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An Approach for Coordinating of the Cooperative Mapping in a Self-Adaptive Formation System Based on a Modification of the Ant Colony Algorithm

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***Abstract*—In this work, an approach for cooperative and distributed mapping in a self-adaptive formation system based on a modified version of the ant colony optimization algorithm is proposed. The strategy is distributed, decentralized, real time and it is applied to tasks in which formation characteristic is an essential requirement. The coordination system's design is inspired by the biological mechanisms that define a social organization in collective systems, specifically, the ant colony system. Voronoi tessalation and Delaunay triangulation techniques are used to model the formation strategy. The approach is adaptable for scenarios with suffer changes in the structure of the environment. The performance of the system is evaluated using a simulator. Simulation results show that the cooperative mapping is efficient, the trials are performed considering an indoor environment. Besides results show that the proposed formation approach is able to rearrange spatially the robots as they navigate, changing the relative robot distances according to the spatial environment restrictions.**

***Keywords*-Cooperative mapping; formation; self-adaptive system; ant colony algorithm;**

I. INTRODUCTION

The manner in which individuals integrate a group autonomously to generate a complex dynamic captivates the attention of scientific community. The discovery of rules that lead this dynamic movement and how physical principles are related to individuals are of interests of mathematicians, physicists, biologists and computer scientists [1]. Another issue is the reason biological agents of the same species tend to move closer each other. By observations of real and simulated situations of biological agents, Partridge [2] firms that animals are able to perform more complex tasks, increasing their ability for defense and likelyhood for surviving, when they move in group.

There are some applications in which multiple agent systems are adopted, such as: mapping, rescue operations in catastrophic events, fire extinction and exploration in hostile environment [3], [4], [5]. For the mentioned applications, spatial position of robots is essential. In the present paper, this characteristic is named robots formation. Multiple agent systems endowed formation strategy place robots in order to establish a geometric shape - dynamic or static - or maximize the coverage of an area during robots move, considering that area coverage is observed if the region perceived by the robots at each instant is totally contiguous, i.e., there are no spaces among robots without sensoring [6].

In the present work, a cooperative mapping approach that uses a bioinspired coordination strategy of multiple robot considering the formation problem is addressed. The coordination strategy is named *Self-Adaptive Formation System* (SAFS) and it was proposed by Calvo [7]. SAFS is designed according to a modified version of the ant system algorithm presented in [8]. Similar to biological agents, the robots in SAFS strategy are able to realize indirect communication. The movement direction of agents in the SAFS strategy is defined in order to guide them preferably to regions of the environment where is low amount of pheromone.

The topology of formation is determined by fundamentals of Voronoi tesselation and Delaunay triangulation [9], [10], [11]. An intrinsic characteristic of this strategy is the ability to increase the covered area by the robots group whenever the topology changes. Results show that the strategy does not depend on the knowledge of the environment, where the agents act in environments with different arrangement of obstacles. This strategy is able to adapt the group topology for any environment configuration, avoiding one or more robots to be pulled away from the group. However SAFS has a relevant limitation when it is considered the cooperation among the robots. Individually each robot builds a map that represents the visited region of the environment, but this information is underutilized since each robot keep it only to itself.

A cooperative mapping approach is proposed for building maps in a collaborative way. It is defined by the development of a local map integration method based on inter-robot observations. The proposed integration method is an extension of the approach elucidated in [12], where the mobile robots spread out across certain area and share information through an ad hoc wireless network.

The remainder of the paper is organized such as it follows. The related works are presented in Section 2. In Section 3, it is provided a description of the multiple robot coordination strategy SAFS. The map integration method based on inter-robot observations, proposed by us, is the focus of Section 4. In Section 5, it is shown simulation results obtained from a set of experiments. The main contributions of the paper as well as expectations for the future works are highlighted in Section 6.

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II. RELATED WORKS

One of the first works to reproduce the movement pattern of animals was presented in [13]. Inspired by behavior of fish schools, this model was the basis for the approach that intends to solve complex tasks through basic principles of fishes movement [14]. In [15], Reynolds defined criteria to a robots group to estabilish a formation independent of adopted strategy, such as: separation, alignment and cohesion. For separation criterium, an agent of a group is repelled by those that are near in order to avoid collision. The alignment criterium requires that all robots assume the same orientation. Finally, to satisfy the cohesion criterium, the robots formation should be maintained as long as the group moves.

The multiple robot mapping [16], [17], [18], [19] has attracted attention because the parallel exploration effectiveness using multiple robots and the reduction of time for the large areas mapping. The main challenge of multiple robot mapping tasks is the integration process of the maps produced individually by each robot. The most of proposed methods in literature deal with this problem taking the center of each robot and performing the integration according to relative positions of the robots which, consequently, need to be known. The integration problem becomes challenging when the coordinate transformation, between the initial poses of the robots, is unknown. This transformation is called *initial correspondence*. A method for merging independently created maps with unknown initial correspondence, while robots assume a formation, is proposed in this paper.

The integration of local maps in a global map might be obtained by two different ways. The first way is the search of landmarks alignments between two maps [20], [21], [22]. In this case, the most likely transformation is that producing the higher correspondence between the landmarks. As a fundamental presupposition, the transformation based on landmarks has the existence of overlapping areas between the local maps. The second way is the use of inter-robot observations, referred as rendez-vous measures, for computing the coordinate transformation [23], [24]. When two robots meet each other and compute their relative positions, this information can be used to define the transformation required for integrating their maps.

The method proposed here integrates maps using the information sharing model proposed by [12]. The local map integration is defined by transformation matrices, represented by the distance between two robots and their relative positions. It is worth to note that the method proposed by Tan et al. [12] is not responsible for mapping the environment. It does not build a map that represents the environment, as it is being proposed here.

III. SELF-ADAPTIVE FORMATION SYSTEM (SAFS)

In the SAFS strategy, the robots tend to stay close each other to explore a region. In this coordination strategy, the robots are able to deposit pheromone with repulsive and attractive properties in order to perform exploration and formation behaviors, respectively. There are two kinds of robots: leaders and followers. Considering a leader-follower paradigm, the leader is the unique robot that releases repulsive and attractive pheromone to indicate explored areas and the path to be coursed by followers, respectively. However, the leader just detects repulsive pheromone to guide the followers to areas not recently traveled, i.e., areas with low amount of repulsive pheromone. The followers, on the other hand, deposit only repulsive pheromone to mark explored areas and detect attractive pheromone to follow the trajectory of the leader.

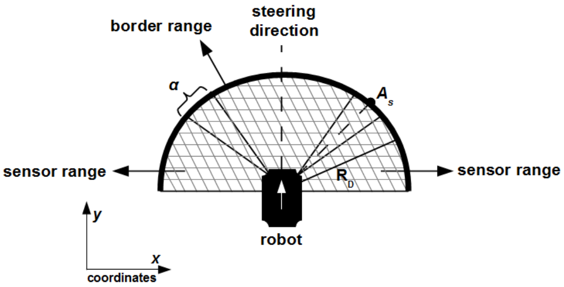

Figure 1. Robot and sensor model

Robots in the SAFS strategy are equipped by sensorial fields: repulsive and attractive pheromone sensors and antenna. Before describing the sensors models, consider the assumption that there are N identical mobile robots R_k, $k = 1, \ldots, N$, $N \in \mathbb{N}$ that move in a planar space $Q \subset \mathbb{R}^2$ and an arbitrary point in Q denoted by q. Repulsive and attractive pheromone sensors present the same physical structure. The model of the sensors is such that it detects pheromone stimuli at a specific distance R_D, as shown in Fig. 1, from -90 degrees to 90 degrees, corresponding to the average of the amount of pheromone deposited in an angle interval. The total range of 180 degrees is divided in identical angle intervals, such that the sensor detects stimuli corresponding to different angles A_s, such that: $(2S + 1)\alpha = 180$ and $A_s = s\alpha$, where $s \in [-S, S]$ and $s \in \mathbb{N}$. Hence, it is said that the amount of pheromone - repulsive and attractive - in angle A_s corresponds to concentration of substance in its interval. The terms s and S are a generalization, such that, $s \in \{sr, sa\}$ and $S \in \{S_r, S_a\}$, for the repulsive and attractive pheromone sensors, respectively.

The direct communication is provided by a device named antenna. It emits and detects information, or messages, around robot with communication radius $R_C > 0$. Considering only the SAFS strategy, there are two kinds of message emitted and detected by antenna. They are modeled as two pairs, such as: (msg_id, msg_level) and $(msg\_id, msg\_pheromone)$. The first term in both of them, msg_id is the identification of a robot and the second, msg_level and $msg\_pheromone$, corresponds to the hierarchical level in the formation topology and the amount of perceived repulsive pheromone, respectively. The first pair is emitted continuously, whereas the second one is only emitted when a new leader is required. Considering now the cooperative mapping process, a third kind of message is emitted and detected, modeled as (msg_id, msg_map). This last message is responsible for transmitting information about the local map of a robot and its position in this map. Finally, robots are equipped with an obstacle distance sensor. It allows robots to avoid collisions in risk situations when they are very close to an obstacle. Another feature of this sensor is to map the environment.

In order to detect the instant when a robot must deviate an obstacle, SAFS analizes all readings of the obstacle sensor. If at least one of them indicates that the distance between the robot and the closest obstacle is lower than $\eta > 0$, then it is used a mechanism to avoid obstacle based on fuzzy logic. A robot R_k is able to detect its neighbors through messages received from antenna. These information are organized in two set of pairs, $D_{A1} = \{(msg\_id_i), (msg\_level_i) | 0 \leq i \leq n\}$ and $D_{A2} = \{(msg\_id_i), (msg\_pheromone_i) | 0 \leq i \leq n\}$, where n is the number of detected neighbors, that is, robots are at distance from R_k lower than R_C. Thus, the position of each neighbor can be estimated.

A robot receives information about the level of its neighbors - D_{A1} - continuously. If the robot is a follower, the detected information is useful to determine its level in the topology of the robots group. If the robot does not receive this information for a long time, then either it is dispersed from the group or it is in a group which there is no leader. Hence, a leader must be defined. Since the leader is guided to regions with low amount of repulsive pheromone, the new leader is that detects lower concentration of this substance, considering the transmission of D_{A2} among the robots. The functions attributed to a leader are: (1) attract followers, leaving a trail of attractive pheromone to be cursed; (2) start the process for defining hierarchical levels of group, through the transmission of D_{A1}; and (3) promote group movement towards to regions with low amount of repulsive pheromone. On the other hand, followers are supported by three behaviors: (1) follow the attractive pheromone trail left by the leader; (2) follow the neighbors; and (3) disperse from them. A complete description of the formulas and process of accessing the pheromones is presented in [7].

IV. MAP INTEGRATION METHOD

In the proposed approach, each robot is responsible for managing its local occupancy map. Since each robot keeps a local map of the environment, a process of integration must be realized. For a better comprehension of the map integration method, consider two adjacent robots R_i and R_j and their respective coordinate systems $\sum_i$ and $\sum_j$. Robot R_i sends to R_j its coordinate system $\sum_i$ and its position (x_i, y_i) inside $\sum_i$. On the other hand, R_j sends to R_i its coordinate system $\sum_j$ and its position (x_j, y_j) inside $\sum_j$. It is worth to notice that $\sum_i$ and $\sum_j$ are static.

Based on the perception between robots R_i and R_j, α_{ij} and α_{ji} are known for both robots, where α_{ij} is the orientation of R_i in the coordinate system of R_j and α_{ji} is the orientation of R_j in the coordinate system of R_i. The distance d_{ij} between robots R_i and R_j is also known. The relative orientation between robots R_i and R_j is denoted by θ_{ij}.

Next, a description of the integration process is presented. Consider a position P_k belonging to $\sum_i$, $P_k = (x_{P_k}, y_{P_k})$. Initially, according to (1), the position P_k undergoes a rotation process based on the relative angle between robots in order to establish a new position $P_k^{'} = (x_{P_k}^{'}, y_{P_k}^{'})$.

$$\begin{aligned} x_{P_k}^{'} &= x_{P_k} \cos\theta_{ij} - y_{P_k} \sin\theta_{ij} \\ y_{P_k}^{'} &= x_{P_k} \sin\theta_{ij} + y_{P_k} \cos\theta_{ij} \end{aligned} \quad (1)$$

Since the position P_k is rotated, it is required to define the position of the robot R_i according to this rotation. In similar way to the previous equation, the rotated position of the robot R_i is defined by:

$$\begin{aligned} x_i^{'} &= x_i \cos\theta_{ij} - y_i \sin\theta_{ij} \\ y_i^{'} &= x_i \sin\theta_{ij} + y_i \cos\theta_{ij} \end{aligned} \quad (2)$$

Considering the rotation of the coordinate system of R_i and according to the relative angle between robots, it is needed to define the distance on the *x*-axis and the *y*-axis from the position P_k to the position of the robot R_i on the new coordinate system, that is:

$$\begin{aligned} x_{P_k}^{i} &= x_{P_k}^{'} - x_i^{'} \\ y_{P_k}^{i} &= y_{P_k}^{'} - y_i^{'} \end{aligned} \quad (3)$$

Since the rotation is done and the distance from the position P_k to the robot R_i is established on the rotated coordinate system, the next step is compute the translation from the position $P_k^{'}$ to the coordinate system $\sum_j$, as presented in (4). The position $P_k^{*} = (x_{P_k}^{*}, y_{P_k}^{*})$ obtained from this translation defines the position in coordinate system $\sum_j$ which will be integrated with the coordinate of position P_k belonging to $\sum_i$.

$$\begin{aligned} x_{P_k}^{*} &= x_j + d_{ij} \cos\alpha_{ji} + x_{P_k}^{i} \\ y_{P_k}^{*} &= y_j + d_{ij} \sin\alpha_{ji} + y_{P_k}^{i} \end{aligned} \quad (4)$$

According to the previous definition, the position P_k^{*} is established by three factors: 1) the position of robot R_j in its coordinate system; 2) the distance between two robots; and 3) the distance from the position P_k to the position of the robot R_i, considering the relative rotation of its coordinate system. It is worth to emphasize that the system proposed by Tan et al. [12] does not consider the position of the robot R_j during the translation process of coordinates of the robot R_i.

In Algorithm 1, it is presented the general operation of the map building process. The variable *number_iterations* indicates the total amount of iterations for execution the exploration and mapping methods. The variable id_{R_i} indicates the identifier of the robot R_i and the variables $pose_{R_i}$ and map_{R_i} are the position and the map of the robot R_i, respectively. Note that $pose_{R_i}$ indicates the position of robot R_i in its map map_{R_i}.

The function **atualizeMap** updates the occupancy grid through sensor readings captured by the robot. Functions

sendData and **receiveData** are responsible for transmitting and receiving, respectively, the information about the local map of a robot and its localization. If the robot R_i identifies a robot R_j, considering their communication radius, the function **integrate** is called. This function realizes the integration of their local maps.

Algorithm 1 ∘ Map building method

```
main()
BEGIN
  FOR $it_{current} \leftarrow 1$ TO $number\_iterations$
    atualizeMap();
    sendData($pose_{R_i}, map_{R_i}, id_{R_i}$);
    IF (receiveData($pose_{R_j}, map_{R_j}, id_{R_j}$) == TRUE)
      integrate($it_{current}, pose_{R_j}, map_{R_j}, id_{R_j}$);
    END-IF
    detectPheromone();
    adjustMovementDirection();
    releasePheromone();
    move();
  END-FOR
END
```

The function **detectPheromone** detects pheromone concentration at the border of the sensor range. It is worth to remember that a leader detects only repulsive pheromone, whereas followers detect only attractive pheromone. The adjustment of the steering direction is determined by the function **adjustMovementDirection**, according to attraction forces defined by the leader, neighbors and centroids of the Voronoi tesselation. Since the direction is defined, the robot deposits pheromone on the environment (**releasePheromone**) and moves to the specified direction (**move**). If a robot is the leader, then it leaves attractive and repulsive pheromone. Otherwise, the robot leaves only repulsive pheromone.

V. EXPERIMENTS AND RESULTS

Experiments are carried out in Player/Stage platform that models various robots and sensors simulating their dynamics simultaneously. The robot model used is the Pioneer 2DX equipped with a laser range-finder SICK LMS 200 able to scan the environment (general obstacles, e.g., walls and objects). Robots are able to map the environment using the method Occupancy Grid [25] coordinated by the proposed approach presented here. For simplification the localization problem is not considered in this work. Although the information about the robot's localization is very important for the proposed method.

All experiments were executed 10 times. Thus, the average of the explored regions is computed to evaluate them. The discrete time is adopted in simulation and it is equivalent to the number of iterations. Each simulated experiment takes 1000 iterations. The environment models adopted are illustrated in Fig. 2. The environment, in which the multiple robot system carries out the exploration, is divided in connected small regions called here *sectors*. The simulated environment was divided in 25 sectors of equal size. A sector is said to be visited if it is reached by any robot. It is worth to note that if a robot is physically in a sector C_i and its sensors detect both sector C_i and C_j, it has been considered the robot visited only the sector C_i. The environment was set up for a dimension $40m \times 20m$. Each iteration of the simulation spends $5.4s$, then it was considered $90min$ as the simulation total time.

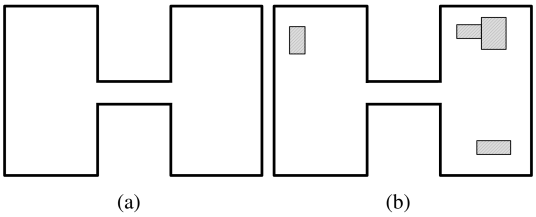

Figure 2. Simulated environment models: (a) Environment A; (b) Environmet B

Considering the integration process, the local map integrating is only started at iteration $t >= 100$. This restriction ensures that the robots obtain the minimal information about the environment before to start the integration process. Since the robot R_i joins its coordinate system with the robot R_j, it is defined that robot R_i will wait 50 iterations to share again its local map with the robot R_j. This strategy decreases the redundant information exchange.

The performed experiments in this paper were executed for reproducing the tests proposed by Calvo [7]. Hence, it was used 10 robots located in left room of the environment shown in Fig. 2, using a communication radius equal to $16m$. As reported by Calvo, in the begining, there is no leader in the group. Then, SAFS detects the absence of transmission of level information and activates the process to define the leader robot. Initially, robots are very close each other and far from their centroid then, the first observed behavior is the dispersion, establishing a formation. After that, the formation is maintained with the same topology up to the leader robot reaches the corridor. At this moment, the topology is reorganized in order to the group can cross the corridor. Leaving corridor, the group finds again a wide area. Thus, centroid attraction force propels robots to border of the group again to maximize the covered area.

At every instant, robots detect their adjacent neighbors, build Voronoi cells, compute their centroids and move towards them. If a robot is far from its centroid, then it is close an Voronoi edge. In other words, it is in a collision risk with other robot or an obstacle. In this case, the intensity of centroid attraction force is high. As a robot approximates to its centroid, this intensity decreases, because the robot is near to a safe local. The equilibrium state is achieved at moment when all robots are exactly, or near, at their respective centroid position. Thus, the intensity of the force is null. The main advantages of this process are to provide a collision-free navigation and a

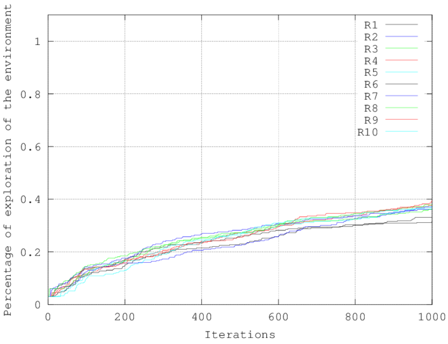

Figure 3. Percentage of exploration of the environment without integration (module inactive) - Environment A

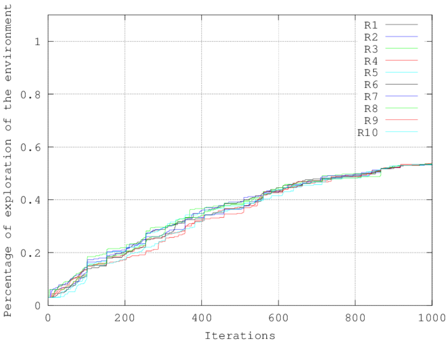

Figure 4. Percentage of exploration of the environment with integration (module active) - Environment A

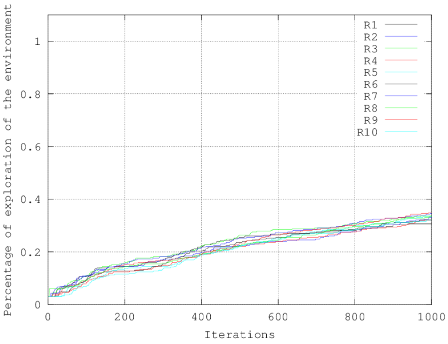

Figure 5. Percentage of exploration of the environment without integration (module inactive) - Environment B

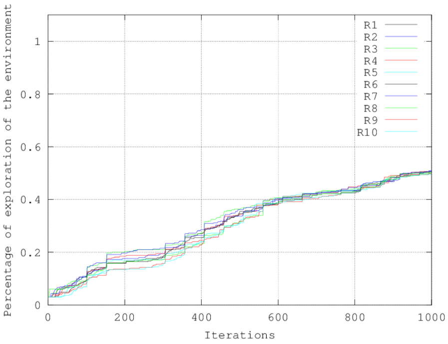

Figure 6. Percentage of exploration of the environment with integration (module active) - Environment B

limited dispersion of the robots.

It was compared the mapping task without and with integration. The percentage of explored area of the environment shown in Fig. 2(a), using the SAFS strategy, can be seen in Fig. 3 and 4, respectively. For both graphics, the *x*-axis represents the iterations of the exploration process and the *y*-axis represents a variable on the interval [0,1] that denotes the percentage of explored regions of the environment. This variable denotes the relation between the amount of the sectors represented by the map of each robot and the total amount of sectors of the environment.

In Fig. 3, it is presented the average of explored area obtained with the SAFS strategy without using the map integration method, i.e., the integration module is inactive and the robots do not exchange maps with the others. Thus, the robots acquire information only about the sectors visited by themselves. On the other hand, in Fig. 4, it is presented the average of explored area obtained with the SAFS strategy using the map integration method, i.e., the integration module is active.

Considering the mapping task without and with integration, the exploration process of the environment of Fig. 2(b), using the SAFS strategy, is shown in Fig. 5 and 6, respectively. In Fig. 5, it is presented the average of performance obtained with the SAFS strategy without using the map integration method. In Fig. 6, it is presented the average of performance obtained with the SAFS strategy using the map integration method.

In Table I, it is presented the average and the pattern deviation among all robots, considering the final average of exploration of each scenario, i.e., with the integration module active and inactive for both environments.

As it can be observed by the presented results, the mapping of the environment using the map integration method is better than the mapping without integration. According to various situations presented to the robots group, SAFS strategy adapted the topology of formation, such that no robot drove away from the group. The SAFS strategy is able to reach the three desired characteristics defined by Reynolds [15] and mentioned in Section 2 through by equilibrium provided by combination of attraction forces.

VI. CONCLUSION

In this work, it was described a bioinspired distributed coordination strategy, named SAFS, for multiple agent systems applied to robots formation. The method is not dependent on the knowledge of the environment structure or initial positions of the robots. The strategy was tested to evaluate its ability of formation in distinct situations

Table I
AVERAGE OF EXPLORATION SAFS

Environment	Integration Module	Average (μ)	Pattern Deviation (σ)
A	Inactive	0.36	0.03
A	Active	0.53	0.04
B	Inactive	0.50	0.07
B	Active	0.33	0.05

and in all cases it was observed the Reynolds criteria.

A set of experiments were conducted for performance analysing. Two mechanisms are considered and compared, one of them is the SAFS strategy without map integration, and the other one is the SAFS with map integration. The SAFS with integration is significantly superior, since the percentage of explored are using the local map integration is higher than that without the integration.

As future works some aspects of the exploration system will be considered for analysis. First one is related to the communication mechanism, the multiple robot exploration with limited communication range restricts the communication abilities of the robots and, naturally, the task of map integration with limited communication range is harder than without this constraint. The second one deals with the abilities of the SAFS strategy, as the leader changing and addition will be investigated.

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