Departamento de Engenharia Elétrica - EESC/SEL | Comunicações em Eventos - EESC/SEL

2013-09

Towards the development of a two-time scale CUEP/BCU method

North American Power Symposium (NAPS 45), 2013, Manhattan, KS, USA
http://www.producao.usp.br/handle/BDPI/48389

Towards the Development of a Two-Time Scale CUEP/BCU Method

Edson A. R. Theodoro, Luís F. C. Alberto, and Hsiao-Dong Chiang,

Abstract—**This paper proposes a new Two-Time Scale (TTS) BCU method, and reports the first ever known use of the TTS-CUEP method in a multi-machine power system. The proposed TTS-BCU method is a numerical algorithm to correctly compute the slow and fast CUEPs of the TTS-CUEP method. It also provides a more robust algorithm to compute the CUEP of the original system.**

Index Terms—**Transient Stability, Direct Methods, CUEP/BCU Methods, Two-Time Scale Decomposition, Singularly Perturbed Systems.**

I. Introduction

The Controlling Unstable Equilibrium Point (CUEP) method [1] has been recognized as the most effective direct method for transient stability assessment of power systems since mid 90's [2]. The search for the correct CUEP is a challenge in many computational aspects [1]: (i) the CUEP is a unstable equilibrium point of a nonlinear differential-algebraic system, which provides a small and often irregular region of convergence for Newton's based methods, (ii) the search space is large, typically having hundreds of variables, and (iii) the identification of the exit-point (the point at which the fault-on trajectory crosses the stability boundary) is a numerical approximation and is generally far from the CUEP.

The BCU method [3] is a robust numerical algorithm that explores the properties of an artificial reduced gradient system to enable the computation of the CUEP of the original system. Among its advantages, one has [1]: (i) the search space for the CUEP is diminished, once the artificial gradient system is a reduced system, (ii) the exit-point identification can be properly addressed by the use of an energy function, once the crossing of the stability boundary of a gradient system occur at a point of maximum potential energy, and (iii) it uses the characterization of the stability boundary (as the union of the stable manifolds of all equilibrium points that lie in the stability boundary) to calculate the CUEP.

Recently, a Two-Time Scale (TTS) CUEP method was proposed in [4] (a sound theoretical foundation for this method was provided a year later in [5]) with the aim to explore time-scale properties, already present in the power system models, to improve the direct stability assessment. This method relies on a relationship between the CUEP of the original system and the CUEPs of the fast and slow subsystems to improve the CUEP calculation. Taking into account the time-scale properties in the CUEP method has several advantages: (i) speeding up CUEP calculation, (ii) obtaining less conservative estimations of critical clearing time (CCT), and (iii) providing a deeper insight into the unstable modes of the system.

This paper proposes a new TTS-BCU method, and reports the first ever known use of the TTS-CUEP method to assess stability of a multi-machine power system. The proposed TTS-BCU method is a numerical algorithm to correctly compute the slow and fast CUEPs of the TTS-CUEP method, and it also provides a more robust algorithm to compute the CUEP of the original system.

The paper is organized as follows: in Section II the two-time scale problem formulation is presented; in section III the multi-machine power system model, as well its energy functions are presented; in section IV the TTS-CUEP method is revised; in section V the proposed TTS-BCU method is discussed; in section VI tests and results are discussed; and finally in section VII the main conclusions are depicted.

II. A Two-Time Scale Problem Formulation

Many power system models present two-time scale properties, i.e., variables with slow and fast dynamics coexist in the system. These systems can be modeled in the form of a singularly perturbed system (*Two-Time Scale (TTS) system*), which can be expressed in the slow and fast time-scales, where $\mathbf{x} \in \mathbb{R}^n$ is a vector of slow variables, $\mathbf{z} \in \mathbb{R}^m$ is a vector of fast variables and ε is a positive small real number:

$$(\Sigma_\varepsilon)\begin{cases}\frac{d\mathbf{x}}{dt} = f(\mathbf{x},\mathbf{z})\\ \varepsilon\frac{d\mathbf{z}}{dt} = g(\mathbf{x},\mathbf{z})\end{cases} \overset{\tau=t/\varepsilon}{\longrightarrow} (\Pi_\varepsilon)\begin{cases}\frac{d\mathbf{x}}{d\tau} = \varepsilon f(\mathbf{x},\mathbf{z})\\ \frac{d\mathbf{z}}{d\tau} = g(\mathbf{x},\mathbf{z})\end{cases} \tag{1}$$

Taking the limit $\varepsilon \to 0$ in the previous equations, two simplified decoupled systems are derived, the slow system (Σ_0) and the fast system $(\Pi_{BLS}(\mathbf{x}))$ [4]:

$$(\Sigma_0)\begin{cases}\frac{d\mathbf{x}}{dt} = f(\mathbf{x},\mathbf{z})\\ 0 = g(\mathbf{x},\mathbf{z})\end{cases}, \quad (\Pi_{BLS}(\mathbf{x}))\begin{cases}\frac{d\mathbf{x}}{d\tau} = 0\\ \frac{d\mathbf{z}}{d\tau} = g(\mathbf{x},\mathbf{z})\end{cases} \tag{2}$$

We can notice that for each *frozen* (fixed) value of the slow variables, $\mathbf{x}$, there is a corresponding fast system $(\Pi_{BLS}(\mathbf{x}))$. It is also important to notice that the fast systems have the same stability boundary characterization as the two-time scale system [6].

On the other hand, the slow system (Σ_0) is an algebraic-differential equation system. In the slow system the solution (flow) is constrained to an algebraic manifold $\Gamma = \{(\mathbf{x},\mathbf{z}) \in \mathbb{R}^n \times \mathbb{R}^m : g(\mathbf{x},\mathbf{z}) = 0\}$.

The set Γ is subdivided into several stable (Γ_s) and unstable (Γ_u) components by thin sets of nonhyperbolic points $NH = \{(\mathbf{x},\mathbf{z}) \in \mathbb{R}^n \times \mathbb{R}^m : D_z g(\mathbf{x},\mathbf{z})\ is\ singular\}$ [4].

The stability boundary characterization of the slow system is quite complex, once many different limit sets can be lay on its stability boundary, such as: equilibrium points, pseudo-equilibrium points, periodic orbits, and singular points among others [7].

Besides the obvious complexity of the slow system, the slow system (Σ_0) and the family of fast systems ($\Pi_{BLS}(\mathbf{x})$) can completely describe the dynamics of the two-time scale system for a sufficiently small parameter ε. In [4], a novel TTS-CUEP method for transient stability analysis was proposed, based on the TTS-decomposition of the power system dynamics, and on a relationship between the CUEP of the original two-time scale system (1) and the CUEPs of the slow and fast subsystems (2). Several advantages in terms of CUEP computation and understanding of the power system's dynamic are obtained.

III. The Multi-machine Power System Model

Consider a power system composed of n_b buses, n_g generators, n_l load buses and an infinite bus[1]. The generators are represented by one-axis models (field flux decay model) and loads are modeled as constant impedances:

$$(\Pi_\varepsilon)\begin{cases}\dot{\delta}_i=\omega_i\\ M_i\dot{\omega}_i=P_{m_i}-\sum_{j=1}^{n_g+1}E'_{q_i}E'_{q_j}\,(G_{ij}cos(\delta_i-\delta_j)+\\ \qquad B_{ij}sin(\delta_i-\delta_j))-D_i\omega_i\\ \dot{E}'_{q_i}=c_i\varepsilon\big[E_{FD_i}-E'_{q_i}+(x_{d_i}-x'_{d_i})\sum_{j=1}^{n_g+1}E'_{q_i}E'_{q_j}\\ \qquad \big(B_{ij}cos(\delta_i-\delta_j)-G_{ij}sin(\delta_i-\delta_j)\big)\big]\end{cases}\tag{3}$$

being $i=1,...,n$, $c_i=1/(\varepsilon T'_{do_i})$ and $\varepsilon=max_{i=1,\dots,n_g}(1/T'_{do_i})$ a positive small parameter. G and B are the reduced conductance and susceptance matrices of the system, and all the other machine parameters are defined according to the usual notation, as presented in [1].

In this model, the mechanical variables $(\boldsymbol{\delta},\boldsymbol{\omega})$ are considered fast variables when compared to the equivalent voltage on the quadrature axes $\mathbf{E}'_{\mathbf{q}}$. This is a reasonable consideration when the time constants T'_{do_i} are large.

The fast subsystem ($\Pi_{BLS}(\mathbf{E}'_{\mathbf{q}})$) derived from this model is the classical generator network-reduced model:

$$(\Pi_{BLS}(\mathbf{E}'_{\mathbf{q}}))\begin{cases}\dot{\delta}_i=\omega_i\\ M_i\dot{\omega}_i=P_{m_i}-D_i\omega_i-\sum_{j=1}^{n_g+1}E'_{q_i}E'_{q_j}\\ \qquad (G_{ij}cos(\delta_i-\delta_j)+B_{ij}sin(\delta_i-\delta_j))\end{cases}\tag{4}$$

On the other hand, the simplified slow system (Σ_0) is a algebraic-differential equation system [4], [5]:

$$(\Sigma_0)\begin{cases}\dot{E}'_{q_i}=c_i\varepsilon\big[E_{FD_i}-E'_{q_i}+(x_{d_i}-x'_{d_i})\sum_{j=1}^{n_g+1}E'_{q_i}E'_{q_j}\\ \qquad \big(B_{ij}cos(\delta_i-\delta_j)-G_{ij}sin(\delta_i-\delta_j)\big)\big]\\ 0=\omega_i\\ 0=P_{m_i}-\sum_{j=1}^{n_g+1}E'_{q_i}E'_{q_j}\,(G_{ij}cos(\delta_i-\delta_j)+\\ \qquad B_{ij}sin(\delta_i-\delta_j))\end{cases}\tag{5}$$

IV. The TTS-CUEP Method

For the correctness of the TTS-CUEP method, the following assumptions are made:

(A1) All the equilibrium points of the original TTS ($\Sigma_\varepsilon/\Pi_\varepsilon$), fast ($\Pi_{BLS}(\mathbf{E}'_{\mathbf{q}})$) and slow ($\Sigma_0$) systems are hyperbolic;
(A2) The stable and unstable manifolds of the equilibrium points on the stability boundary of the TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$) satisfy the transversality condition [6];
(A3) There exist energy functions V_ε, V_{fast} and V_{slow} associated with the original TTS ($\Sigma_\varepsilon/\Pi_\varepsilon$), fast ($\Pi_{BLS}(\mathbf{E}'_{\mathbf{q}})$) and slow ($\Sigma_0$)systems, respectively;

The conceptual algorithm for the TTS-CUEP method for transient stability assessment could be described in the following steps [5]:

Step 1: (*Assessing the Stability of the Fast-Fault-on System*) Check if the *Fast Fault-on trajectory* $\phi_0^F(\tau,\mathbf{x}_0,\mathbf{z}_0)$ converges to an asymptotical equilibrium point $(\mathbf{x}_0,\mathbf{z}_{sF})$. If it converges, then proceed to Step 2 for stability assessment of the *Slow system*; otherwise proceed to Step 3 for stability assessment of the *Fast system*.

Step 2:(*Assessing the Stability of the Slow System*)

Step 2.1: Calculate the CUEP $(\mathbf{x}_{coS},\mathbf{z}_{coS})$ of the *Slow system*.

Step 2.2: Calculate the critical energy of the *Slow system* by computing the slow energy at the CUEP of the *Slow system*, that is, $V_{slow_{cr}}=V_{slow}(\mathbf{x}_{coS},\mathbf{z}_{coS})$.

Step 2.3: Check whether the *Slow Fault-on trajectory* encounters a singularity on Γ_s^F before the fault is cleared. If it does, the analysis is terminated with the conclusion that the two-time scale system may be unstable; otherwise compute the slow energy of the *post-fault Slow system* at the clearing time t_{cl} along the *projected fault-on trajectory*[2], i.e., $V_{slow_{cl}}=V_{slow}(\Phi_0^P(t_{cl},\mathbf{x}_0,\mathbf{z}_s^F)$.

Step 2.4: If $V_{slow_{cl}}<V_{slow_{cr}}$, the the *Slow system* is stable and the *two-time scale system* is stable for sufficiently small ε and stop; otherwise proceed to Step 2.5 to refine the stability assessment of the *Slow system*.

Step 2.5: Numerically integrate the *Slow system* starting from the projected initial condition of the *post-fault system*. If the *Slow system* is stable, then, for sufficiently small ε, the two-time scale system is stable.

Step 3: (*Assessing the Stability of the Fast System*)

[1]Any other reference could be adopted, like one-machine or the center of inertia, without any further complications to the developed theory.

[2]It is important to notice that the fault-on algebraic manifold Γ_s^F is different from the post-fault algebraic manifold Γ_s^{PF}, so in order to calculate the post-fault energy the fault-on trajectory must be projected over Γ_s^{PF} [4].

Step 3.1: Calculate the CUEP $(\mathbf{x}_0, \mathbf{z}_{coF})$ of the *Fast system*.

Step 3.2: Calculate the critical energy of the *Fast system* by computing the fast energy at the CUEP of the *Fast system*, that is, $V_{fast_{cr}} = V_{fast}(\mathbf{x}_0, \mathbf{z}_{coF})$.

Step 3.3: Calculate the fast energy of the Fast post-fault system at the clearing time $\tau_{cl} = t/\varepsilon$, i.e., $V_{fast_{cl}} = V_{fast}(\Phi_0(\mathbf{x}_0, \mathbf{z}_0))$.

Step 3.4: If $V_{fast_{cl}} < V_{fast_{cr}}$ then the Fast system is stable and proceed to Step 4; otherwise the fast system may be unstable and proceed to Step 3.5 to refine the stability assessment of Fast system.

Step 3.5: Numerically integrate the *post-fault fast system*. If the *post-fault fast system* is unstable, then the analysis is terminated with the conclusion that the two-time scale system may be unstable for sufficiently small ε; otherwise, proceed to Step 4.

Step 4: (*Assessing the Stability of the TTS System*)

Step 4.1: Check the existence of a *Uniform*[3] CUEP $(\mathbf{x}_{co}, \mathbf{z}_{co})$ on the type-one component Γ_u of Γ. If it exists then proceed to Step 4.2; otherwise calculate the CUEP $(\mathbf{x}_{coS}, \mathbf{z}_{coS})$ of the *Slow system* which, in this case, is the *Uniform* CUEP of the two-time scale system, i.e., $(\mathbf{x}_{co}, \mathbf{z}_{co}) = (\mathbf{x}_{coS}, \mathbf{z}_{coS})$.

Step 4.2: Calculate the critical energy by computing the energy function, of the two-time scale system, at the *Uniform* CUEP for a fixed small ε, that is, $V_{\varepsilon_{cr}} = V_\varepsilon(\mathbf{x}_{co}, \mathbf{z}_{co})$.

Step 4.3: Calculate the energy function of the post-fault two-time scale system at the clearing time τ_{cl}, i.e., $V_{\varepsilon_{cl}} = V_\varepsilon(\Phi_\varepsilon(\tau_{cl}, \mathbf{x}_0, \mathbf{z}_0))$.

Step 4.4: If $V_{\varepsilon_{cl}} < V_{\varepsilon_{cr}}$ the the *two-time scale system* is stable; otherwise it may be unstable.

In the presented algorithm, there is need for the calculation of slow and fast CUEPs, in Steps 2.1, 3.1 and 4.1. However, no algorithm to accomplish this task was provided before hand.

In this paper, a numerical algorithm, called TTS-BCU method, is proposed for the computation of the fast and slow CUEPs. The new TTS-CUEP/BCU method is an extension of the traditional CUEP/BCU method for power system stability models that exhibit two-time scale properties.

V. The TTS-BCU Method

The TTS-BCU method consists in finding artificial reduced systems to the original TTS system $(\Sigma_\varepsilon/\Pi_\varepsilon)$, and for its fast $(\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'}))$ and slow (Σ_0) subsystems,to enable the fast/slow CUEPs calculation in the TTS-CUEP method.

For the original TTS system we define a novel artificial

[3]Let $(\mathbf{x}_{co\varepsilon}, \mathbf{z}_{co\varepsilon})$ be the CUEP of the two-time scale system (Σ_ε) with respect to the fault-on trajectory $\Phi_\varepsilon^F(t, \mathbf{x}_0, \mathbf{z}_0)$, and consider the map $\varepsilon \rightarrow (\mathbf{x}_{co\varepsilon}, \mathbf{z}_{co\varepsilon})$. If there exists $\varepsilon^* > 0$ such that the map is constant for all $\varepsilon \in (0, \varepsilon^*)$, then $(\mathbf{x}_{co}, \mathbf{z}_{co}) = (\mathbf{x}_{co\varepsilon}, \mathbf{z}_{co\varepsilon})$ is a uniform CUEP with respect to the fault-on trajectory $\Phi_\varepsilon^F(t, \mathbf{x}_0, \mathbf{z}_0)$ for all $\varepsilon \in (0, \varepsilon^*)$.

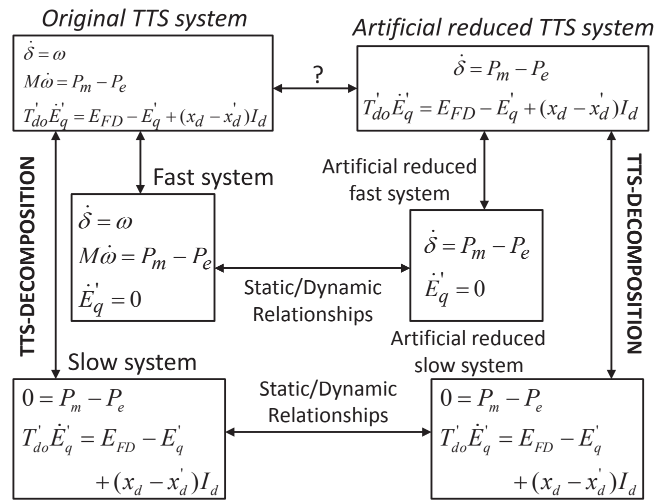

Fig. 1. Scheme of the static and dynamic relationships of the TTS-BCU method.

reduced TTS system $(\Pi_{\varepsilon(AR)})$ as following:

$$(\Pi_{\varepsilon(AR)}) \begin{cases} \dot{\delta}_i = P_{m_i} - \sum_{j=1}^{n_g+1} E_{q_i}^{'} E_{q_j}^{'} \, (G_{ij}cos(\delta_i - \delta_j) + \\ \qquad B_{ij}sin(\delta_i - \delta_j)) - D_i\omega_i \\ \dot{E}_{q_i}^{'} = c_i\varepsilon\big[E_{FD_i} - E_{q_i}^{'} + (x_{d_i} - x_{d_i}^{'}) \sum_{j=1}^{n_g+1} \\ \quad E_{q_i}^{'} E_{q_j}^{'} \big(B_{ij}cos(\delta_i - \delta_j) - G_{ij}sin(\delta_i - \delta_j)\big)\big] \end{cases} \tag{6}$$

Applying the same TTS-decomposition, the artificial fast $(\Pi_{BLS(AR)}(\mathbf{E}_\mathbf{q}^{'}))$ and slow $(\Sigma_{0(AR)})$ subsystems can be derived:

$$(\Pi_{BLS(AR)}(\mathbf{E}_\mathbf{q}^{'})) \begin{cases} \dot{\delta}_i = P_{m_i} - D_i\omega_i - \sum_{j=1}^{n_g+1} E_{q_i}^{'} E_{q_j}^{'} \\ \qquad (G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)) \end{cases} \tag{7}$$

$$(\Sigma_{0(AR)}) \begin{cases} \dot{E}_{q_i}^{'} = c_i\varepsilon\big[E_{FD_i} - E_{q_i}^{'} + (x_{d_i} - x_{d_i}^{'}) \sum_{j=1}^{n_g+1} \\ \quad E_{q_i}^{'} E_{q_j}^{'} \big(B_{ij}cos(\delta_i - \delta_j) - G_{ij}sin(\delta_i - \delta_j)\big)\big] \\ 0 = P_{m_i} - \sum_{j=1}^{n_g+1} E_{q_i}^{'} E_{q_j}^{'} \, (G_{ij}cos(\delta_i - \delta_j) + \\ \quad B_{ij}sin(\delta_i - \delta_j)) \end{cases} \tag{8}$$

The equilibrium points of these artificial fast/slow subsystems exhibit a static (location and type of the equilibrium points) and a dynamic (presence on the stability boundary) relationship with the equilibrium points of the original fast/slow systems. An scheme of these relationship is depicted in Fig. 1 for the proposed multi-machine power system model of section (III).

Firstly, we intend to prove the relationship between the equilibrium points of the original TTS system $(\Sigma_\varepsilon/\Pi_\varepsilon)$ and the equilibrium points of its fast $(\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'}))$ and slow (Σ_0) subsystems.

Before that, we will state some facts about the equilibrium points of these systems:

- the slow system (Σ_0) has the same equilibrium points of the original TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$);
- the equilibrium points of the fast system ($\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'})$), generally, are not equilibrium points of the slow system (Σ_0);
- the equilibrium points of the fast system ($\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'})$) lie on the set Γ.

The static relationship between the type of the hyperbolic equilibrium points of the slow (Σ_0) and original TTS ($\Sigma_\varepsilon/\Pi_\varepsilon$) systems are stated in the next theorem.

Theorem 1: [4] If a hyperbolic type-j equilibrium point, say $(\mathbf{x}^*, \mathbf{z}^*)$ of (Σ_0) lies on a type-k component Γ_i of Γ, then there exists $\varepsilon^* > 0$ such that $(\mathbf{x}^*, \mathbf{z}^*)$ is a hyperbolic type-(j+k) equilibrium point of ($\Sigma_\varepsilon/\Pi_\varepsilon$) for all $\varepsilon \in (0, \varepsilon)$.

Thus, as the CUEP is generally a type-one hyperbolic equilibrium point [1], theorem 1 assures that the uniform CUEP of the TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$), considering that $\|Re(\boldsymbol{\lambda})\| > \alpha > 0$, where $\boldsymbol{\lambda}$ is the eigenvalue vector of the Jacobian matrix of the original TTS system, is or a type-one hyperbolic equilibrium point of the slow system (Σ_0) on a type-zero (stable) component Γ_s of Γ, or on a type-zero (asymptotic stable) equilibrium point of the slow system (Σ_0) on a type-one (unstable) component Γ_u of Γ.

The dynamic relationship of the original TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$) and the slow ($\Sigma_0$) and fast ($\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'})$) are stated by the next two theorems.

Theorem 2: [4] Suppose $(\mathbf{x}_s, \mathbf{z}_s)$ and $(\mathbf{x}_u, \mathbf{z}_u)$ are respectively asymptotic stable and unstable equilibrium points of (Σ_0) on the stable component Γ_s. Also, suppose that for each ε, the associated TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$) has an energy function and its equilibrium points are isolated. Then, there exists an $\varepsilon^* > 0$ such that for all $\varepsilon \in (0, \varepsilon^*)$, the unstable equilibrium point $(\mathbf{x}_u, \mathbf{z}_u)$ lies on the stability boundary $\partial A_0(\mathbf{x}_s, \mathbf{z}_s)$ of (Σ_0) if and only if $(\mathbf{x}_u, \mathbf{z}_u)$lies on the stability boundary $\partial A_\varepsilon(\mathbf{x}_s, \mathbf{z}_s)$ of ($\Sigma_\varepsilon/\Pi_\varepsilon$).

Theorem 3: [4] Suppose $(\mathbf{x}_s, \mathbf{z}_s)$ is an asymptotic stable equilibrium point of (Σ_0) on the stable component Γ_s, $(\mathbf{x}_u, \mathbf{z}_u)$ is a unstable equilibrium point of (Σ_0) on the unstable component Γ_u, and consider $\|Re(\boldsymbol{\lambda})\| > \alpha > 0$, where $\boldsymbol{\lambda}$ is the eigenvalues' vector of the original TTS system's Jacobian matrix evaluated in any equilibrium point of ($\Sigma_\varepsilon/\Pi_\varepsilon$). Also, suppose that $(\mathbf{x}_u, \mathbf{z}^*)$ lies on the stability region $A_0(\mathbf{x}_s, \mathbf{z}_s) \subset \Gamma_s$ of (Σ_0), and $(\mathbf{x}_u, \mathbf{z}_u)$ lies on the stability boundary $\partial A_{BLS}(\mathbf{x}_u, \mathbf{z}^*)$ of ($\Pi_{BLS}(\mathbf{x}_u)$). . Then, there exists $\varepsilon^* > 0$ such that for all $\varepsilon \in (0, \varepsilon^*)$, the unstable equilibrium point $(\mathbf{x}_u, \mathbf{z}_u)$ is a type-one unstable equilibrium point of ($\Sigma_\varepsilon/\Pi_\varepsilon$) lying on the stability boundary $\partial A_\varepsilon(\mathbf{x}_s, \mathbf{z}_s)$ of ($\Sigma_\varepsilon/\Pi_\varepsilon$).

Exploiting these static and dynamic relationships, the CUEP of the original TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$) can be obtained by computing the CUEPs of the fast ($\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'})$) and slow ($\Sigma_0$) subsystems.

The same procedure can be made to prove the static and dynamic relationships between the artificial reduced TTS system ($\Sigma_{\varepsilon(AR)}/\Pi_{\varepsilon(AR)}$) and its fast ($\Pi_{BLS(AR)}(\mathbf{E}_\mathbf{q}^{'})$) and slow ($\Sigma_{0(AR)}$) subsystems.

Now, to finish the theoretical foundation of the TTS-BCU method, it is necessary to establish a static and dynamic relationship between the slow and fast subsystems derived from the original TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$) and those derived from the artificial reduced TTS system ($\Sigma_{\varepsilon(AR)}/\Pi_{\varepsilon(AR)}$).

Both the slow systems (Σ_0) and ($\Sigma_{0(AR)}$) have the same form, so the verification of the static and dynamic relationships between their equilibrium points is trivial.

On the other hand, the verification of the static/dynamic relationships for the fast systems ($\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'})$) and ($\Pi_{BLS(AR)}(\mathbf{E}_\mathbf{q}^{'})$) follows the same steps that the traditional BCU method [1], [8].

Once the TTS-BCU has been proved, it provides an efficient way to calculate the uniform CUEP of the original TTS system by the calculus of the slow/fast CUEPs in the TTS-CUEP method. In the next section, the TTS-CUEP/BCU method will be applied for transient stability assessment of multi-machine power system model.

VI. Tests and Discussions

The energy functions, used by the TTS-CUEP/BCU method, for the original TTS system ($\Sigma_\varepsilon/\Pi_\varepsilon$) and for the slow system (Σ_0) are taken as numerical ones [1], [8], while the energy function for the fast subsystems ($\Pi_{BLS}(\mathbf{E}_\mathbf{q}^{'})$) is taken from [9]. The path-dependent integral terms are computed along a straight line, as follows:

$$\begin{aligned} V_\varepsilon \;=\; & -\sum_{i=1}^{n_g} \int_{\delta_i^0}^{\delta_i} \left[P_{m_i} - P_{e_i}\right] d\delta_i - \\ & \sum_{i=1}^{n_g} \int_{E_{q_i}^{'0}}^{E_{q_i}^{'}} \left[E_{FD_i} - E_{q_i}^{'} + (x_{d_i} - x_{d_i}^{'}) I_{d_i}\right] dE_{q_i}^{'} \end{aligned} \quad (9)$$

$$V_{slow} \;=\; -\sum_{i=1}^{n_g} \int_{E_{q_i}^{'0}}^{E_{q_i}^{'}} \left[E_{FD_i} - E_{q_i}^{'} + (x_{d_i} - x_{d_i}^{'}) I_{d_i}\right] dE_{q_i}^{'} \quad (10)$$

$$\begin{aligned} V_{fast} \;=\; & \sum_{i=1}^{n_g} \frac{M_i \omega_i^2}{2} - \sum_{i=1}^{n_g} (P_{m_i} - \|E_{q_i}^{'}\|^2 G_{ii}) - \\ & \sum_{i=1}^{n_g} \sum_{j=i+1}^{n_g+1} E_{q_i}^{'} E_{q_j}^{'} B_{ii} (cos(\delta_i - \delta_j) - cos(\delta_i^0 - \delta_j^0)) + \\ & \sum_{i=1}^{n_g} \sum_{j=i+1}^{n_g+1} \big[E_{q_i}^{'} E_{q_j}^{'} \frac{(\delta_i - \delta_i^0) + (\delta_j - \delta_j^0)}{(\delta_i - \delta_i^0) - (\delta_j - \delta_j^0)} \\ & G_{ii} (sin(\delta_i - \delta_j) - sin(\delta_i^0 - \delta_j^0))\big] \end{aligned} \quad (11)$$

With this energy functions, and considering the proposed multi-machine power system model, two small power system models will be studied by the traditional CUEP/BCU method and by the novel TTS-CUEP/BCU method for transient stability assessment in the next subsections.

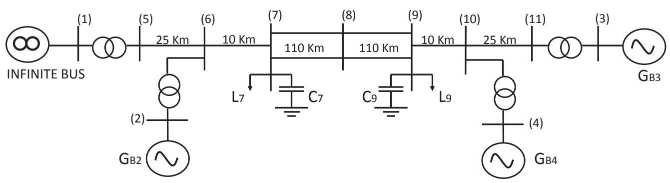

Fig. 2. Four-generator system.

TABLE I
TESTS RESULTS AND COMPARISONS FOR THE TTS-CUEP/BCU METHOD APPLIED TO MODIFIED KUNDUR SYSTEM.

Fault on bus #	Line Tripped	CCT (TTS-BCU)	CCT (Tradit. BCU)	CCT (Step-by-step)
6	—	104 ms	104 ms	149 ms
5	—	204 ms	204 ms	285 ms
8	8-9	121 ms	121 ms	132 ms
3	—	73 ms	—	100 ms
7	—	138 ms	138 ms	191 ms

A. Four-generators System

For our first example, consider the system of Fig. 2. This system is a modification of the system presented in [10], considering the multi-machine power system model (3) studied in this paper.

Table I presents the results of the proposed TTS-CUEP/BCU method for CCT estimations, as well its comparison with the results of the traditional CUEP/BCU method.

The CCT estimates of the proposed TTS-CUEP/BCU are the same as the traditional estimates obtained via CUEP/BCU method. As expected they are conservative estimates of the true CCT (presented in the fifth column of Table I).

Analyzing the results in Table I, we observe that for the fourth contingency the traditional CUEP/BCU method does not provide a CCT estimate. It occurs because the CUEP obtained by this procedure does not lie on the stability boundary, which does not occur when using the TTS-CUEP/BCU method.

It should be pointed out that there are numerical methods that could be used in the traditional CUEP/BCU method to improve its effectiveness, like the “BCU - Exit Point” method, but it is also remarkable that a TTS-version of the BCU - Exit Point method also can be obtained. Therefore, the fact that the TTS-CUEP/BCU method can perform a correct calculation of the CUEP in a case where the traditional CUEP/BCU method does not, without relying on more sophisticated numerical algorithms, is a prime contribution of the proposed methodology.

B. Modified IEEE 14 Bus System

Consider the slightly modified IEEE 14 bus system, as shown in Fig. 3.

The results of the proposed TTS-CUEP/BCU method for the calculus of the uniform CUEPs, including CCT estimations, and a comparison with the traditional CUEP/BCU method are presented in Table II.

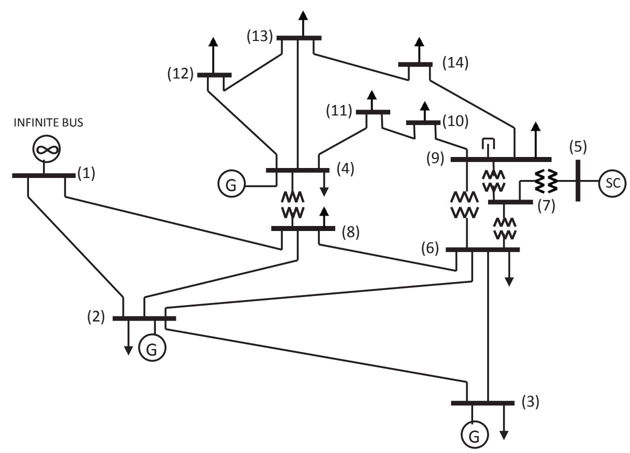

Fig. 3. Modified IEEE 14 bus system.

TABLE II
TESTS RESULTS AND COMPARISONS FOR THE TTS-CUEP/BCU METHOD APPLIED TO MODIFIED IEEE 14 BUS SYSTEM.

Fault on bus #	Line Tripped	CCT (TTS-BCU)	CCT (Tradit. BCU)	CCT (Step-by-step)
2	1-2	76 ms	76 ms	136 ms
4	4-13	318 ms	318 ms	365 ms
6	6-8	121 ms	121 ms	190 ms
7	7-9	235 ms	235 ms	524 ms
9	9-14	255 ms	—	672 ms

The accuracy of the proposed TTS-CUEP/BCU method can be evaluated observing that its CCT estimates matches with the results of the traditional CUEP/BCU method (the CUEP calculated by both methods for the first four contingencies are the same). Also, as expected [3], the estimates are conservative when compared with the real CCT (presented in the fifth column of Table II).

Despite its accuracy, more important is to highlight that the TTS-CUEP/BCU also succeeded in calculating the CUEP for the last contingency in Table II, while the traditional CUEP/BCU method could not find the ’exit-point’ [1] along the fault-on trajectory, and consequently the CUEP.

We also note that both BCU-based methods give very conservative CCT estimations for contingencies at bus 7 and 9. After further study, it was concluded that the correct CUEPs were obtained but the numerical energy functions give rise to the conservativeness in CCT estimation.

VII. CONCLUSION

In this paper a novel TTS-BCU method was proposed to compute the slow and fast CUEPs in the TTS-CUEP method. Its theoretical basis was enlightened, and the first known application of the TTS-CUEP/BCU method in multi-machine power systems was made.

The proposed TTS-CUEP/BCU method was demonstrated to be able to compute the correct uniform CUEP in cases

TABLE III
PARAMETERS FOR THE FOUR-GENERATORS SYSTEM
(V_{base} = 230 KV, S_{base} = 100 MVA).

Bus	V	Ang	P_g	Q_g	P_l	Q_l	B_{sh}
1	1.000	0.00°	-0.5475	-0.1168	0.00	0.00	0.00
2	1.030	13.11°	12.7400	2.8062	0.00	0.00	0.00
3	1.030	30.84°	10.4000	2.9707	0.00	0.00	0.00
4	1.030	15.33°	10.0100	4.1786	0.00	0.00	0.00
5	1.002	0.52°	0.00	0.00	0.00	0.00	0.00
6	1.006	1.29°	0.00	0.00	1.5000	0.00	0.00
7	1.000	-4.81°	0.00	0.00	9.6700	1.0000	2.0000
8	0.990	-7.66°	0.00	0.00	1.5000	0.00	0.00
9	0.968	-5.55°	0.00	0.00	17.6700	1.0000	3.5000
10	0.976	5.78°	0.00	0.00	1.5000	0.00	0.00
11	0.996	21.11°	0.00	0.00	0.00	0.00	0.00

Line	R (p.u.)	X (p.u.)	b_c (p.u.)	Tap
1 - 5	0.0000	0.017	0.000	1.00
2 - 6	0.0000	0.017	0.000	1.00
3 - 11	0.0000	0.017	0.000	1.00
4 - 10	0.0000	0.017	0.000	1.00
5 - 6	0.0025	0.025	0.044	1.00
6 - 7	0.0010	0.010	0.018	1.00
7 - 8	0.0110	0.110	0.193	1.00
7 - 8	0.0110	0.110	0.193	1.00
8 - 9	0.0110	0.110	0.193	1.00
8 - 9	0.0110	0.110	0.193	1.00
9 - 10	0.0010	0.010	0.018	1.00
10 - 11	0.0025	0.025	0.044	1.00

Gen. Bus	r_{arm} (p.u.)	x_d (p.u.)	x'_d (p.u.)	T'_{do} (s)	M (s^2)
2	$2.01.10^{-6}$	$1.51.10^{-3}$	$2.52.10^{-4}$	8.00	0.310
3	$2.01.10^{-6}$	$1.51.10^{-3}$	$2.52.10^{-4}$	8.00	0.295
4	$2.01.10^{-6}$	$1.51.10^{-3}$	$2.52.10^{-4}$	8.00	0.295

TABLE IV
PARAMETERS FOR THE MODIFIED IEEE 14 BUS SYSTEM
(V_{base} = 100 KV, S_{base} = 100 MVA).

Bus	V	Ang	P_g	Q_g	P_l	Q_l	B_{sh}
1	1.060	0.00°	0.9275	0.0539	0.00	0.00	0.00
2	1.045	1.61°	0.4000	0.0312	0.2170	0.1270	0.00
3	1.010	-2.53°	1.0000	-0.1889	0.9420	0.1900	0.00
4	1.070	-7.03°	0.3000	0.3376	0.1120	0.0750	0.00
5	1.090	-7.53°	0.00	0.2089	0.00	0.00	0.00
6	1.033	-4.94°	0.00	0.00	0.4780	-0.0390	0.00
7	1.056	-7.53°	0.00	0.00	0.00	0.00	0.00
8	1.040	-4.24°	0.00	0.00	0.0760	0.0160	0.00
9	1.048	-8.87°	0.00	0.00	0.2950	0.1660	0.1900
10	1.044	-8.83°	0.00	0.00	0.0900	0.0580	0.00
11	1.053	-8.06°	0.00	0.00	0.0350	0.0180	0.00
12	1.055	-7.97°	0.00	0.00	0.0610	0.0160	0.00
13	1.049	-8.12°	0.00	0.00	0.1350	0.0580	0.00
14	1.030	-9.55°	0.00	0.00	0.1490	0.0500	0.00

Line	R (p.u.)	X (p.u.)	b_c (p.u.)	Tap
1 - 2	0.019	0.059	0.053	1.00
1 - 8	0.054	0.223	0.049	1.00
2 - 3	0.047	0.198	0.044	1.00
2 - 6	0.058	0.176	0.034	1.00
2 - 8	0.057	0.174	0.035	1.00
3 - 6	0.067	0.171	0.013	1.00
6 - 8	0.013	0.042	0.00	1.00
6 - 7	0.00	0.209	0.00	1.00
6 - 9	0.00	0.556	0.00	1.00
8 - 4	0.00	0.252	0.00	1.00
4 - 11	0.095	0.199	0.00	1.00
4 - 12	0.0123	0.256	0.00	1.00
4 - 13	0.066	0.130	0.00	1.00
7 - 5	0.00	0.176	0.00	1.00
7 - 9	0.00	0.110	0.00	1.00
9 - 10	0.032	0.084	0.00	1.00
9 - 14	0.127	0.270	0.00	1.00
10 - 11	0.082	0.192	0.00	1.00
12 - 13	0.221	0.200	0.00	1.00
13 - 14	0.171	0.348	0.00	1.00

Gen. Bus	r_{arm} (p.u.)	x_d (p.u.)	x'_d (p.u.)	T'_{do} (s)	M (s^2)
2	0.00	1.39	1.25	8.00	0.053
3	0.00	0.91	0.75	8.00	0.026
4	0.00	1.63	1.5	8.00	0.014
5	0.00	1.32	1.2	8.00	0.067

where the traditional CUEP/BCU method was not. This feature is a prime contribution of the TTS-CUEP/BCU method.

Future applications and developments of the TTS-CUEP/BCU method in power system stability analysis include the simultaneous assessment of transient and voltage stability, and the development of the theoretical basis for network-preserving power system models.

APPENDIX A
POWER SYSTEMS DATA

The buses, lines and generators parameters of the two multi-machine power systems tested in this paper are presented in Tables III and IV.

ACKNOWLEDGMENT

The authors would like to thank FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo - Brazil) for the financial support to this research.

REFERENCES

[1] H.-D. Chiang, Direct Methods for Stability Analysis of Electric Power Systems. Wiley IEEE Press, 2011.

[2] F. Rahimi, M. Lauby, J. Wrubel, and K. Lee, "Evaluation of the transient energy function method for on-line dynamic security analysis," Power Systems, IEEE Transactions on, vol. 8, no. 2, pp. 497–507, 1993.

[3] H.-D. Chiang, F. Wu, and P. Varaiya, "A bcu method for direct analysis of power system transient stability," Power Systems, IEEE Transactions on, vol. 9, no. 3, pp. 1194–1208, 1994.

[4] L. F. C. Alberto and H.-D. Chiang, "Controlling unstable equilibrium point theory for stability assessment of two-time scale power system models," in Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE, 2008, pp. 1–9.

[5] ——, "Theoretical foundation of cuep method for two-time scale power system models," in Power Energy Society General Meeting, 2009. PES '09. IEEE, 2009, pp. 1–9.

[6] H.-D. Chiang, M. Hirsch, and F. Wu, "Stability regions of nonlinear autonomous dynamical systems," Automatic Control, IEEE Transactions on, vol. 33, no. 1, pp. 16–27, 1988.

[7] V. Venkatasubramanian, H. Schattler, and J. Zaborsky, "Dynamics of large constrained nonlinear systems-a taxonomy theory [power system stability]," Proceedings of the IEEE, vol. 83, no. 11, pp. 1530–1561, 1995.

[8] H.-D. Chiang, C.-C. Chu, and G. Cauley, "Direct Stability Analysis of Electric Power Systems using Energy Functions: Theory, Applications, and Perspective," Proceedings of the IEEE, vol. 83, no. 11, pp. 1497 –1529, Nov. 1995.

[9] T. Athay, R. Podmore, and S. Virmani, "A practical method for the direct analysis of transient stability," Power Apparatus and Systems, IEEE Transactions on, vol. PAS-98, no. 2, pp. 573–584, 1979.

[10] M. Klein, G. Rogers, and P. Kundur, "A Fundamental Study of Inter-Area Oscillations in Power Systems," Power Systems, IEEE Transactions on, vol. 6, no. 3, pp. 914 –921, Aug. 1991.