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Intercontact times in opportunistic networks and their impact on forwarding convergence

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## 1 Introduction

The increasing popularity of some new mobile technologies (smartphones for example) has opened new interesting scenarios in communications because of the possibility of a device to communicate with another one without using the wireless (or wired) network interfaces but taking advantages of the mobility of all the devices. In this direction, one of the most important evolution of Mobile ad hoc networks are opportunistic networks, that are self-organizing networks where there are not any guarantee of two devices to be linked with complete multi-hop path in any time. What a node has to do to deliver a certain message, is to find a space-time multi-hop path, that is portions of path that can carry on the message during the time until it reaches the destination. We can see an example in Figure 1: the source $S$ has to deliver a message to the destination D; the message can arrive at $D$ at time $t 3$, even if in $[t 1, t 3] S$ and $D$ are not directly linked. As nodes do not have any knowledges of the network topology, but only




Figure 1: Example of space-time path.
of the destination the massage have to arrive to, this way of delivering needs at any time to make some decisions, that are to whom has to be sent message and how many copies has to be sent.
For this reason many routing and forwarding algorithms were built, that we can classify in two big categories: dissemination-based routing and context based routing. Dissemination-based algorithms are the simplest and are based on the fact that any pieces of information of the networks are available and so, as a consequence, the message has to be sent in all the network. This way of proceeding obviously reduces the message delay, but on the other side can cause a lot of contention between messages and also network congestion. However to limiting these negatives effects, in general some restriction are imposed, such as the number of the copies present in the networks or the maximum of the hops
a message can do. Some examples of these algorithms are Epidemic Routing, MV, Network Coding. The other kind of routing are Context-Based Routing algorithms that, as the name suggests, uses some information about the context in which nodes operate to make a choice for the most suitable next hop. In this way we can reduce the problems about message duplications but the delay grows up. Furthermore it is important to underline that the context-based routing algorithms carry on an higher overhead because at each hop there are more information to take. Examples of these algorithms are Context-Aware Routing, MobySpace Routing.
All of these algorithms have qualities and deviancies, in particular the choice of a certain algorithm causes a delay of the message arrival. For this reason several studies has conducted to better understand which of these are much more reasonable: they have the aim of better describe the networks configurations, that is how often two any nodes are linked, how long they are linked, what time occurs between two links. For this goal, authors agrees to pay attention in an information about nodes that is intercontact time: we define intercontact time (ICT) between two nodes the duration of time between two consecutive direct contact of the nodes. It is obvious to note that shorter is the intercontact time between all nodes and shorter is the delay of the message to arrive to the destinations. Others elements can contribute to increase or reduce the delay, for example the shape of the domain in which nodes move or the velocity of the nodes. Therefore it was analyzing the intercontact time distribution in some experiments with real traces, that the most of parameters can be studied.
In this survey I would like to focus on the developing of the studies of intercontact time distributions that we can find in literature pointing out how this analysis can contribute to a better understanding of network behavior.

## 2 Developing of the analysis of intercontact time individual and aggregate distributions

For studying intercontact time distribution authors analyze several data sets collected in networks with mobile devices; those data sets can be analyzed in two different ways, that is distinguishing individual pairs or considering all together aggregate pairs. Studying the aggregate distribution is more manageable for many reasons, for example because it needs smaller samples or because it can give one compact characterization of all the network. However as the message pass from a node to another one, if we are interested in understanding the behavior of message in network we have to look at the developing of individual intercontact time between pairs. For this reason it is important to be sure that the distribution in aggregate can give us correct information about individual distribution.
The firsts works that analyze intercontact time do not pay so much attention to this fact and some of these pretended aggregate distribution to describe individual distribution. However, as we will see in the following, all the papers I describe point out some results that are interesting themselves and also to better understand the network behavior.
Before entering in more details, in the first paragraph there is a description of data sets that are used in literature to discover intercontact time behavior.

### 2.1 Data sets

To understand the properties of connectivity of devices, several empirical data are used that are collected in different ways, that is with different devices, different duration and different context. We can divide these data sets into three categories: Infrastructure-based data sets, Direct contact data sets and GPSbased data sets.
The first category consist of traces that reflect connectivity with existing infrastructure like Access Points (APs) or wireless mobile devices. As these traces have to be used for studying the device-to-device transmission, some assumptions are made. In APs case, where data are collected by traces between mobile device and an AP, it is assumed that two devices seeing the same AP would be able also to communicate. In the other case, where data are collected by cell phones, it is assumed that two devices seeing the same cell tower would be able to communicate.
The second data set consist of traces collected with special devices (for example iMotes) that communicate via Bluetooth directly each other, and were collected usually in conference or at University.
Finally, the third data set consist of private traces collected by tracking people thought GPS units. In this case, it is assumed that two mobile devices are in contact if their distance is less then or equal to 500 meters. We report for example in the table 1 data sets used in [3].

| Name | Technology | Duration | Devices | Contacts | Mean Inter-contact Time | Year |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCSD | WiFi | 77 days | 275 | 116,383 | 24 hours | 2002 |
| Vehicular | GPS | 6 months | 196 | 9,588 | 20.8 hours | 2004 |
| MITcell | GSM | 16 months | 89 | $1,891,024$ | 3.5 hours | 2004 |
| MITbt | Bluetooth | 16 months | 89 | 114,046 | 87 hours | 2004 |
| Cambridge | Bluetooth | 11.5 days | 36 | 21,203 | 14 hours | 2005 |
| Infocom | Bluetooth | 3 days | 41 | 28,216 | 3.3 hours | 2005 |

TABLE 1: Traces studied in [3]

### 2.2 Aggregate distribution

## Power law behavior

The very first paper that analyzes with attention the intercontact time distribution is [2]. Authors start from the observation of three data sets collected in real experiments: in two of these, data are traces measuring connectivity between clients and access points (AP) in several wireless network; in the last one, data are traces of direct contacts recorded using portable radio devices (iMotes). Observing the plots of the complementary cumulative distribution function (CCDF) of the intercontact time for all the aggregate pairs that we can see in figure 2, authors are suggested to conclude that in until the time of 1 day the plots are best fitted by power law distributions. So studying the quantile-quantile plot comparison between the empirical distribution found and three parametric models (exponential, power law and log-normal) they observe that up to one day the best fitting is with power law distribution.


Figure 2: Aggregated distribution of the intercontact time in eight data set experiments: (a) iMote-based experiments at Cambridge and Hong Kong, and the Toronto experiment, (b) iMote-based experiment at INFOCOM, and (c) data collected at UCSD, Dartmouth, and MIT.

It is important to note that before this paper all the mobility models created were characterized by an exponential tail, that is a delay time finite. On the other hand, the power law distributions have heavy tail; it is for this reason that for the first time authors in [2] conducted a survey to study the consequences of this heavy tail in two mobility models. So, they assume that intercontact time distribution follows a power law for each pair, that all pairs have the same coefficient $\alpha$ (homogeneous model), that the sequence of intercontact time are i.i.d. and that they are independent between pairs.

The first model analyzed is called Two-hop Relaying Algorithm and behave in this way: the source that have a message to send to a destination, forward it to the first node that meets. If it is the destination, the message is delivered in one hop, otherwise the node puts this message in a queue and forwards it to the destination as they meet. If there are more messages for the same destination the node delivers the messages starting from the first that has been forwarded. As in a real situation the amount of information that two nodes can exchange depends on the duration of the contact time, in both the models they make a distinction in the length of the contact time: the short case in which only one data unit can be sent between two nodes during each contact, and the long case in which all the queues in two nodes are completely emptied during each contact.
The result of the study is the following.
Theorem 1. Suppose there are $N$ devices which transmit data according to the two-hop relaying data algorithm. For a pair of source-destination devices $(s, d)$, let $t_{k}^{(s)}$ be the time when the $k$-th bundle is created at $s$ to be sent to $d$ and let $t_{k}^{(d)}$ be the time when it is delivered to d. Letting $D_{k}=t_{k}^{(d)}-t_{k}^{(s)}$ and $X$ the intercontact time between nodes, we have, starting from any initial condition:
(i) if $\alpha<2$,

$$
\lim _{k \rightarrow+\infty} \mathbb{E}\left[D_{k}\right]=+\infty
$$

(ii) if $\alpha>2$ and we assume that all contacts are long,

$$
\lim _{k \rightarrow+\infty} \mathbb{E}\left[D_{k}\right]=\bar{D}<+\infty
$$

(iii) if $\alpha>2$ and we assume that all contacts are short, when each source sends data to a unique and distinct destination with rate $\lambda<\frac{N-1}{2 \mathbb{E}[X]}$, then the delay of a bundle has finite expectation.

The second model is a generalization of the first one: the source sends not only one copy but $m$ copies to $m$ different nodes, that behave like in the first model. In this case authors demonstrate
Theorem 2. Let us consider a source destination pair ( $s, d$ ) and $t_{k}^{(s)}$, $t_{k}^{(d)}$ defined as in Theorem 1. We assume that all contacts are long.
(i) If $\alpha>2$, there exists a forwarding algorithm using only one copy of the data, with a finite expected delay, such that, starting from any initial condition,

$$
\lim _{k \rightarrow+\infty} \mathbb{E}\left[D_{k}\right]=\bar{D}<+\infty
$$

(ii) If $1<\alpha<+\infty, m \in \mathbb{N}$ is chosen such that $\alpha>1+\frac{1}{m}$ and the network contains at least $N \geq 2 m$ devices, there exists an algorithm using $m$ relay devices such that, in steady state,

$$
\mathbb{E}\left[D_{k}\right]=\bar{D}<+\infty
$$

(iii) If $\alpha \leq 1$, for a network containing a finite number of devices and any forwarding algorithm we have, starting from any initial condition,

$$
\lim _{k \rightarrow+\infty} \mathbb{E}\left[D_{k}\right]=+\infty
$$

We can summarize the results in long contact case in this way:

- if $\alpha>2$ both the algorithms converges to a finite expected delay (as in the exponential case);
- if $1<\alpha<2$ the two-hop relaying algorithm does not converge to a finite expected delay, but the delay grows without bound with time. By the way, if the network contains more than $\frac{2}{\alpha-1}$ nodes, we can find a $m$ greater than $\frac{1}{\alpha-1}$ so that the $m$-algorithm converge to a finite expected delay;
- if $\alpha \leq 1$ any forwarding algorithm does not converge to a finite expected delay.


## Power law behavior with exponential decay

The results of Chaintreau et al. are very surprising especially for the case of a power law distribution of intercontact time with exponent $\alpha \leq 1$; in fact in this case every forwarding models bring an infinite mean message delay. On the other hand a paper of Karagiannis et al. [3] analyzes some empirical results and it discovers that there is a dichotomy in CCDF of intercontact time. Authors in [3] studied six data sets and plotted aggregate distribution of CCDF of intercontact time: up to a certain time, in general half a day, their results agree with [2]'s onces, that is that the plot is fitted by a power law distribution; on the contrary, they observe that beyond this specific time, the distribution follows a faster decay. In fact, plotting the same results in a lin-log scale, instead of
a $\log -\log$ one, as we can see in Figure 3, they discover a linear behavior: that shows that the tail of intercontact time distribution is not a power law but an exponential one. Furthermore they underline that in two of these six cases analyzed there are a certain periodicity that seems to be of 24 hours.

Suggested by that observations, authors in [3] would like to show that this


Figure 3: Intercontact time CCDF for the six data sets in lin-log scale.
exponential decay can delete the problem of infinite message delay that, as we saw before, happens as a consequence of the heavy tail of power law distribution in mobility models studied in [2]. To demonstrate this fact they analyze two simple mobility models and find that they can represent the dichotomy discovered in data sets.
The first model is the Simple Random Walk: we assume there are $m$ sites, tagged as $0,1, \ldots, m-1$ where two devices can move according to a simple random walk, precisely if one device is in the site $i$, it can moves or in the site $i-1(\bmod m)$ or in the site $i+1(\bmod m)$ with the same probability. We denote with $X_{1}(n)$ and $X_{2}(n)$ the number of the site where the two nodes are at the time $n$. We than suppose that $m$ is even and that $X_{1}(0)=X_{2}(0)$ but $X_{1}(1) \neq X_{2}(1)$. The intercontact time between the two devices is then

$$
T=\min \left\{n>0: X_{1}(n)=X_{2}(n)\right\}
$$

For this model, authors in [2] prove the following theorem.
Theorem 3. Consider two independent simple random walks on a circuit of $m$ sites, where $m$ is assumed to be even. The intercontact time $T$ between the two random walks has the following properties:
(i) expected intercontact time: is $\mathbb{E}(T)=m-1$;
(ii) power law for an infinite circuit:

$$
P(T>n) \sim \frac{2}{\sqrt{\pi n}} \quad \text { for large } n
$$

(iii) exponentially decaying tail:

$$
P(T>n) \sim \varphi(n) e^{-\beta n} \quad \text { for large } n
$$

where $\varphi(n)$ is a trigonometric polynomial in $n$ and $\beta>0$.
The second model is the Random Waypoint on a Chain: it is a discrete time and space version of a simple random walk. It consists in $m$ sites, where devices move independently and site by site per time instant. In this case, two devices are in contact if they are in the same site at the same instant. This model is analyzed by simulation and we can see the CCDF of intercontact time in Figure 3: the first is plotted in log-log scale, instead the second in lin-log scale. Also these plots confirm the thesis of a power law distribution with exponential tail.


Figure 4: Intercontact time CCDF for random waypoint on a chain of $m=1000$ sites on log-log and lin-log scales.

## Possible causes of power law with exponential decay: bounded and unbounded domain

The most interesting observations about the reasons of the behavior described before are made by Cai et al. in [4] and [5]. In paper [4] they analyze first of all some model algorithms to show that making different assumptions in the domains of nodes, different distributions can be found. Then they uses these results to give a very intuitive interpretation of the empirical results observed.

Suppose in a first time that two nodes move in a finite domain $\Omega \subset \mathbb{R}^{2}$.
We assume then that the two nodes move according to the Random Waypoint algorithm: in the $n$-th step, the node $A$ selects a random waypoint in $\Omega$ and a uniform speed $V_{n}^{A}$ in $\left[v_{\min }^{A}, v_{\max }^{A}\right]$, where $0<v_{\min }^{A} \leq v_{\max }^{A}$. So $A$ goes straight to that destination with velocity $V_{n}^{A}$ and then pauses for a random time that we indicate as $\tau_{n}^{A}$. We suppose that all the random choices are independent. The first result is the following:

Theorem 4. If two nodes $A$ and $B$ move according to the Random Waypoint algorithm with zero time pauses, that is $\tau_{n}^{A}=\tau_{n}^{B}=0$ for all $n$, then there exist $a$ constant $c>0$ such that

$$
P\{X>t\} \leq e^{-c t}
$$

for sufficiently large $t$.
To arrive at the same result for the case with non-zero time pauses, there are a technical assumption that has to be make. Let $D$ the diameter of $\Omega$ and indicate with $T_{n}^{A}$ the longest time it takes for the node $A$ to reach two consecutive destination, precisely from the $(2 n-1)$-th up to the $2 n$-th one; in expression

$$
T_{n}^{A}=\frac{D}{V_{2 n-1}^{A}}+\tau_{2 n-1}^{A}+\frac{D}{V_{2 n}^{A}}+\tau_{2 n}^{A}
$$

We can call it as the "renewal" interval for $A$. Then we build $S_{n}$ by induction: $S_{0}=0$; for $n \geq 1$, starting from $S_{n-1}$ we skip two consecutive renewal intervals for $A$ and then one renewal interval for $B$, this time instant is $S_{n}$. We can call $Z_{n}=S_{n}-S_{n-1}$. (An illustration is available in Figure 5)
We are ready to give the result for non-zero time pauses.


Figure 5: Illustration for the construction of $Z_{n}$. Start from $S_{n-1}$, first skip two consecutive "renewal" points of node $A$, and then wait for the next following renewal point of node B . This defines $Z_{n}$.

Theorem 5. If two nodes $A$ and $B$ move according to the Random Waypoint algorithm with non-zero time pauses, supposing that for all $n$ it is $\mathbb{E}\left[Z_{n}\right]<+\infty$, the intercontact time $X$ between node $A$ and $B$ has exponential decay.

Another model analyzed is the Random Walk Mobility one, that is a discrete time and space model. Consider an unit square and divide it into $N=\frac{1}{(d \sqrt{2})^{2}}$ cells as we can see in Figure 6. We can numerate cells by $1, \ldots, N$ starting from the first row and going on: the cell $(n, m)$ have the number $\sqrt{N}(n-1)+m$ with $n, m=1, \ldots, \sqrt{N}$. At the beginning of a time slot one node jumps from $i$ cell to $j$ cell with probability $p(i, j)$, so we can see that $i$ can directly arrive to $j$ if $p(i, j)>0$; we assume that two cells are quite strongly connected: not only we require that exists a path to arrive from a cell to another one in finite steps, but also that the same propriety remains true if we delete one cell. We say that in this case the cells are 2-connected, note that this imply that for every couple of cells there are at least two different paths that link them. So we have:


Figure 6: A trajectory of our RWM model: The node can jump from a cell to any other cell with certain probability, and the boundary condition can be arbitrary ( $x_{1} \rightarrow x_{2}$ wrapping; $y_{1} \rightarrow y_{2}$ reflecting).

Theorem 6. Suppose that node $A$ moves according to the Random Walk Mobility model and that a node $B$ moves according to any arbitrary model; suppose that for any trajectory of node $B$, node $A$ meets node $B$ with positive probability, so that $P\{X>M\}<1$ for all $M<+\infty$. If the cells are also 2-connected, then there exists a constant $\gamma>0$ such that

$$
P\{X<t\} \leq e^{-\gamma t}
$$

for $T$ sufficiently large.
So far we supposed to be in a bounded domain $\Omega$ and we have ever found exponential decay. Now suppose to be in an unbounded domain that is $\Omega=\mathbb{R}^{2}$ and we will see, as in paper [4], that in this case we have a heavy tail instead of a an exponential tail.
Authors analyze as a model a Discrete-Time Isotropic Random Walk model (IRW): at the beginning of each time slot $k$, a node $A$ choses a random angle $\theta_{k}^{A} \in[0,2 \pi]$ and a random length $R_{k}^{A}$. In this case they prove the following:

Theorem 7. Suppose that two independent nodes $A$ and $B$ move according to the Discrete-Time Isotropic Random Walk Model. Then, there exists constant $C>0$ such that the intercontact time of nodes $A$ and $B$ satisfies

$$
P\{X>t\} \geq C t^{-1 / 2}
$$

for all sufficiently large $t$.
As asserts in the paper, all these results are quite intuitive. In a unbounded domain if two nodes do have not met for a long time, it is possible that they are going in different direction an it is possible that they will not meet for a long time; so, in this case we can say that the intercontact time has memory. Instead in a bounded domain node can hit the bound and that ensure that the choice has been "reset" by the change of the direction that hitting the bound causes; so in this case we can say that the intercontact time has not memory.

We have now to understand in which cases of real situation we can model the space like a bounded domain or an unbounded domain and especially where the switch from power law an exponential happens. In [4] it is underlined that
the question depends on the relationship between the timescale and and the size of the boundary. Considering the Random Walk Model, if we denote with $A(t)$ the position of the node $A$ at the time $t$, we have

$$
\mathbb{E}\left[|A(t)|^{2}\right]=\sum_{h, k=1}^{t} \mathbb{E}\left[R_{h} R_{k} e^{i(h-k)}\right]=\sum_{k=1}^{t} \mathbb{E}\left[R_{k}^{2}\right]=\xi^{2} t
$$

because the random variables are independent for $k \neq h$. So, during a time scale of $t$ the average displacement of the node $A$ would be $O(\sqrt{t})$. If the maximum of timescale is $t_{0}$ and the diameter of the domain is $D$, we have for $D \ll \sqrt{t_{0}}$ that the node rarely will hurt the boundary, so we can say that we are in a finite domain; instead $D \gg \sqrt{t_{0}}$ the node will not hurt the boundary many times, so we can say that we are in a infinite domain. We can see this situation in Figure 7. There we can see drown the trajectories of five nodes at first (a) in the interval $\left[0,10^{4} s\right]$ and then (b) in the interval $\left[0,10^{6}\right]$. Considering as domain $\Omega=[-500,500] \times[-500,500]$ so that $D=500$, in the first case as we have $\sqrt{t_{0}}=\sqrt{10^{4}}=100$ the domain appears unbounded (in fact rarely the boundary is hit), in the second case, as we have $\sqrt{t_{0}}=\sqrt{10^{6}}=1000$ the domain appears bounded (in fact the boundary is hit many times). We can also see from (a) to (b) the movements increase of about 10 times in the distances that is exactly $\sqrt{100}$ where 100 is the the increase of time.



Figure 7: Sample trajectories of five nodes following standard 2-D Brownian motion observed over time durations (a) $\left[0,10^{4} s\right]$ and (b) $\left[0,10^{6} s\right]$. All nodes start from the origin at $t=0$. The average displacement of each node scales as $O(\sqrt{t})$ from both (a) and (b).

## Analysis of the time of switch

A more detailed analysis of the characteristic time $\tau_{0}$ of switching between power law and exponential distribution, is made by the same authors in [5]. They call $\tau_{0}$ regenerative period, because, as we saw in the previous part, it represents the mean time takes to a node to hurt the boundary, after which it forget his previous position.
To study the relationship between $\tau_{0}$ and the diameter of the domain $D$, first of all we can look at Figure 8, where we can see two mobile nodes that move according to Isotropic Random Walk in a domain of different size (with diameter $D)$ with constant speed of $1 \mathrm{~m} / \mathrm{s}$ in a communication range of $d=50 \mathrm{~m}$ and whose step-length distribution is exponential with mean 100 m .
We can note in (a) that as the domain grows up, so $\tau_{0}$ increases. Let us call

$$
\sigma(t)=\sqrt{\mathbb{E}[|A(t)|]}=\xi \sqrt{t}
$$



Figure 8: Effect of domain size on the regenerative period $\tau_{0}$. IRW is used. Step length distribution is exponential with mean 100 m and speed is set to constant $(1 \mathrm{~m} / \mathrm{s})$. Communication range $d=50 \mathrm{~m}$. Simulation time: 106 seconds. (a) $P\left\{T_{I}>t\right\}$ in log-log scale; the inset is drawn in linear-log scale to show the power-law behavior more clearly. (b) shows the rescaled complementary cumulative distribution function, i.e., $P T_{I}>\eta t$ with factors $\eta=1,4,16$, proportional to $D^{2}$.
so, in order to the first argument of this subsection, we can say that $D=\sigma\left(\tau_{0}\right)$. As $\sigma(t) \sim \sqrt{t}$ for this model, we have that $\tau_{0} \propto D^{2}$, that explain our observation.
To confirm this fact, authors plotted the same results of Figure 8(a) using a rescaled CCDF, that is plotting $P\left\{T_{I}>\eta t\right\}$ instead of $P\left\{T_{I}>t\right\}$ where $\eta$ is chosen proportional to $D^{2}$ that is such that $\eta / d^{2}$ remain constant. Wee can see that in Figure 8(b): after the rescaling the three distribution almost coincide and so the thesis is confirmed.

Authors in [5] analyzed also the effect of mobility pattern on the location of $\tau_{0}$ studying the movement of two nodes in a fixed domain of size $D=2000 \mathrm{~m}$ but changing the mean step-length $\lambda=10 \mathrm{~m}, 50 \mathrm{~m}, 100 \mathrm{~m}$; the velocity of nodes is $1 \mathrm{~m} / \mathrm{s}$.
We can see in Figure 9(a) that longer is the mean step-length and smaller is $\tau_{0}$ : that is quite predictable as if the lengths of the travel node are longer in a same domain and earlier a node will hurt the boundary. Analyzing the model in a similar way to the other once for $D$, authors find easy that increasing the mean step-length by $k$ times, we have that $\tau_{0}$ becomes $1 / \sqrt{k}$ smaller. To confirm this fact, authors plotted the same results of Figure 9(a) using a rescaled CCDF, that is plotting $P\left\{T_{I}>\eta t\right\}$ instead of $P\left\{T_{I}>t\right\}$ where $\eta$ is chosen inversely proportional to $\sqrt{\lambda}$ that is such that $\eta^{2} \lambda$ remain constant. Wee can see that in Figure 8(b): after the rescaling the three regenerative periods $\tau_{0}$ almost coincide and so the thesis is confirmed.
We can summarize these two observation saying that $\tau_{0}$ increase as the domain increase and decrease as the tendency of moving straight increase; we call this tendency of moving straight degree of correlation.


Figure 9: Effect of correlations of the mobility model on the regenerative period $\tau_{0}$. Domain size is fixed to $2000 \mathrm{~m} \times 2000 \mathrm{~m}$. We use IRW models with exponential step-length distribution with means $\lambda=10,50,100$ to represent different degrees of "correlation" (stronger tendency of moving straight for longer duration) (a) $P T_{I}>t$ in log-log scale; (b) shows the rescaled CCDF, $P T_{I}>\eta t$ with factors $\eta=\sqrt{10}, \sqrt{2}, 1$, such that $\eta^{2} \lambda=$ Const.

## Analysis of the head of intercontact time

Even several studies agree power law to be the head of CCDF intercontact time, the first paper that analyses its shape in relation to several parameters of the model is [5]. To do this, authors use the Correlated Random Walk on Grid model both in one and two dimension; as they want to analyze the head of intercontact time, they suppose the domain is unbounded. At each time step a mobile node moves to one of its 2 or 4 neighbor sites respectively: the first step direction is randomly chosen; the following onces are chosen in the same direction of the previous with probability $p$, in the opposite with probability $q$, in others (only in two dimensional case) with probability $r$. So we have that in two dimensional case that $p+q=1$, in the other $p+q+2 r=1$.
Consider the two dimensional case and call $\rho=p-q$ : this correspond exactly to the degree of correlation, in fact larger is $\rho$ and more is the probability of the node to go straight. The result of [5] is:

Theorem 8. If $A$ and $B$ are two devices that moves independently in $\Omega=$ $\mathbb{R}$ according to the Correlated Random Walk on Grid one dimensional, being $\rho \in[0,1)$ the degree of correlation, for any $t>0, P\left\{T_{I}>t\right\}$ is an increasing function of $\rho$.

So bigger is the degree of correlation and heavier is the head of intercontact time CCDF.
In the two dimensional case, authors present the plot of a simulation that we can see in Figure 10. We can see that larger is $p$ and heavier is the tail as in theorem 8.

Another important result described in [5] is a list of several properties that are invariant from $\rho$. Consider three devices $A, B$ and $C$ that moves in a bounded domain according to CRW: in the one dimensional case, the domain is


Figure 10: Effect of correlations of the mobility model on the CCDF of intercontact time. CRW on 2-D grid $200 \times 200$ is used. $p=0.5,0.7,0.9$ cases are simulated with $q=r=(1-p) / 3$. (a) $P\left\{T_{I}>t\right\}$ in a log-log scale; (b) $P\left\{T_{I}>t\right\}$ in a linear-linear scale.
a ring of $N$ sites, where $N$ is odd; in the two dimensional case, the domain is a torus of $\sqrt{N} \times \sqrt{N}$ sites, where $\sqrt{N}$ is odd. Then we have the following results:

Theorem 9. Let $A, B$ be two independent mobile nodes following 1-D CRW on a ring with $N$ sites ( $N$ odd), and $C$ be a static node sitting on an arbitrary site on this ring. Let $T_{H}$ be the intercontact time between $A$ and $C$ that we can call also inter-hitting time of node $A$ to $C$ because $C$ is static; $T_{I}$ be the intercontact time time of nodes $A$ and $B ; T_{C}$ be the contact time of nodes $A$ and $B$, that is the time be the length of time interval in which $A$ and $B$ are in contact. Then, the average of $T_{H}, T_{I}$ and $T_{C}$ are all invariant with respect to the correlation coefficient $\rho \in(-1,1)$. Specifically,

$$
\mathbb{E}\left[T_{H}\right]=N-1 ; \quad \mathbb{E}\left[T_{I}\right]=2(N-1) ; \quad \mathbb{E}\left[T_{C}\right]=2
$$

Theorem 10. Let $A, B$ be two independent mobile nodes following 2-D $C R W$ on a ring with $N$ sites ( $\sqrt{N}$ odd), and $C$ be a static node sitting on an arbitrary site on this ring. Let $T_{H}$ be the intercontact time between $A$ and $C$ that we can call also inter-hitting time of node $A$ to $C$ because $C$ is static; $T_{I}$ be the intercontact time time of nodes $A$ and $B ; T_{C}$ be the contact time of nodes $A$ and $B$, that is the time be the length of time interval in which $A$ and $B$ are in contact. Then, the average of $T_{H}, T_{I}$ and $T_{C}$ are all invariant with respect to the correlation coefficient $\rho, p, q, r$. Specifically,

$$
\mathbb{E}\left[T_{H}\right]=N-1 ; \quad \mathbb{E}\left[T_{I}\right]=\frac{4}{3}(N-1) ; \quad \mathbb{E}\left[T_{C}\right]=2
$$

### 2.3 Individual distribution

Until now we focused on the aggregate intercontact time distribution, as most of authors made. In fact, over-interpreting the results of Chaintreau et al. and of Karagiannis et al. that derive from their hypothesis of an homogeneous network, most of literature assumes that distribution of aggregate and individual can be interchangeable. That is not true in general, especially for heterogeneous networks.

## Individual log-normal behavior

The firsts who point the focus on the individual distribution are Conan et al. in [6], who re-analyze previous data sets and find more precise results abouts pairwise distribution. The aim of their work is to demonstrate that in general the networks appear heterogeneous. To do this, being $\tau$ the process of intercontact time for a given pair, they plotted the CCDF of $\mathbb{E}(\tau)$ of three collected data sets: the first one is an Infrastructure-based data set, the other two are Direct contact data sets. We can see the plot in Figure 11
As shown, the pairwise intercontact time processes should not be considered


Figure 11: CCDF of mean inter-contact times $\mathbb{E}(\tau)$.
homogeneous, because the means span over three order of magnitude: 280.6 hours for Dartmouth, with a standard deviation of 210.5 hours; 4.9 hours for iMote, with a standard deviation of 5.6 hours; and 387.1 hours for MIT, with a standard deviation of 377.3 hours. This lead to the conclusion that results about fitting of intercontact time CCDF with power law with exponential decay has to be rediscussed.
In fact authors in [6] test intercontact time of each pair to know which is the best fitting for these distribution among exponential, log-normal and power law. They use the Cramer-Smirnov-Von-Mises test with an high confidence level, that is $\alpha=0.01$. Precisely, for each pair of node $(i, j)$ that have at least 4 contacts, they compare the CCDF of $I_{N}^{i, j}$, the $N$ intercontact times observed with the three functions:

- exponential distribution: $F_{i j}(x)=1-e^{-\lambda_{i j} x}$, where $\lambda_{i j}$ is the constant rate of exponential distribution;
- Pareto distribution: $F_{i j}(x)=1-\left(\frac{x_{m} i j}{x}\right)^{k_{i j}}$,
where $k_{i j}$ is the shape parameter of Pareto distribution;
- $\log$-normal distribution: $F_{i j}(x)=\frac{1}{2}-\frac{1}{2} \operatorname{erf}\left(\frac{\ln (x)-\mu_{i j}}{\sigma_{i j} \sqrt{2}}\right)$,
that is a distribution whose logarithm follows a normal distribution with parameters $\mu_{i j}$ and $\sigma_{i j}$.

In table 2 we can see the results of the test whose result suggest the best fitting is log-normal distribution, but exponential one is also a good fitting. A similar result is found by Tournoux et al. in [7] with other data.

|  | Dartmouth | iMote | MIT |
| :--- | :---: | :---: | :---: |
| Number of pairs tested | 20,211 | 755 | 2,174 |
| Exponential | $42.8 \%$ | $7.9 \%$ | $56.3 \%$ |
| Pareto | $34.2 \%$ | $12.3 \%$ | $26.5 \%$ |
| Log-normal | $85.8 \%$ | $99.4 \%$ | $96.9 \%$ |
| None | $12.9 \%$ | $0.4 \%$ | $2.7 \%$ |

TABLE 2: Fitting results

At first it seems quite strange that while intercontact time between pairs behave as log-normal or exponential distribution, the aggregate intercontact time follows a power law whith or without exponential cutoff. Authors in [6] prove that this apparent ambiguity has a mathematical reason. For proving analytical results, they use a model of heterogeneous network composed of $n$ nodes. For each pair of nodes $(i, j),\left(t_{i j}^{(n)}\right)_{n}$ is the sequence of time contact between $i$ and $j$. So let assume that the intercontact times $T_{i j}$ between $i$ and $j$ are i.i.d, that is independent identically distributes random variables. The intercontact times aggregated for all pair $\Theta$ is

$$
\Theta=\sup _{i<j} T_{i j} .
$$

We assume that the intercontact time distributions are stationary.
The fist case is the exponential one. Suppose there are $n$ nodes so that we have $K=n(n-1) / 2$ pairs; suppose that each $k$-th pair follow a intercontact time distribution that is a Poisson process with parameters $\lambda_{k}$. So let be $p_{k}$ the proportion of pair with parameter $\lambda_{k}$. So we have:

Theorem 11. For a very large network, replacing the rate $p_{k}$ with the analogous continuous $p(\lambda)$, for every $\alpha>0$ and $b>0$, there exist a function $p$ that is

$$
p(\lambda)=\frac{\lambda^{\alpha-1} b^{\alpha} e^{-b \lambda}}{\Gamma(\alpha)},
$$

such that the CCDF of intercontact time of the aggregate $\Theta$ is

$$
P\{\Theta>t\}=\left(\frac{b}{t+b}\right)^{\alpha}
$$

a power law.
In the same model hypothesis, replace the individual exponential laws with parameters $\lambda_{k}$ by log-normals with parameters $\mu_{k}$ and $\sigma_{k}$ and denote with $p_{\Theta}$ the probability density function of the aggregate $\Theta$.

Theorem 12. There exist a function $p_{\Theta}$ that is

$$
p_{\Theta}(t) \sim C_{1} t^{-1-C_{2}},
$$

such that the CCDF of intercontact time of the aggregate follows a a power law.

Authors in [6] also uses simulations to confirm these results. Their results are very significant because they proved that even if all individual intercontact times follow a exponential or log-normal distribution (but heterogeneous in parameters) the aggregate one can supply different aspect; in particular, as pairwise intercontact patterns are the key for routing and forwarding algorithms, it can be a mistake pretending aggregate analysis to explain individual pairs properties.

The results of [6] are examined in depth by [8], the last contribute that I analyze. In this paper we can find a complete characterization of the aggregate distribution, that Passerella et al. attribute not only at the single individual pair intercontact time distribution, but also at the distribution of the contact rates between pairs, that is the reciprocal of the pair average intercontact time. Before going on, it seems relevant to describe the analysis they make with contact rate traces. They start from three data sets collected using Bluetooth scanning and check the fitting with four different distributions:

- Pareto:

$$
f(x)=\left(\frac{b}{b+x}\right)^{\alpha}
$$

for $\alpha, b, x>0$;

- Pareto0:

$$
f(x)=\left(\frac{b}{x}\right)^{\alpha}
$$

for $\alpha, b>0$ and $x>b$;

- Gamma:

$$
f(x)=\frac{r^{\xi}}{\Gamma(\xi)} x^{\xi-1} e^{-r \xi}
$$

for $\xi, r, x>0$;

- exponential:

$$
f(x)=e^{-\mu x}
$$

for $\mu, x>0$.
At first, they use Maximum Likelihood method to derive the fitting parameters for the candidate distribution, and using these values, they use the Cramer-VonMises test with level 0.01 to decide if the fitting is appropriate or not. They look at the head of the contact rate distribution, because nearer is the rate to zero and more are the pairs with long intercontact times, that characterize a heavy tail in intercontact time distribution. We can see results in table 3: they indicate indicate that Gamma distribution cannot be rejected for all traces. However it is still possible that contact rates may follow other kind of distributions.

## The contact rate in aggregate distribution

For studying analytically the relationship between aggregate distribution on one side, and individual and contact rate on the other side, authors propose their own model. The starting point is a result of Karagiannis et al. that is the following:

|  | Traces |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Roller | Inf05 | Inf06 | Real Min |
| $\Gamma \quad \begin{aligned} & \text { Param. } \\ & \text { CvM } \end{aligned}$ | $\begin{gathered} \xi=4.43, r=1088 \\ \text { NR } \end{gathered}$ | $\begin{gathered} \xi=0.81, r=2469 \\ \text { NR } \end{gathered}$ | $\begin{gathered} \xi=7.17, r=63564 \\ \text { NR } \end{gathered}$ | $\begin{gathered} \xi=8.37, r=6.99 * 10^{6} \\ \text { NR } \end{gathered}$ |
| $\begin{array}{lc} \text { Exp } & \begin{array}{c} \text { Param. } \\ \text { CvM } \end{array} \\ \hline \end{array}$ | $\begin{gathered} r=245 \\ \mathrm{R} \end{gathered}$ | $\begin{gathered} r=3038 \\ \mathrm{R} \end{gathered}$ | $\begin{gathered} r=8866 \\ \mathrm{R} \end{gathered}$ | $\begin{gathered} r=8.35 * 10^{5} \\ \mathrm{R} \end{gathered}$ |
| Pareto0Param. <br> CvM | $\begin{gathered} \xi=0.67, r=0.001 \\ \mathrm{R} \end{gathered}$ | $\xi=0.29, r=5.87 * 10^{-6}$ | $\xi=0.68, r=3.16 * 10^{-5}$ | $\xi=0.82, r=4.86 * 10^{-7}$ |
| $\text { Pareto } \begin{gathered} \text { Param. } \\ \text { CvM } \end{gathered}$ | $\begin{gathered} \xi=0.83, r=0.001 \\ \mathrm{R} \end{gathered}$ | $\xi=0.30, r \underset{\mathrm{R}}{r=5.87} * 10^{-6}$ | $\xi=0.83, r \underset{\mathrm{R}}{=3.16 * 10^{-5}}$ | $\xi=1.19, r \underset{\mathrm{R}}{=} 4.86 * 10^{-7}$ |
| Head | 94\% | 50\% | 65\% | 34\% |

TABLE 3: Goodness of fit results for contact rate distributions

Theorem 13. In a network where $P$ pairs of nodes exist for which inter-contact times can be observed, the CCDF of the aggregate inter-contact times is:

$$
F(x)=\lim _{T \rightarrow+\infty} \sum_{p=1}^{P} \frac{n_{p}(T)}{N(T)} F_{p}(x)=\sum_{p=1}^{P} \frac{\theta_{p}}{\theta} F_{p}(x),
$$

where $T$ is the time during which intercontact times are observed, $n_{p}(T)$ and $N(T)$ are the number of intercontact times of pair $p(p \in\{1, \ldots, P\})$ and the total number of inter-contact times over $T$, respectively, $F_{p}(x)$ the CCDF of inter-contact times of pair $p, \theta_{p}$ the rate of intercontact times for pair $p$ and $\theta=\sum_{p} \theta_{p}$ the total rate of inter-contact times.

This result is quite intuitive: the aggregate distribution is a mixture of individual pair distribution, where each pair weights proportionally to the number of intercontact time observed.
In [8] authors extend this result in a continuous setting. Suppose we are in a network where contact rates are i.i.d and are distributed according to a continuous random variable $\Lambda$ with density $f(\lambda), \lambda \geq 0$. We assume that all pair follows the same individual intercontact time distribution. Then for every pair $p$, denote with $\lambda_{p}$ its rate; so the distribution parameters are set such that the resulting rate is $\lambda_{p}$. Note that in this way we can model an heterogeneous network, because every pair has different rate. For all pair with rate $\lambda, F_{\lambda}(x)$ is the CCDF of intercontact time; the CCDF of aggregate is then:

Theorem 14. In a network where the contact rates are distributed with density $f_{\lambda}$, the CCDF of the aggregate intercontact time is

$$
F(x)=\frac{1}{\mathbb{E}[\lambda]} \int_{0}^{\infty} \lambda f(\lambda) F_{\lambda}(x) d \lambda
$$

This result is similar to one shown in [6], but there does not appear the variable $\lambda$ in the integral (and so it is wrong). The direct dependence on $\lambda$ is quite intuitive: consider that a distribution with only two rates $\lambda_{1} \ll \lambda_{2}$, with the same probability. Even if the two components have the same weight (because of the probability is the same) the number of pairs whose contact rate is $\lambda_{2}$ would be higher that the onces related to $\lambda_{1}$.

## Networks with exponential individual intercontact time

Thanks to theorem showed before, authors in [8] analyses the scenarios that can presents with exponential and power-law individual distribution.
For exponential case we have:
Theorem 15. When individual pair inter-contact times are exponentially distributed, aggregate inter-contact times are distributed according to a Pareto law with parameters $\alpha>1$ and $b>0$ iff the contact rates follow a Gamma distribution $\Gamma(\alpha-1, b)$, i.e.

$$
F(x)=\left(\frac{b}{b+x}\right)^{\alpha} \Longleftrightarrow f(\lambda)=\frac{b^{\alpha-1} \lambda^{\alpha-2} e^{-b \lambda}}{\Gamma(\alpha-1)}
$$

Note that it is an important theorem because the analysis of traces has shown that individual intercontact time distribution can be exponential ([6]) and contact rates can be Gamma (as we showed before).
The next step is suppose that the contact rate follows Pareto or Pareto0 distribution: intuitively, as the latter takes value close to zero, while the other one not, according to the heuristic idea shown before, in Pareto0 case we should have a heavy tail, instead in Pareto not. In fact we have:

Theorem 16. When individual pair intercontact times are exponentially distributed and rates are drawn from a Pareto distribution whose CCDF is $F(\lambda)=$ $\left(\frac{k}{\lambda}\right)^{\gamma}$ for $\lambda>k$, the tail of the aggregate intercontact times decays as a power law with exponential cutoff, i.e.:

$$
F(\lambda)=\left(\frac{k}{\lambda}\right)^{\gamma}, \lambda>k \Longrightarrow F(x) \sim \frac{e-k x}{k x} \text { for large } x .
$$

This result confirm our intuition. Moreover this fact tell us that, in contrary of what literature has supposed, an aggregate distribution with an exponential decay can derive also when individual intercontact time are exponential. For Pareto distribution we have:

Theorem 17. When individual pair intercontact times are exponentially distributed and rates are drawn from a Pareto0 distribution whose CCDF is $F(\lambda)=$ $\left(\frac{k}{k+\lambda}\right)^{\gamma}$ for $\lambda>0$, the tail of the aggregate inter-contact times decays as a power-law with shape equal to 2, i.e.:

$$
F(\lambda)=\left(\frac{k}{k+\lambda}\right)^{\gamma}, \lambda>0 \Longrightarrow F(x) \sim \frac{1}{x^{2}} \text { for large } x .
$$

In this case, while Pareto0 is close to zero, we obtain a heavy tail. Authors use simulation to prove their result: we can see the result in Figure 14 for Pareto and in Figure 15 for Pareto0.

## Networks with power law individual intercontact time

The next part of [8] contains the analysis of what happens if the individual CCDF of intercontact time follows a power law.


Figure 12: $F(x)$, contact rates $\Lambda \sim \operatorname{Pareto}(\gamma, k)$.


Figure 13: $F(x)$, contact rates $\Lambda \sim \operatorname{Pareto} 0(\gamma, k)$.

The first case is the Pareto0 one that is

$$
F_{\lambda}(x)=\left(\frac{q}{q+x}\right)^{\eta}, \eta, q, x>0
$$

As in this case we have two parameters $q, \eta$, authors study two different cases in each of them they fix one of those parameters.

Theorem 18. When individual pair intercontact times follow a Pareto0 distribution whose CCDF is in the form $F_{\lambda}(x)=\left(\frac{q}{q+x}\right)^{\eta}$ and the shape parameter $\eta$ is the same across all pairs, irrespective of the distribution of contact rates, the tail of the distribution of aggregate intercontact times decays, for large $x$, as a power law with exponent $\eta$, i.e. $F(x) \sim x^{-\eta}$ provided $\eta>1$ and the following condition holds true:

$$
\int_{0}^{\infty} \lambda f(\lambda)\left(\frac{\eta-1}{\lambda}\right)^{\eta} d \lambda<+\infty
$$

where $f(\lambda)$ is the density of the contact rate distribution.
So in the case of Pareto0 with same the same shape parameter, for every distribution of contact rates the aggregate distribution presents a heavy tail with the same exponent. Fixing the scale parameter, we obtain the following.
Theorem 19. When individual pair intercontact times follow a Pareto0 distribution whose CCDF is in the form $F_{\lambda}(x)=\left(\frac{q}{q+x}\right)^{\eta}$ and the scale parameter $q$ is the same across all pairs, if contact rates follow a Gamma, Pareto0, or Pareto distribution, the tail of the distribution of aggregate intercontact times decays, for large $x$, as a power law. Specifically, the following holds true:

- if contact rates follow a Gamma distribution $\Gamma(\alpha, b)$ then

$$
F(x) \sim \frac{C}{x(\ln x)^{\alpha+1}}
$$

holds true for large x, $C$ being a constant greater than 0 . Moreover, it can also be shown that $\lim _{x \rightarrow \infty} F(x)>\frac{C}{x^{1+\beta}}$, for any $\beta>0$;

- if contact rates follow a Pareto0 distribution $\operatorname{Pareto} 0(\gamma, k)$ then

$$
F(x) \sim \frac{C}{x g(x)}
$$

holds true for large $x, C$ being a constant greater than 0 and $g(x)$ being a function that, for large $x$, goes to 0 more slowly than $x^{-\beta}$ for any $\beta>0$. Therefore, $\lim _{x \rightarrow \infty} F(x)>\frac{C}{x^{1+\beta}}$ holds true for any $\beta>0$;

- if contact rates follow a Pareto distribution Pareto $(\gamma, k)$ then

$$
F(x) \sim \frac{C}{x^{k q+1} \ln x}
$$

for large $x, C$ being a constant greater than 0 . Therefore, $\lim _{x \rightarrow \infty} F(x)>$ $\frac{C}{x^{1+k q+\beta}}$ holds true for any $\beta>0$.
So in the case of Pareto0 with the same scale parameter and the contact rates follows Gamma, Pareto0 and Pareto distribution, the aggregate intercontact time presents a heavy tail.
Using the same approach that Pareto0, now consider the case in which individual intercontact time follow a Pareto distribution with parameters $q, \eta$, that is

$$
F_{\lambda}(x)=\left(\frac{q}{x}\right)^{\eta}, \eta, q>0 x>q
$$

Fixing the shape parameter $\eta$ we have
Theorem 20. When individual pair intercontact times follow a Pareto distribution whose CCDF is in the form $F_{\lambda}(x)=\left(\frac{q}{x}\right)^{\eta}$ and the shape parameter $\eta$ is the same across all pairs, irrespective of the distribution of contact rates, the tail of the distribution of aggregate intercontact times decays, for large $x$, as a power law with exponent $\eta$, i.e. $F(x) \sim x^{-\eta}$, provided $\eta>1$ and the following condition holds true:

$$
\int_{0}^{\infty} \lambda f(\lambda)\left(\frac{\eta-1}{\lambda \eta}\right)^{\eta} d \lambda<+\infty
$$

where $f(\lambda)$ is the density of the contact rate distribution.
This case is similar to the Pareto0 once, as the aggregate has the same shape of individual intercontact time. Unfortunately, fixing the scale parameter authors in [8] cannot find any closed form for aggregate distribution.

The last case is when the individual distribution follows a Pareto with exponential tail, that is

$$
F_{\lambda}(x)=\frac{\Gamma(1-\eta, \mu x)}{\Gamma(1-\eta, \mu q)}, \eta>1, \mu, q>0
$$

where $\eta$ and $q$ are the shape and scale parameters of Pareto, while $\mu$ represents the rate of exponential part. As Gamma function is not a closed form, the only cases that we can analyze precisely are $\mu q \sim 0$ or $\mu q \sim+\infty$; however, the meaning of these parameters suggest that the only cases to analyze are $\mu \rightarrow+\infty$ and $q \rightarrow 0$.
When $\mu \rightarrow+\infty$, it means that exponential decay comes quickly. The significant case is when $\eta$ is fixed between pairs.

Theorem 21. When individual pair intercontact times follow a Pareto distribution with exponential cutoff with shape, scale and rate parameters $\eta, q$ and $\mu$, if $\mu$ is very large and $\eta$ is the same across all pairs, then the CCDF of the aggregate intercontact times $F(x)$ decays, for large $x$, as a Pareto distribution with exponential cutoff with the same shape and rate parameters $\eta$ and $\mu$, i.e. $F(x) \sim(\mu x)^{-\eta} e^{-\mu x}$, provided the following condition holds true

$$
\int_{0}^{\infty} \frac{\lambda f(\lambda)}{\Gamma\left(1-\eta, \frac{\mu}{\lambda}\right)} d \lambda<+\infty
$$

Also in this case, the aggregate and individual intercontact time distribution is the same.

The other case is when $q \rightarrow 0$ that corresponds to the possibility of intercontact time close to 0 . Using again the approach of fixing one of the two parameters $\mu$ and $\eta$, we have that if $\mu$ is fixed no closed forms are obtained. Fixing $\eta$ there is the following result.

Theorem 22. When individual intercontact times follow a Pareto distribution with exponential cutoff, whose scale parameter tends to 0 and whose shape parameter is fixed across all pairs, the distribution of aggregate intercontact times $F(x)$ presents, for large $x$, a heavy tail, provided $\eta \in(0,1)$ holds true. Specifically:

- if contact rates follow a Gamma distribution $\Gamma(\alpha, b)$ then

$$
\lim _{x \rightarrow \infty} F(x)=C x^{-(\alpha+1)},
$$

$C$ being a constant greater than 0;

- if contact rates follow a Pareto0 distribution then

$$
\lim _{x \rightarrow \infty} F(x)=C x^{-2}
$$

$C$ being a constant greater than 0.
In this case we have no heavy tail in individual but heavy tail in aggregate. These results are validate by simulations as we can see in figure 16 and 17 .

## 3 Conclusions

Routing and forwarding algorithms need an accurate analysis of intercontact time distribution of nodes for understanding the properties of delay message.


Figure 14: $F(x)$, individual ICT $X \sim$ Pareto0 with fixed shape. Contact rates are Pareto $(2,0.01)$ or $\Gamma(2,1)$.


Figure 15: $F(x)$, , individual ICT $X \sim$ Pareto with exponential cutoff with small scale and fixed shape. Contact rates are $\Gamma$.

In this survey we recalled the most important contributes in this argument trying to underline the different approaches, that are aggregate and individual one. By analysis of aggregate homogeneous network we could find some important features, as for example the relationship between the space where nodes move and their velocity. However the last contributes showed that the networks appear homogeneous; for this reason, there are cases in which looking at aggregate intercontact time distribution can not say anything about individual intercontact time, instead in other looking at aggregate and individual intercontact time is the same.
In table 4 there is a summary of the cases that can be obtained in aggregate by different individual pairs; the table is made by Passerella et al. in [8]. All this accurate analysis tells us that the study of aggregate can not be always taken as representative of individual intercontact time; when it is true it is possible to analyze only aggregate distribution to study the routing and forwarding algorithm. In other cases, because of the different behavior, we can not say anything about message delay using the aggregate distribution, but it is necessary to collect additional information about individual pairs to understand if algorithm can be used or not.

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| Individual ICT distributions | Contact rate distribution | Aggregate ICT distribution | Can use the aggregate only? |
| :---: | :---: | :---: | :---: |
| Pareto, fixed shape $\alpha$ | whatever | Power-law, shape $\alpha$ | Y |
| Pareto0, fixed shape $\alpha$ | whatever | Power-law, shape $\alpha$ | Y |
| Pareto with cutoff, fixed shape $\alpha$ | whatever | Power-law with cutoff, shape $\alpha$ | Y |
| Exponential | Gamma, shape $\alpha$ | Power-law, shape $\alpha+1$ | N |
| Exponential | Pareto0 | Power-law, shape 2 | N |
| Exponential | Pareto | Power-law with cutoff | N |
| Pareto, fixed scale $q$ | Gamma, Pareto0 | Power-law, shape 1 | N |
| Pareto, fixed scale $q$ | Pareto, scale $k$ | Power-law, shape $1+k q$ | N |
| Pareto with cutoff | Gamma, shape $\alpha$ | Power-law, shape $\alpha+1$ | N |
| Pareto with cutoff | Pareto0 | Power-law, shape 2 | N |

TABLE 4: Summary of the main results of [8]
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