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Dong-Joon Lim  
Portland State University

Timothy R. Anderson  
Portland State University, tim.anderson@pdx.edu

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Incorporating Value Judgment in Technology Forecasting Using Data Envelopment Analysis

Dong-Joon Lim, Timothy R. Anderson
Dept. of Engineering and Technology Management, Portland State University, USA

Abstract—Technology Forecasting using Data Envelopment Analysis (TFDEA) has been employed to a wide range of applications because of its ability to model complex tradeoffs. The very name indicates that it is based on Data Envelopment Analysis (DEA) which has strength that it doesn't require fixed a priori weighting scheme. Instead, it adopts dynamic weighting scheme that each data point can choose their best possible weights. However, it is well known that this flexibility may result in extreme weights that may be considered unrealistic in certain applications and has been one subject of DEA researches. This paper extends the standard TFDEA model to incorporate value judgment in assessment to refine the analysis framework. The proposed model is applied to the Liquid Crystal Display (LCD) industry to address impact of various weight restrictions on the technology forecast results.

I. INTRODUCTION

Frontier analysis (or best practice) methods that model the frontier of the technology rather than model the average use of the technological possibilities have become popular in modern benchmarking studies [1]. As an example, Technology Forecasting using Data Envelopment Analysis (TFDEA) has shown its usefulness in a wide range of applications since the first introduction in PICMET ’01 [2]. This approach has a strong advantage in capturing technological advancement from the state of the arts (SOAs) rather than being influenced by the inclusion of mediocre technologies.

Data Envelopment Analysis (DEA), which underlies TFDEA process, is unique in that it allows each Decision Making Unit (DMU) freely choose its own weighting scheme, and as such, the efficiency measure will show it in the best possible light [3]. Consequently, the weights chosen by DEA in assessing one unit’s efficiency may be completely different from the weights selected for another unit [4]. This dynamic weighting scheme has shown practical advantages in a wide range of applications especially when the efficiency measures involve complex tradeoffs that are difficult to model by one universal set of weights. In addition, this approach can generate a reference set, or convex combinations, that can be used as reasonable benchmarks for each DMU to improve its efficiency.

However, in some cases, the analyst may wish to incorporate some prior views that the application area provides about the relative worth of inputs and outputs in the assessment. Allen et al. [5] categorized such cases motivating the use of value judgments in DEA as follows;

- To incorporate prior views on the value of individual inputs and outputs
- To relate the values of certain inputs and/or outputs
- To incorporate prior views on efficient and inefficient DMUs
- The assessed efficiency needs to respect the economic notion of input/output substitution
- To enable discrimination between efficient units

Theoretical expansion of DEA can deal with foregoing situations by imposing additional constraint called Weights Restriction (WR) in the multiplier model in various ways which are discussed in the next section.

Note that WR constitutes additional constraints to the original formulation, and therefore, the efficiency scores obtained with the WR will never be improved by their imposition. Since TFDEA iterates DEA process to capture the change of efficiency scores over time, imposing WR may render parts of the technology frontier no longer represent SOA. As a result, it is expected that the calculation of average Rate of Change (RoC) would be affected by dropping technologies that could have had influential RoCs (either high or low) without WR in corresponding years. The purpose of this paper is to develop multiplier TFDEA model that can employ WR and to address its impact as well as possible usages.

II. WEIGHTS RESTRICTIONS

This section briefly reviews well known WR implementation methods in DEA multiplier model. There are three broad types of methods; restricting weights, restricting virtual weights, and altering Production Possibility Set (PPS). Mathematical notations of first two methods are shown in (1) which assumes Input-orientation and Variable Returns to Scale (VRS) DEA model. The variable $x_{ij}$ represents the $i$th input and $y_{rj}$ represents the $r$th output of technology $j$. The variables for the linear program underlying DEA are $v_i$, $u_r$, $w$. The variables $v_i$, and $u_r$ represent the weights that DMU assigns to each one of its inputs and outputs so that its efficiency will be maximized. The value of variable $w$, which is dual to the convexity constraint in the envelopment model, reflects the impact of scale size on the productivity of a DMU. Here it is set as free to assess efficiency under VRS.

A. Restricting weights

There are three major techniques belong to this method; Absolute WR, Assurance regions of type I (AR-I), and Assurance regions of type II (AR-II).
Absolute WR was first introduced by Dyson and Thanassoulis in 1988 [6]. This technique simply restricts weight for an input or output to vary within a specific range defined by lower and upper bounds, $\delta_t, \tau_t, \beta_t, \eta_t$, without relating the weights of one input or output to another input or output. In spite of several difficulties associated with bounds selection and infeasibility [7], Absolute WR is widely used for its intuitive managerial sense.

AR-I was first used by Thompson et al. in 1986 [8] in a well-known case of site selection for the Super Conducting Super Collider to deal with a limited number of decision making units (sites). This technique links input and output weights using ratio bound of $\gamma_t$. However, AR-II is not prevalent in practical applications due to its vulnerability to infeasibility and less straightforward managerial sense than other WR techniques.

### B. Restricting virtual weights

It should be noted that the weights ($v_i, u_r$) from the DEA are unit dependent, and therefore, a larger or smaller weights does not necessarily mean that a high or low importance is attached to a given input or output. In this sense, using restrictions on virtual (or weighted) inputs and outputs has a significant advantage of being independent from units of measurements. Wong and Beasley linked this approach in their study in 1990 [11]. This technique, however, suffers from computational complexity since the restriction can be to hold for each DMU and for a number of its input-output variables. This technique also shares problems of Absolute WR, with orientation sensitivity and potential infeasibility [3].

These weight restriction techniques can be readily applied to the DEA multiplier model. This is illustrated by the standard DEA input-oriented VRS multiplier model [12] in the following formulation with appropriate constraints added for each of the discussed weight restriction techniques.

$$
Max \quad h_o = \sum_t u_r \cdot y_{r0} + w \\

s.t. - \sum_i v_i \cdot x_{ij} + \sum_j u_r \cdot y_{rj} + w \leq 0, \quad j = 1, ..., n \\
\sum_i v_i \cdot x_{io} = 1, \quad \delta_t \leq v_i \leq \tau_t, \quad \text{Absolute WR} \\
\beta_t \leq u_r \leq \eta_t, \quad \text{Absolute WR} \\
$$

$$
\begin{align*}
\frac{k_i \cdot v_i + k_{i+1} \cdot v_{i+1}}{\sum_i v_i \cdot x_{ij}} & \leq v_{i+2}, & \text{AR - I} \\
\frac{\omega_r \cdot u_r + \omega_{r+1} \cdot u_{r+1}}{\sum_j u_r \cdot y_{rj}} & \leq u_{r+2}, & \text{AR - I} \\
\alpha_t \leq \frac{v_i}{u_r} & \leq \beta_t, & \text{AR - I} \\
\gamma_t \cdot v_i & \geq \varepsilon, & \text{AR - II}
\end{align*}
$$

### C. Altering PPS

The previous methods incorporate value judgment by imposing bounds directly on weights (or virtual weights) within the original PPS model. As a result, the frontier of PPS that represents the marginal rates of substitution becomes modified. One can reverse this process that first artificially alters the original PPS such that traditional radial DEA models can then be used to yield the equivalent results. One can also view adding constraints in the multiplier model as being analogous to adding variables in the dual (envelopment) model by duality.

Cone Ratio (CR) approach is the best-known technique that acts on data transformation to reflect prior views in assessment. The original idea of replacing ordinal relationships among weights had been introduced by Golany [13] and Ali et al. [14]. However, the generalized procedure of this approach was coined as CR by Charnes et al. [15]. In CR approach, new input-output vectors ($x', y'$) are defined by transformation matrixes ($A, B$) such that ($x', y' = (A \cdot x, B \cdot y)$). Similar to direct WR methods, transformation matrixes can be specified in a number of ways, using expert opinion, economic notion, or the set of optimal weights of preferable DMUs found through an unrestricted DEA model [16], [17]. It was also shown by Charnes et al. that AR constraints of the form $D \cdot u$ correspond to a CR model where the transformation matrix $B$ is such that $B^T = (D^T \cdot D)^{-1} \cdot D^T$ where $D$ is obtained from the upper and lower limits of the assurance region [3], [18].

Another way to incorporate value judgments by explicitly changing the production possibilities set is through the addition of unobserved DMUs (UDMUs) in the reference set. This technique was first introduced by Golany and Roll in 1994 [19] and generalized by Allen and Thanassoulis in 2004 [20]. The basic idea is to identify Pareto-efficient DMUs, non-enveloped DMUs, and Anchor DMUs (ADMUs) from which UDMUs are constructed by determining which outputs to adjust.

### III. TFDEA FORMULATION

The various WR techniques have their own strengths and weaknesses, and therefore, the selection of suitable technique depends on the application area. This study adopts AR-I since this technique is less likely to suffer from infeasibility as well as can be readily integrated into the current TFDEA procedures. It should be noted that AR-I is unit dependent as it acts directly on weights of inputs or outputs. Therefore, normalization of dataset by dividing each input-output by its respective mean, which is a commonly used normalization
process in DEA [21], must be preceded by TFDEA procedures to implement WR as intended meaning.

Figure 1 shows TFDEA process with AR-I implementation in a multiplier model. Specifically, the variable \( g_k^{tr} \) serves as the objective function and represents weighted sum of inputs using the most favorable set of weights for technology \( k \) at time period \( t_f \). Since each reference set only includes technologies that had been released up to \( t_f \), \( g_k^{tr} \) indicates how superior (or efficient) the technology \( k \) is at the time of release. The RoC, \( y_k^{tr} \), may then be calculated by taking all DMUs that were efficient at the time of release, \( g_k^{tr} = 1 \), but were superseded by technology at time \( t_f \), \( g_k^{tr} > 1 \). Note that effective year, \( E_k \), is set as a certain year from which the forecast is made since current study assumes static frontier year. For a more comprehensive treatment of TFDEA, the interested reader is referred to earlier studies [22–24].

IV. ILLUSTRATIVE EXAMPLES

DEA studies frequently suffers from the occurrence of extreme weight solutions, which becomes a motivation for applying the weight restrictions [25], [26]. While this is well known to occur in DEA, it has not been previously recognized in TFDEA. Therefore, this section provides a numerical example demonstrating not only how dynamic weighting scheme could end up extreme weight solutions but also how different weight restriction bounds can significantly affect the forecasting results. For the sake of illustration, a recently published application of the Liquid Crystal Display (LCD) industry that includes 389 display panels from 1997 to 2012 has been revisited [27]. This application used two structural characteristics as inputs (weights and bezel size) and three functional characteristics as outputs (screen size, resolution, and contrast ratio) to analyze the technology advancement trend through TFDEA. Replicating this study using the proposed multiplier model, it is possible to get a set of weights that each LCD technology had assigned to its inputs and outputs.

Figure 2 shows how many technologies assigned zero-weight(s) (or lower bound \( i.e. \varepsilon \)) to corresponding outputs to obtain efficiency score of 1 (100%). It turned out that 56 technologies out of 60 which were the SOAs at the time of release took advantage of assigning zero-weight(s) to one of their outputs. Moreover, 14 technologies chose only one output to show them in the best possible light. Note that, by definition, it is impossible to have zero-weights for all three outputs. This result reconfirms an issue that unbounded radial DEA model can allow variables to be omitted from the assessment despite they may have been advisedly included to be considered. This prevents the model from investigating various tradeoffs amongst variables which is one of benefits from DEA. In the TFDEA context, technologies identified as being efficient under this unreasonable value system may not represent SOAs of each time period. Therefore, it is important to incorporate value judgment into the free weighting scheme so as to render the assessment in line with a prior view of the application area by preventing such unrealistic evaluation criteria.
To illustrate the impact of WR in TFDEA, six different WRs were imposed and the results are summarized in Table 1. Note that all the calculation was based on lpSolveAPI with 15 decimal points using software developed by Lim and Anderson [28]. It should be also noted here that backtesting was used to validate the forecasting results. Backtesting runs the forecasting model up to a certain point in time and calculates how it would have performed had it been applied in the past. Here the dataset has been divided into a training set and testing set based on year 2007. Also, the six different WRs used were selected for illustration purposes to show their impact. The selection of actual WR ranges requires careful collaboration with industry experts.

Model 1 corresponds to the standard, unbounded TFDEA, i.e. without WR, that identifies 60 SOA technologies at the time of release and 7 SOA technologies at the frontier year of 2007. Average Rate of Change (Avg RoC) was found to be 1.101331 which means the overall performance of LCD technologies has improved by an average of 10.13% a year. The Mean Absolute Deviation (MAD) of 2.140325 indicates that this unbounded TFDEA model trained by technologies up to 2007 made an average forecasting error of 25 months when it was applied to the test set of post 2007 technologies.

In the same manner, the six bounded models (models 2 to 7) each yielded different results under their own corresponding WRs. The numbers of SOA technologies identified both at the time of release (R) and frontier (F) are not greater than those from unbounded model as readily expected. Model 3 may describe the scenario in which decision maker sets higher priority for screen size ($u_1$) over contrast ratio ($u_3$) and resolution ($u_2$). The smallest MAD, 1.839589, from this value system has an implication that a performance measure reflecting this tradeoff can better explain post 2007 technologies than others. This is consistent with industry analyses that the tendency to develop larger LCD TVs coupled with falling price has been a major driver [29], [30].

One might notice that all three models, 5, 6, and 7, which prioritize contrast ratio ($u_3$) over screen size ($u_1$) performed worse than the others. This can be attributed to the fact that relatively rapid development of contrast ratio made technologies more likely to be superseded by future technologies. This may enlarge RoC each year, and eventually, raise Avg RoC that future technologies are to be aggressively forecasted. Therefore, it may be reasonable to impose a restriction between $u_1$ and $u_3$ so that limited weights can be assigned to the latter.

V. CONCLUSION

This study is the first TFDEA application using the multiplier model to incorporate value judgment in technology assessment. The rationale behind imposing WR in TFDEA is that traditional radial efficiency model can allow zero-weighting scheme that diverse tradeoffs among inputs or outputs may not be considered.

This potential problem and impact of different WR models are addressed by revisiting LCD application that lately published. The results suggest that the technology advancement trend of flat panel display industry could be better explained by putting higher weights in order of screen size, contrast ratio, and resolution in the time period studied.

As an early stage model, this study can suggest several subsequent research topics.

First of all, the model can be elaborated by incorporating parameter estimation methods into WR techniques. This study shows varying results from 6 different WR models, i.e. which is more important, for the demonstration purpose. However, each WR scenario can be further specified, i.e. how much more important is, by reflecting actual managerial view.

### TABLE 1 RESULTS FROM SIX DIFFERENT WRs

<table>
<thead>
<tr>
<th>Model</th>
<th>WR</th>
<th>SOA at R$^1$</th>
<th>SOA at F$^2$</th>
<th>Avg RoC$^3$</th>
<th>MAD$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unbounded</td>
<td>60</td>
<td>7</td>
<td>1.101331</td>
<td>2.140325</td>
</tr>
<tr>
<td>2</td>
<td>$u_1 \geq u_2 \geq u_3$</td>
<td>55</td>
<td>6</td>
<td>1.137559</td>
<td>2.074173</td>
</tr>
<tr>
<td>3</td>
<td>$u_1 \geq u_3 \geq u_2$</td>
<td>53</td>
<td>7</td>
<td>1.104285</td>
<td>1.839589</td>
</tr>
<tr>
<td>4</td>
<td>$u_2 \geq u_1 \geq u_3$</td>
<td>25</td>
<td>5</td>
<td>1.078029</td>
<td>1.941379</td>
</tr>
<tr>
<td>5</td>
<td>$u_2 \geq u_3 \geq u_1$</td>
<td>35</td>
<td>6</td>
<td>1.144800</td>
<td>2.410615</td>
</tr>
<tr>
<td>6</td>
<td>$u_3 \geq u_1 \geq u_2$</td>
<td>32</td>
<td>4</td>
<td>1.194921</td>
<td>2.283140</td>
</tr>
<tr>
<td>7</td>
<td>$u_3 \geq u_2 \geq u_1$</td>
<td>27</td>
<td>4</td>
<td>1.198903</td>
<td>2.403865</td>
</tr>
</tbody>
</table>

$^1$SOA at R: State-of-the-art at the time of release  
$^2$SOA at F: State-of-the-art at the frontier (2007)  
$^3$Avg RoC: Average Rate of Change  
$^4$MAD: Mean Absolute Deviation
One may use weights from unbounded DEA, expert opinion, price information, etc.

In addition, it is well known that optimal solution for extreme efficient units are often highly degenerate, and consequently, have alternate optima [31]. This makes it possible that there exist different weighting schemes resulting in the same efficiency score depending on, for instance, the software used. These alternate optimal solutions can generate arbitrary results especially when the dynamic frontier year is used since current TFDEA model tries to set each target year based on dual lambda values [32]. Therefore, it is required to resolve this issue to ensure reproducible results.

REFERENCES