that the above becomes simply $v_4 \geq 0$.

An interesting aside is that, in our examples of profitable vs. non-profitable ring, both conditions hold for *any* $v_1, v_2 \geq 0$. So that, for the two bidders participating in the ring, the profitability of their collusion depends on the external environment.

Finally we note that the same reasoning (with the same conclusions), can be carried out for the case in which bidder 2 is proposing the ring. In this case, the condition for the ring to be profitable will be that $u'_2 - u_2 \geq u_1$.

5.2 Quality Scores

When we introduce non-CTR quality scores in the model, the situation is similar. For brevity we omit the relevant expressions, since they are just adjusted versions of the ones for the classical setting.

An interesting difference in this model is that, once two or more bidders collude, we may assume that they will use the bidder with the highest value and the ad with the highest quality score, thus potentially incurring in greater profits. Even in this case, however, as in the previous section, a ring might not be profitable.

To give an idea of this possibility we consider the numerical values in Section 4.3, and show the revenue of the (always profitable) ring between players 2 and 3 in Figure 2.

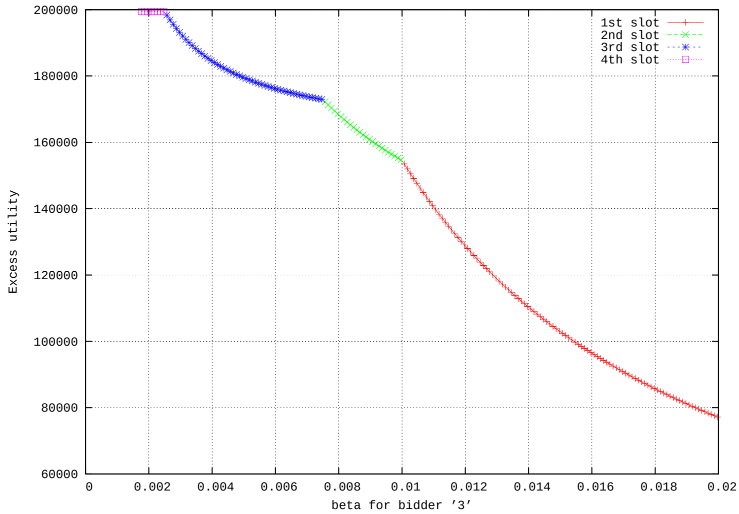

Figure 2: Excess revenue of a 2-3 ring as the quality score for bidder 3 increases.

6 Conclusions

In this paper we give an extended definition of quality scores, that take into account also components not directly related to clickthrough rates. This changes all the equilibrium and efficiency properties of the model, which now completely depend on quality scores.

Although there is little information available on how exactly search engines use these values, it is clear that the ability to directly and arbitrarily affect the auction ranking cannot be ignored. As can be seen by our example, without imposing limitations and conditions on the values the quality scores can assume, little can be said on the effects on these markets, since the numbers are completely arbitrary.

It would be interesting to include quality scores in a model considering a wider time frame, in which the search engine's would not only be interested in short-term revenue, but also on long-term goals, such as advertisements free of spammers or a more frequent cycling of advertisers on very popular keywords. In this model, the role of quality scores would be even more important, and their actual usefulness for the search engine might be theoretically verified.

References

[1] G. Aggarwal, A. Goel, and R. Motwani. Truthful auctions for pricing search keywords. In *Proceedings of the 7th ACM conference on Electronic commerce*, pages 1–7. ACM, 2006.

[2] S. Amer-Yahia, S. Lahaie, and D.M. Pennock. Towards a generic bidding standard for online advertising. In *Proceedings of the 9th ACM conference on Electronic commerce*. ACM, 2008.

[3] I. Ashlagi, D. Monderer, and M. Tennenholtz. Mediators in position auctions. *Games and Economic Behavior*, 67(1):2–21, 2009.

[4] M. Babaioff and T. Roughgarden. Equilibrium efficiency and price complexity in sponsored search auctions. In *Proceedings of the 6th Workshop on Ad Auctions*, 2010.

[5] E.H. Clarke. Multipart pricing of public goods. *Public choice*, 11(1):17–33, 1971.

[6] B. Edelman, M. Ostrovsky, and M. Schwarz. Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords. 2005.

[7] T. Groves. Efficient collective choice when compensation is possible. *The Review of Economic Studies*, 46(2):227–241, 1979.

[8] S. Lahaie. An analysis of alternative slot auction designs for sponsored search. In *Proceedings of the 7th ACM conference on Electronic commerce*, pages 218–227. ACM, 2006.

[9] N. Nisan, T. Roughgarden, E. Tardos, and V.V. Vazirani. *Algorithmic Game Theory*. Cambridge University Press New York, NY, USA, 2007.

[10] H.R. Varian. Position auctions. *International Journal of Industrial Organization*, 25(6):1163–1178, 2007.

[11] W. Vickrey. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance*, 16(1):8–37, 1961.