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2013

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European Control Conference - ECC, 2013, Zürich.
http://www.producao.usp.br/handle/BDPI/48349

2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

An extension of the invariance principle for switched T-S fuzzy systems*

Michele C. Valentino[1] michele.valentino@usp.br
Flávio A. Faria[1] flaviof15@yahoo.com.br
Vilma A. Oliveira[1] vilma@sc.usp.br

***Abstract*— In this work, we propose an extension of the invariance principle for switched T-S fuzzy systems using an auxiliary function V for the convex combination of the subsystems which applies to unstable subsystems. The results are given in terms of LMIs. Numerical examples illustrate the effectiveness of the proposed design methods.**

I. INTRODUCTION

Switched nonlinear systems arise in practice when modeling the operation of many engineering systems [1], [2], [3], [4]. Although switching is not a new concept in engineering, in the past decade the theory of switched systems has attracted the attention of many researchers. As a consequence, the stability theory for switched nonlinear systems has significantly developed in this period. Despite the important advances in stability theory, the attractor of many switched systems may not be an equilibrium point. A classical example is the on–off temperature control system. For this class of problems, we are not interested in studying the stability of a particular equilibrium point but the asymptotic behavior of solutions [5], [6], [7], [8], [9].

An extension of the invariance principle for switched nonlinear systems was presented in [8]. The results in [8] can be applied to switched systems with ultimately bounded subsystems. In this paper, invariance results for switched systems with subsystems which are not ultimately bounded or unstable are given. This is accomplished by allowing the derivative of an auxiliary function V along the solutions of the convex combination of the subsystems [10] to be positive on some sets. The result is extended to switched T-S fuzzy systems and then we can analyze the asymptotic behavior of the solution just by checking properties of some sets and if a set of linear matrix inequalities (LMIs) is feasible.

II. PRELIMINARIES

Let us consider the following switched nonlinear system

$$\dot{x}(t) = f_{\sigma(x(t))}(x(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\sigma(x) : \mathbb{R}^n \to \mathcal{P} = \{1, 2, ..., N\}$ is a piecewise constant function of the state, called switching signal with N the number of subsystems and f_p is a $\mathcal{C}^1$ nonlinear function for all $p \in \mathcal{P}$. We assume that the state of (1) does not jump at the switching instants, that is, the switched system solution $\varphi(t, x_0)$ is everywhere continuous. The $\mathcal{C}^1$ nonlinear function f_p can be exactly represented by a T-S fuzzy model in the following subset of the state space [11], [12]:

$$S_p := \{x(t) \in \mathbb{R}^n : |x_\upsilon(t)| \le \bar{x}_{p\upsilon},\ \upsilon \in \mathcal{I} \text{ and } p \in \mathcal{P}\} \tag{2}$$

where $\mathcal{I} \subset \{1, 2, \cdots, n\}$ and $\bar{x}_{p\upsilon}$ is a positive real number for all $\upsilon \in \mathcal{I}$, $p \in \mathcal{P}$. Then, the switched nonlinear system (1) can be described by fuzzy IF-THEN rules, as follows [10], [13]:

Model rule k for subsystem p:
IF $x_1(t)$ is M_{pk1} and $x_2(t)$ is M_{pk2} and $\cdots$ and $x_q(t)$ is M_{pkq}
THEN $\dot{x}(t) = \mathbf{A}_{pk}x(t) + \mathbf{B}_{pk}u(t), \qquad k = 1, 2, \cdots, r_p$
where $M_{pkj},\ j = 1, 2, \cdots, q,\ q \le n$, are the fuzzy sets. The overall fuzzy subsystem p is obtained by fuzzy blending the rules k as follows:

$$\dot{x}(t) = \sum_{k\in\mathcal{R}_p} h_{pk}(x(t))\mathbf{A}_{pk}x(t) \tag{3}$$

where $\mathbf{A}_{pk} \in \mathbb{R}^{n\times n}$ is the matrix of the local models, $\mathcal{R}_p = \{1, ..., r_p\}$ with r_p the number of model rules of the subsystem p and

$$h_{pk}(x(t)) = \frac{w_{pk}(x(t))}{\sum_{k\in\mathcal{R}_p} w_{pk}(x(t))}$$

with $w_{pk}(x(t)) = \prod_{j=1}^{q} M_{pkj}(x_{pj}(t))$ the normalized weight function for each local model. We assume that $h_{pk}(x(t))$ is a $\mathcal{C}^1$ function for all $p \in \mathcal{P}$ and $k \in \mathcal{R}_p$.

Remark 1 If no constraints on the state are needed for some p, then $S_p = \mathbb{R}^n$.

From the properties of membership functions we have:

$$h_{pk}(x(t)) \ge 0 \quad \text{and} \quad \sum_{k\in\mathcal{R}_p} h_{pk}(x(t)) = 1 \tag{4}$$

for all $p \in \mathcal{P}$ and $k \in \mathcal{R}_p$. Using (4), it follows that

$$\sum_{k\in\mathcal{R}_\beta} h_{\beta k}(x(t)) - \frac{1}{N-1}\left(\sum_{\substack{p\in\mathcal{P}\\ p\neq\beta}} \sum_{k\in\mathcal{R}_p} h_{pk}(x(t))\right) = 0 \tag{5}$$

*This work was supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) from Brazil under grants 142246/2010-7, 150838/2012-3 and 304985/2009-0.

[1] Departamento de Engenharia Elétrica e de Computação, Escola de Engenharia de São Carlos, USP - Universidade de São Paulo, 13566-590, São Carlos, São Paulo, Brazil

with $\beta \in \mathcal{P}$. When convenient, arguments of $h_{pk}(x(t))$ and $x(t)$ will be omitted.

In this paper, we study the solutions of system (1) under a particular class of switching signals, that is, the solutions that have a nonvanishing dwell time and satisfy a switching condition. For easy reference, some preliminary definitions and propositions, which were taken from [10], [6] (see also [14] and [15]), are presented.

Definition 1 The solution $\varphi(t, x_0)$ of (1) has a non vanishing dwell time if there exists $\gamma > 0$ such that

$$\inf_k (\tau_{k+1} - \tau_k) \geq \gamma \tag{6}$$

where $\{\tau_k\}$ is the sequence of switching times associated to $\varphi_{\sigma(x)}(t, x_0)$. The number γ is called a dwell time for $\varphi(t, x_0)$ and the set of all switched solutions possessing a nonvanishing dwell time is denoted by $\mathcal{S}_{dwell}$.

Definition 2 A compact set $\mathcal{M}$ is weakly invariant with respect to the switched system (1) if for each $x_0 \in \mathcal{M}$ there exists an index $p \in \mathcal{P}$, a solution $\varphi(t, x_0)$ of the vector field $f_p(x)$ and a real number $b > 0$ such that $\varphi(t, x_0) \in \mathcal{M}$ for either $t \in [-b, 0]$ or $t \in [0, b]$.

Let $x, a \in \mathbb{R}^n$, then $d(x, a) = ||x - a||_2$. A switched solution $\varphi(t, x_0)$ of (1) is attracted to a compact set $\mathcal{M}$ if for each $\epsilon > 0$ there exists a time $T > 0$ such that

$$\varphi(t, x_0) \in B(\mathcal{M}, \epsilon) \text{ for } t \geq T \tag{7}$$

where $B(a, \epsilon) = \{x \in \mathbb{R}^n : d(x, a) < \epsilon\}$ and $B(\mathcal{M}, \epsilon) = \cup_{a \in \mathcal{M}} B(a, \epsilon)$. Clearly $\varphi(t, x_0)$ is attracted to $\mathcal{M}$ if and only if

$$\lim_{t \to \infty} dist(\varphi(t, x_0), \mathcal{M}) = 0. \tag{8}$$

Definition 3 Let $\varphi(t, x_0) : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ be a continuous curve. A point q is a limit point of $\varphi(t, x_0)$ if there exists a sequence $\{t_k\}_{k \in \mathbb{N}}$,with $t_k \to \infty$, as $k \to \infty$ such that $\lim_{k \to \infty} \varphi(t_k, x_0) = q$. The set of all limit points of $\varphi(t, x_0)$ will be denoted by $\omega^+(x_0)$.

Proposition 1 Let $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ be a bounded switched solution of (1) for $t \geq 0$. Then, $\omega^+(x_0)$ is nonempty, compact and weakly invariant. Moreover, $\varphi(t, x_0)$ is attracted to $\omega^+(x_0)$.

Proof. See [6].

Let α_p be a real number such that

$$\alpha_p \geq 0, \quad \forall p \in \mathcal{P} \quad \text{and} \quad \sum_{p \in \mathcal{P}} \alpha_p = 1. \tag{9}$$

Proposition 2 Let $V : \mathbb{R}^n \to \mathbb{R}$ be a smooth function. If there exist real numbers $\alpha_p,\ p \in \mathcal{P}$ satisfying (9) such that

$$\frac{\partial V}{\partial x} \left[\sum_{p=1}^{N} \alpha_p f_p(x(t)) \right] < 0 \tag{10}$$

then, there exists a switching law that assures that function V decreases along the switched solution of system (1).

Proof. Following [10], let V be a smooth function. If there exist positive numbers $\alpha_p,\ p \in \mathcal{P}$ satisfying (10), then for all t there exists at least one $p \in \mathcal{P}$ such that

$$\frac{\partial V}{\partial x} [f_p(x(t))] < 0. \tag{11}$$

Therefore, function V decreases along the switched solution of system (1).

From Proposition 2, a stabilizing switching law can be established via the following condition.

Switching Condition 1: The switched system (1) with N subsystems can be switched to or can stay at subsystem p if at time t

$$\frac{\partial V}{\partial x} f_p(x(t)) < 0. \tag{12}$$

III. MAIN RESULTS

The results given in [8] can be applied to switched systems with ultimately bounded subsystems. To overcome this limitation, in this section, invariance results for switched systems with subsystems which are not ultimately bounded or unstable are given. This is accomplished by allowing the derivative of an auxiliary function V along the solutions of the convex combination of the subsystems to be positive on some sets. The result is extended to switched T-S fuzzy systems and then we can analyze the asymptotic behavior of the solution only by checking properties of some sets and if a set of linear matrix inequalities (LMIs) is feasible.

We use the auxiliary system

$$\dot{x}(t) = \sum_{p=1}^{N} \alpha_p f_p(x(t)) := f(x(t)) \tag{13}$$

where $\alpha_p,\ p \in \mathcal{P}$ are known numbers satisfying (9). The solution of system (13) starting in x_0 is denoted by $\varphi_1(t, x_0)$. Let $V : \mathbb{R}^n \to \mathbb{R}$ be a smooth function and define the following sets along the solution of system (13):

$$\begin{aligned} C &= \{x \in \mathbb{R}^n : \nabla V(x) f(x) > 0\} \\ \mathcal{E} &= \{x \in \mathbb{R}^n : \nabla V(x) f(x) = 0\} \\ \Omega_\ell &= \{x \in \mathbb{R}^n : V(x) \leq \ell\} \end{aligned} \tag{14}$$

with $\sup_{x\in C} V(x) < \ell < \infty$. Now, the main result of this section is presented.

Theorem 1 Consider switched system (1), real numbers α_p satisfying (9) and $V(x) : \mathbb{R}^n \to \mathbb{R}$ be a $\mathcal{C}^1$ function. If there exists a real number ℓ such that $\sup_{x\in C} V(x) < \ell < \infty$, then there exists a switching law satisfying Condition 1 for all $x \notin \Omega_\ell$ such that every bounded solution $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$.

Proof. First, let $x_0 \in \Omega_\ell$ and $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ be a solution satisfying Condition 1 for all $x \notin \Omega_\ell$. Suppose there exists $\tau > 0$ such that $\varphi(\tau, x_0) \notin \Omega_\ell$. Then, there exist $\bar{\tau} \in (0, \tau)$ such that $V(\varphi(\bar{\tau}, x_0)) = \ell$ (by the continuity of V and $\varphi(t, x_o)$) and $V(\varphi(t, x_0)) > \ell, \forall t \in (\bar{\tau}, \tau]$, but this is a contradiction, since $\sup_{x\in C} V(x) < \ell < \infty$ and $C \in \Omega_\ell$. Then $V(\varphi_1(t, x_0))$ decreases outside of Ω_ℓ and by Proposition 2, there exists a switching law that assures that the function V decreases along the switched solution of system (1) out of Ω_ℓ. Therefore, $\varphi(t, x_0)$ is a bounded solution and by Proposition 1, $\omega^+(x_0)$ is nonempty, compact, weakly invariant and $\omega^+(x_0) \subset \Omega_\ell$. Moreover, $\varphi(t, x_0)$ is attracted to $\omega^+(x_0)$. Then, the solution is attracted to a weakly invariant set inside Ω_ℓ.

Now, let $x_0 \notin \Omega_\ell$ and $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ be a solution satisfying Condition 1 for all $x \notin \Omega_\ell$. If $\varphi(t, x_0)$ enters Ω_ℓ at some t, then the result follows from the first part of this proof. Suppose the bounded solution $\varphi(t, x_0) \notin \Omega_\ell, \forall t \geq 0$. Since $\ell > \sup_{x\in C} V(x)$, $\varphi_1(t, x_0) \notin C \subset \Omega_\ell, \forall t \geq 0$, that is, $\Delta V(\varphi_1(t, x_0)) f(\varphi_1(t, x_0)) \leq 0, \forall t \geq 0$. Then by Proposition 2, there exists a switching law, such that V decreases along the switched solution of system (1). We conclude that $V(\varphi(t, x_0))$ is a lower bounded non-increasing function of t. Then, there exists $r \in R$ such that $r = \lim_{t\to\infty} V(\varphi(t, x_0))$. Since the solution is bounded, $\omega^+(x_0)$ is nonempty. Let $a \in \omega^+(x_0)$, then there exists a sequence $\{t_k\}$ with $t_k \to \infty$ as $k \to \infty$ such that $\varphi(t_k, x_0) \to a$. The continuity of V ensures that $V(\varphi(t_k, x_0)) \to V(a)$ as $k \to \infty$, then $V(a) = r$, $\forall a \in \omega^+(x_0)$.

Finally, by Proposition 1, $\omega^+(x_0)$ is a weakly invariant set. Thus there exists an interval $[\alpha, \beta]$ containing the origin and a function $\upsilon(t)$ such that $\upsilon(0) = a$, $\upsilon(t) \in \omega^+(x_0), \forall t \in [\alpha, \beta]$ and $\exists j \in \mathcal{P}$ such that $\dot{\upsilon}(t) = f_j(\upsilon(t)), \forall t \in [\alpha, \beta]$. Then, $V(\upsilon(t)) = V(a) = r$, $\forall t \in [\alpha, \beta]$ and $\nabla V(\upsilon(t)) f_j(\upsilon(t)) = 0 \; \forall t \in [\alpha, \beta]$. Particularly, for $t = 0$ we have $\nabla V(\upsilon(0)) f_j(\upsilon(0)) = \nabla V(a) f_j(a) = 0$. Hence $a \in \mathcal{E}$. Then $\omega^+(x_0) \subset \mathcal{E}$ and the solution is attracted to a weakly invariant set in $\mathcal{E}$. Therefore, there exists a switching law satisfying Condition 1 for $x \notin \Omega_\ell$ such that every bounded solution $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$.

Example 1 *We consider switched system (1) with $\mathcal{P} = \{1, 2\}$ and*

$$f_1(x) = \begin{bmatrix} x_2 + x_1 x_2 \\ x_1^2 - 2x_2 x_1^2 \end{bmatrix}, \quad f_2(x) = \begin{bmatrix} -2x_1 x_2 \\ -x_1 - 2x_2^3 + 2x_2 \end{bmatrix}$$

Let $V(x) = \frac{x_1^2 + x_2^2}{2}$ and $\alpha_1 = \alpha_2 = 0.5$. Then

$$\begin{aligned} \nabla V(x) f(x) &= x_1(0.5x_2 + 0.5x_1x_2 - x_1x_2) \\ &+ x_2(0.5x_1^2 - x_2x_1^2 - 0.5x_1 - x_2^3 + x_2) \\ &= x_2^2(-x_1^2 - x_2^2 + 1). \end{aligned}$$

Therefore, $C = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 < 1\}$, $\mathcal{E} = \{x \in \mathbb{R}^2 : x_2 = 0$ ou $x_1^2 + x_2^2 = 1\}$ and $\ell > 1$. By Theorem 1, any bounded solution $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ satisfying Condition 1 for all $x \in \mathbb{R}^n$ with $x \notin \Omega_\ell$ is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$. Fig. 1 illustrates the time-domain simulation for $x_0 = [-2 \; -2.2]$ and Fig. 2 shows the switching law. This simulation confirms the results of Theorem 1 by showing an attractor inside circle of radius 2.2 since $\ell = 1.1$ (Fig. 3). Observe in Fig. 4 that function V increases in Ω_ℓ.

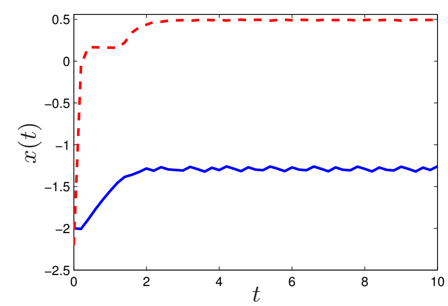

Fig. 1. Switching solution for initial condition $x_0 = [-2 \;\; -2.2]$ for Example 1.

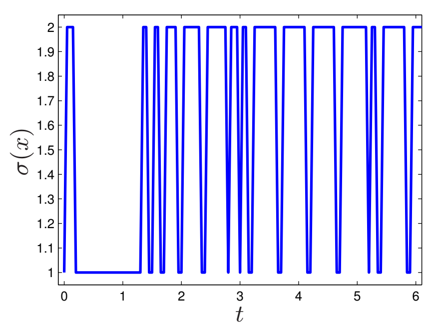

Fig. 2. Switching law $\sigma(x)$ for Example 1.

A. An extension of the invariance principle for switched T-S fuzzy systems

In what follows, an extension of Theorem 1 for switched T-S fuzzy systems is presented. The result allows the analysis of the asymptotic behavior of switched system solution just by verifying the feasibility of LMIs and some properties of the sets defined.

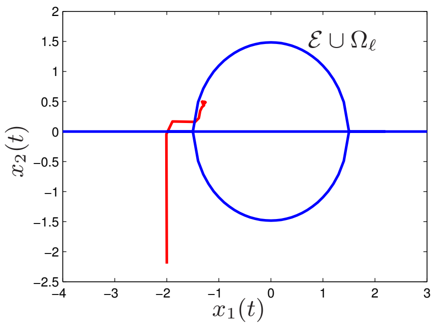

Fig. 3. Phase portrait for Example 1.

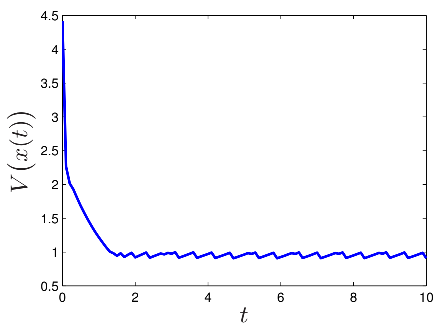

Fig. 4. Function V along the switched system solution for Example 1.

Now, f_p in the auxiliary system (13) is described by a T-S fuzzy model, that is

$$\dot{x}(t) = \sum_{p \in \mathcal{P}} \alpha_p \left(\sum_{k \in \mathcal{R}_p} h_{pk} \mathbf{A}_{pk} \right) x(t). \tag{15}$$

The result is developed using the following scalar function:

$$V(x) = x' \left(\sum_{p \in \mathcal{P}} \sum_{k \in \mathcal{G}_p} h_{pk} \mathbf{P}_{pk} \right) x \tag{16}$$

where $\mathcal{G}_p$ is a subset of $\mathcal{R}_p$ for all $p \in \mathcal{P}$, which is previously chosen.

We define the following sets

$$\begin{aligned} Z &= \bigcap_{p \in \mathcal{P}} S_p, & (17) \\ D &\supseteq \bigcup_{\substack{p \in \mathcal{P} \\ k \in \mathcal{G}_p}} \{x \in Z : \nabla h_{pk}(x) f(x) > 0\}. & (18) \end{aligned}$$

Theorem 2 Consider system (15) and real numbers α_p satisfying (9). If set D is bounded, closed and there exist matrices positive definites $\mathbf{P}_{pk} \in \mathbb{R}^{n \times n}$ satisfying (19)-(25), then there exists a switching law satisfying Condition 1 for all $x \notin \Omega_\ell$ such that every bounded solution $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ of switched system (1) is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$.

Proof. Consider the property (5) and real numbers α_p satisfying (9). The derivative of function (16) along the solution of the system (15) is given by (28), where $\mathbf{P}_\phi = \sum_{p \in \mathcal{P}} \sum_{k \in \mathcal{G}_p} \dot{h}_{pk} \mathbf{P}_{pk}$. If there exist matrices positive definites $\mathbf{P}_{pk} \in \mathbb{R}^{n \times n}$ such that the LMIs (19)-(25) are feasible, then $\mathbf{P}_\phi$ is the only term that can make (28) positive. As P_{pk} is positive definite for all $p \in \mathcal{P}$ and $k \in \mathcal{G}_p$ then $C \subseteq D$. Since set D is compact and V is continuous, there exists $\ell_1 \in R$ such that $\max_{x \in D} V(x) = \ell_1$. Thus, there exists a real number ℓ such that $\sup_{x \in C} V(x) < \ell \leq \ell_1 < \infty$. Therefore, by Theorem 1 there exists a switching law satisfying Condition 1 for all $x \notin \Omega_\ell$ such that every bounded solution $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$.

Example 2 *We consider system (1) with the following T-S models:*

$$\begin{aligned} \mathbf{A}_{11} &= \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}, \quad \mathbf{A}_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ \mathbf{A}_{21} &= \begin{bmatrix} -3 & 0 \\ 1 & -10 \end{bmatrix}, \quad \mathbf{A}_{22} = \begin{bmatrix} -3 & 0 \\ 0 & -10 \end{bmatrix} \end{aligned} \tag{26}$$

and membership functions

$$\begin{aligned} h_{11} &= \frac{x_1^2 + x_2^2}{10}, \ h_{12} = 1 - h_{11}, \\ h_{21} &= \frac{x_1^2}{25}, \ h_{22} = 1 - h_{21} \end{aligned} \tag{27}$$

in the set $Z = \{x \in \mathbb{R}^2 : |x_1| \leq 5 \ \text{ and } \ |x_2| \leq 5\}$. Using MATLAB to solve (19)-(24) with parameters $\alpha_1 = 0.8$ and $\alpha_2 = 0.2$ for all $p \in \mathcal{P}$ and $k \in \mathcal{R}_p$, we obtain the following matrices:

$$\mathbf{P}_{11} = -\begin{bmatrix} 1.031 & 0.003 \\ 0.003 & 1.273 \end{bmatrix}, \ \mathbf{P}_{12} = \begin{bmatrix} 0.375 & 0.008 \\ 0.008 & 0.127 \end{bmatrix},$$
$$\mathbf{P}_{21} = \begin{bmatrix} -0.777 & 0.005 \\ 0.005 & -0.528 \end{bmatrix}, \ \mathbf{P}_{22} = \begin{bmatrix} 0.897 & 0.024 \\ 0.024 & 0.316 \end{bmatrix}.$$

Let $\mathcal{G}_1 = \{2\}$ and $\mathcal{G}_2 = \{2\}$, then

$$\dot{h}_{12} = \frac{x_1^2(-0.2 - 0.08 x_1 x_2) - 0.4 x_2^2 (x_1^2 + x_2^2 - 2.5)}{5},$$
$$\dot{h}_{22} = \frac{-0.2 x_1^2}{25}$$

thus, $D = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 2.5\}$, which is compact. Therefore, by Theorem 2, there exists a switching law satisfying Condition 1 for all $x \notin \Omega_\ell$ such that every bounded solution $\varphi(t, x_0) \in \mathcal{S}_{dwell}$ is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$. Fig. 5 illustrates the time-domain simulation for $x_0 = [-2 \quad 2]$ and Fig. 6 shows the switching law. This simulation confirms the results of Theorem 2 by showing that the solution is attracted to the largest weakly invariant set of $\mathcal{E} \cup \Omega_\ell$, which was obtained numerically for $\ell = 0.2$ (Fig. 7). Observe in Fig. 8 that function V increases in Ω_ℓ.

$$\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{\beta k}+\mathbf{P}_{\beta k}\mathbf{A}_{\beta k})+\mathbf{Q}_\alpha \prec \mathbf{0}, \quad k\in\mathcal{G}_\beta, \tag{19}$$

$$\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{\beta k})+\mathbf{Q}_\alpha \prec \mathbf{0}, \quad k\in\mathcal{R}_\beta-\mathcal{G}_\beta, \quad i\in\mathcal{P}, \quad j\in\mathcal{G}_i, \tag{20}$$

$$\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{\beta j}+\mathbf{P}_{\beta j}\mathbf{A}_{\beta k}+\mathbf{A}'_{\beta j}\mathbf{P}_{\beta k}+\mathbf{P}_{\beta k}\mathbf{A}_{\beta j})+2\mathbf{Q}_\alpha \prec \mathbf{0}, \quad j,k\in\mathcal{G}_\beta, \quad j<k, \tag{21}$$

$$\alpha_p(\mathbf{A}'_{pk}\mathbf{P}_{pk}+\mathbf{P}_{pk}\mathbf{A}_{pk})-\frac{1}{N-1}\mathbf{Q}_\alpha \prec \mathbf{0}, \quad p\in\mathcal{P}-\{\beta\}, \quad k\in\mathcal{G}_p, \tag{22}$$

$$\alpha_p(\mathbf{A}'_{pk}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{pk})-\frac{1}{N-1}\mathbf{Q}_\alpha \prec \mathbf{0}, \quad p\in\mathcal{P}-\{\beta\}, \quad k\in\mathcal{R}_p-\mathcal{G}_p, \quad i\in\mathcal{P}, \quad j\in\mathcal{G}_i, \tag{23}$$

$$\alpha_p(\mathbf{A}'_{pk}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{pk})+\alpha_i(\mathbf{A}'_{ij}\mathbf{P}_{pk}+\mathbf{P}_{pk}\mathbf{A}_{ij})-\frac{2}{N-1}\mathbf{Q}_\alpha \prec \mathbf{0}, \quad i,p\in\mathcal{P}-\{\beta\}, \quad k\in\mathcal{G}_p, \quad j\in\mathcal{G}_i, \tag{24}$$

$$\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{\beta k})+\alpha_i(\mathbf{A}'_{ij}\mathbf{P}_{\beta k}+\mathbf{P}_{\beta k}\mathbf{A}_{ij})+\frac{N-2}{N-1}\mathbf{Q}_\alpha \prec \mathbf{0}, \quad k\in\mathcal{G}_\beta, \quad i\in\mathcal{P}, \quad i<\beta, \quad j\in\mathcal{G}_i, \tag{25}$$

where

$$\mathbf{Q}_\alpha=\sum_{p\in\mathcal{P}}\sum_{k\in(\mathcal{R}_p-\mathcal{G}_p)}\alpha_p\big(\mathbf{A}'_{pk}\mathbf{P}_{pk}+\mathbf{P}_{pk}\mathbf{A}_{pk}\big).$$

$$\begin{aligned}
\dot V(x(t)) \quad &= \quad x(t)'\Bigg\{\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}\dot h_{pk}\mathbf{P}_{pk}+\left(\sum_{p\in\mathcal{P}}\alpha_p\sum_{j=1}h_{pk}\mathbf{A}_{pk}\right)'\left(\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}\mathbf{P}_{pk}\right)\\
&+ \quad \left(\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}\mathbf{P}_{pk}\right)\left(\sum_{p\in\mathcal{P}}\alpha_p\sum_{k\in\mathcal{G}_p}h_{pk}\mathbf{A}_{pk}\right)\\
&+ \quad \left(\left(\sum_{k\in\mathcal{R}_\beta}h_{\beta k}\right)-\frac{1}{N-1}\left(\sum_{\substack{p\in\mathcal{P}\\p\neq\beta}}\sum_{k\in\mathcal{R}_p}h_{pk}\right)\right)\left(\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}\right)\mathbf{Q}_\alpha\Bigg\}x(t)\\
&= \quad x(t)'\Bigg\{\sum_{k\in\mathcal{G}_\beta}\sum_{\substack{i\in\mathcal{P}\\i<\beta}}\sum_{j\in\mathcal{G}_i}h_{\beta k}h_{ij}\left(\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{\beta k})+\alpha_i(\mathbf{A}'_{ij}\mathbf{P}_{\beta k}+\mathbf{P}_{\beta k}\mathbf{A}_{ij})+\frac{N-2}{N-1}\right)\\
&+ \quad \sum_{\substack{p\in\mathcal{P}\\p\neq\beta}}\sum_{k\in\mathcal{G}_p}\sum_{\substack{i\in\mathcal{P}\\i\neq\beta}}\sum_{j\in\mathcal{G}_i}h_{pk}h_{ij}\left(\alpha_p(\mathbf{A}'_{pk}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{pk})+\alpha_i(\mathbf{A}'_{ij}\mathbf{P}_{kp}+\mathbf{P}_{kp}\mathbf{A}_{ij})-\frac{2}{N-1}\mathbf{Q}_\alpha\right)\\
&+ \quad \sum_{k\in\mathcal{G}_\beta}h^2_{\beta k}(\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{\beta k}+\mathbf{P}_{\beta k}\mathbf{A}_{\beta k})+\mathbf{Q}_\alpha)\\
&+ \quad \sum_{k\in\mathcal{G}_\beta}\sum_{\substack{j\in\mathcal{G}_\beta\\j<k}}h_{\beta k}h_{\beta j}(\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{\beta j}+\mathbf{P}_{\beta j}\mathbf{A}_{\beta k}+\mathbf{A}'_{\beta j}\mathbf{P}_{\beta k}+\mathbf{P}_{\beta k}\mathbf{A}_{\beta j})+2\mathbf{Q}_\alpha)\\
&+ \quad \sum_{\substack{p\in\mathcal{P}\\p\neq\beta}}\sum_{k\in\mathcal{G}_p}h^2_{pk}(\alpha_p(\mathbf{A}'_{pk}\mathbf{P}_{pk}+\mathbf{P}_{pk}\mathbf{A}_{pk})-\frac{1}{N-1}\mathbf{Q}_\alpha)\\
&+ \quad \sum_{k\in(\mathcal{G}_\beta-\mathcal{R}_\beta)}\sum_{i\in\mathcal{P}}\sum_{j\in\mathcal{G}_i}h_{\beta k}h_{ij}\left(\alpha_\beta(\mathbf{A}'_{\beta k}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{\beta k})+\mathbf{Q}_\alpha\right)\\
&+ \quad \sum_{\substack{p\in\mathcal{P}\\p\neq\beta}}\sum_{k\in(\mathcal{R}_p-\mathcal{G}_p)}\sum_{i\in\mathcal{P}}\sum_{j\in\mathcal{G}_i}h_{pk}h_{ij}\left(\alpha_p(\mathbf{A}'_{pk}\mathbf{P}_{ij}+\mathbf{P}_{ij}\mathbf{A}_{pk})-\frac{1}{N-1}\mathbf{Q}_\alpha\right)+\mathbf{P}_\phi\Bigg\}x(t)
\end{aligned} \tag{28}$$

IV. CONCLUSIONS

An invariance principle for switched T-S fuzzy systems, which can be applied to unstable systems was presented. In this result, we are not interested in analysing the stability of a particular equilibrium point but the asymptotic behavior of solutions. However, many existing results using Lyapunov direct method can not be used for this analysis. Hence, we first presented an invariance principle for switched nonlinear systems and after this result was extended to switched T-S fuzzy systems. Theorem 2 can be improved by using properties of the membership functions [16] to obtain less conservative LMIs.

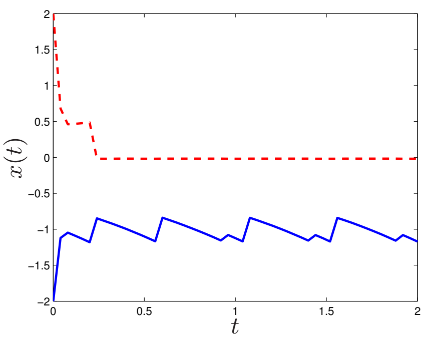

Fig. 5. Switching solution for initial condition $x_0 = [-2\ \ 2]$ for Example 2.

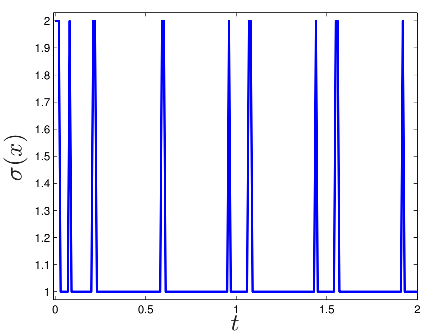

Fig. 6. Switching law $\sigma(x)$ for Example 2.

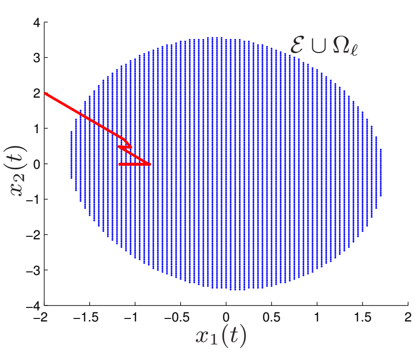

Fig. 7. Phase portrait for Example 2.

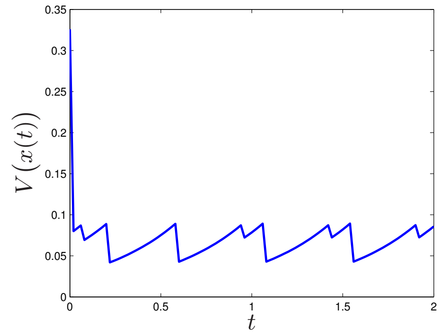

Fig. 8. Function V along the switched system solution for Example 2.

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