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When less is better: Insights from the product mix dilemma from the Theory of Constraints perspective

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Perhaps due to its origins in a production scheduling software called Optimised Production Technology (OPT), plus the idea of focusing on system constraints, many believe that the Theory of Constraints (TOC) has a vocation for optimal solutions. Those who assess TOC according to this perspective indicate that it guarantees an optimal solution only in certain circumstances. In opposition to this view and founded on a numeric example of a production mix problem, this paper shows, by means of TOC assumptions, why the TOC should not be compared to methods intended to seek optimal or the best solutions, but rather sufficiently good solutions, possible in non-deterministic environments. Moreover, we extend the range of relevant literature on product mix decision by introducing a heuristic based on the uniquely identified work that aims at achieving feasible solutions according to the TOC point of view. The heuristic proposed is tested on 100 production mix problems and the results are compared with the responses obtained with the use of Integer Linear Programming. The results show that the heuristic gives good results on average, but performance falls sharply in some situations.

Keywords: Theory of Constraints; optimization; production mix; excess capacity; Integer Linear Programming

1. Introduction

Several researchers and papers have discussed and compared the Theory of Constraints (TOC) to exact and heuristic approaches regarding their applicability and effectiveness. The studies that demonstrate the weaknesses of TOC in choosing the best production mix make use of examples grounded in deterministic data, without taking into account the variability inherent in the real world. As will be conceptually and appropriately discussed, when considering that the real world is not deterministic, solutions that maximise end results would not always be feasible and therefore the discussions derived from them would have little practical value. This perspective, shared by TOC, according to which variations or the presence of noise makes it impossible to reach optimal results (in general, not only in production mix problems) is the focus of this article and has not been effectively examined by previous works. The only work found that explicitly addressed this issue was that of Barnard (2006), presented at the TOCICO (Theory of Constraints International Certification Organisation) conference in 2006. More specifically, from a classic production mix problem perspective, the present article will discuss that the importance of keeping some level of excess capacity, or protective capacity, is critical to the feasibility of the proposed solutions. Furthermore, this work proposes and assesses an algorithm based on Barnard (2006) and applies it in 100 different production mix problems. For comparison purposes, ILP models are used to provide optimal solutions, which may or may not ensure excess capacities.

In this sense, this paper is not intended to present a wide theoretical review on TOC or optimisation methods, or even to investigate ways to optimise solutions for a better production mix. In fact, this article is intended to achieve three objectives.

(1) Show, conceptually, why the TOC cannot be characterised as an extension of methods aimed at optimising results.
(2) Propose an algorithm based on Barnard (2006).
(3) Based on ILP models, assess the results achieved by the algorithm proposed.

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With these objectives, this paper initially shows and discusses, from a conceptual point of view, the main concepts and assumptions on which the present work is based and on which the main conclusions are built. To illustrate the concepts and the proposal presented, a classic production mix optimisation problem – known as a *P&Q* problem and first shown by Goldratt (1990) – is used. Although this problem is defined by deterministic data, the discussion and analysis are carried out from the perspective of the existence of variability, especially in the capacity of resources. Such statistical variations, however, will not be made explicit nor be considered in the calculations. Results obtained from the application of the *P&Q* problem, including a new problem adapted from it, are then exploited in order to show TOC assumptions and its different point of view when compared with approaches focused on maximising throughput in the absence of variability. Finally, a heuristic based on Barnard (2006) is developed and its results analysed and compared with those obtained from the application of ILP models.

2. The Theory of Constraints and the production mix problem

The TOC is a management philosophy based on a systemic approach, aiming to provide its practitioners with not only answers to the question 'what' (the fundamental principles to improve organisations continuously and sustainably), but also 'how' – the processes of analysis, policies, practices and measures required to implement, in a practical and effective way, a systems approach to achieve continuous improvement in their organisations (Barnard 2010).

Based on the different roles of constraint and non-constraint resources, the TOC provides five core steps that must be followed in order to ensure effective on-going improvement.

(1) Identify the system constraint(s).
(2) Decide how to exploit the system's constraint(s).
(3) Subordinate everything else to that decision.
(4) Elevate the system constraint(s).
(5) If, in a previous step, a constraint was broken, return to the first step, but prevent inertia from becoming the system's constraint.

Goldratt and Cox (2004) affirm that if capitalist enterprises must be evaluated in terms of net profits (*NP*) and return on investment (*ROI*), each local action should be judged according to its impact on the *NP* and the *ROI*. For that, the TOC suggests three performance measures.

- *Throughput* (*T*): The rate at which the system generates money by means of sales, i.e. all the money entering the enterprise minus the amount paid to suppliers (Total Variable Costs – *TVC*).
- *Investment* (*I*): All the money captured by the system.
- *Operating Expense* (*OE*): All the money the system spends on transforming investment into throughput.

According to Goldratt (1990), the relationship between the three TOC measurements (*T*, *I* and *OE*), Net Profit (*NP*) and Return on Investment (*ROI*) becomes immediate. *NP* is Throughput minus Operational Expenses, while *ROI* is Throughput minus Operational Expenses, divided by Investment.

In combination with these measures and derived from the logic present in TOC's five steps, Goldratt (1990) recommends a fourth measure that may be used in production mix decisions: throughput of an individual product divided by the unit of constraint consumed (Ti/UC). Implementation of this measure was illustrated by the author in a problem that became known as *P&Q*, from which many works discussed its effectiveness for these kinds of decisions. The production mix problem literature that considers TOC as part of the solution or as an element of comparison is relatively extensive and is summarised in Table 1. Some of these studies are discussed in the following.

Chaharsooghi and Jafari (2007) divide the studies carried out to solve production mix problems or to improve previously introduced approaches to this problem into three categories. The first group contains the exact approaches, such as the linear programme (LP). The LP approach is generally used for small-scale problem instances, because of the long computational time for larger problems. The second group involves the proposition of the heuristics methods in order to improve the solutions presented by TOC, especially in situations of multiple constraints. The third group involves meta-heuristic approaches geared to deal with large-scale problems.

The unfavourable results allowed by the Ti/UC rule in multiple bottleneck environments have been discussed by several authors. Lee and Plenert (1993) and Plenert (1993), by means of the Integer Linear Programming (ILP)

Table 1. Synthesis of articles on production mix problems.

Authors	Research summary
Luebbe and Finch (1992)	Compare TOC and Linear Programming (LP) using examples, and clarify the relationships between TOC's five-step focusing process and LP
Patterson (1992)	Suggests the effectiveness of the TOC method for the product mix decision when compared with labour-based management accounting
Plenert (1993)	By means of an example, compares the LP method and the TOC philosophy, pointing out the differences between them
Lee and Plenert (1993)	Demonstrate the process used by TOC to determine the product mix that maximises the throughput as a very simple series of steps and that the LP can be used to calculate the optimal solution
Maday (1994), Posnack (1994)	Claim that TOC heuristic's solution would be optimal even without an integer solution
Finch and Luebbe (1995)	Identify weaknesses in the TOC methodology when there are multiple constraints
Zegordi et al. (1995), Mishra et al. (2005), Chaharsooghi and Jafari (2007)	Apply Simulated Annealing to product mix problems
Lee and Plenert (1996)	Compare three product mix approaches: traditional accounting, TOC and LP. They conclude that TOC and LP present the same performance and are superior to traditional accounting
Fredendall and Lea (1997)	Revise the TOC product mix heuristic to identify conditions where the original TOC heuristic fails
Hsu and Chung (1998)	Indicate the use of an explicit algorithm when a plant has multiple resource constraints, but do not compare its effectiveness with the TOC-h of Fredendall and Lea (1997)
Mabin and Gibson (1998)	Reveal how the TOC and LP spreadsheet approaches can complement each other and may be integrated in a simple way
Finch and Luebbe (2000)	Criticise the comparison of TOC with LP performed by Balakrishnan and Cheng (2000); they affirm that they have different natures
Balakrishnan and Cheng (2000)	Contrary to the conclusions of Luebbe and Finch (1992), they show that using LP is preferable to the $ return/constraint unit method
Onwubolu and Mutingi (2001a,b)	Apply a genetic algorithm for the product mix decision
Onwubolu (2001)	Employs a Tabu-search-based algorithm to product mix problems
Boyd and Cox (2002)	Demonstrate that TOC outperforms three accounting systems when making some decisions, including product mix problems
Mabin and Davies (2003)	Study the product mix problem according to a variety of TOC approaches that complement traditional treatments such as LP
Aryanezhad and Komijan (2004)	Present and compare TOC-h and LP, and recommend an algorithm to determine the product mix
Bhattacharya and Vasant (2007), Bhattacharya et al. (2008)	Describe an alternative to the LP solution for a TOC product mix using a fuzzy linear programming approach in order to maximise the degree of satisfaction of the decision maker
Linhares (2009)	Reinforces the fragility of the TOC heuristic, pointing out some facts that go against the TOC literature, including showing some particular cases where the application of TOC to define the production mix fails even for small problems and with a single bottleneck
Wang, Sun, and Yang (2009)	Propose an immune algorithm as a means to solve product mix problems
Ray, Sarkar, and Sanyal (2010)	Demonstrate the combined use of TOC and the analytic hierarchy process (AHP) in the product mix definition
Sobreiro and Nagano (2012)	Evaluate some constructive heuristics and propose a new and better (according to their findings) constructive heuristic based on the Theory of Constraints and the Knapsack Problem

technique, evaluated the TOC heuristic in instances where its solution left idle time in the bottleneck. The optimal product mix fully utilised the bottleneck. They concluded that the ILP solution was more efficient than the TOC heuristic. Maday (1994) and Posnack (1994) disagreed with Lee and Plenert (1993) and Plenert (1993), stating that the TOC heuristic solution would be optimal even without an integer solution.

Fredendall and Lea (1997) proposed a constructive heuristic based on TOC to solve production mix problems with multiple bottlenecks that would provide a better solution than the use of the initial proposals presented by Goldratt (1990). Based on this line of research indicated by Fredendall and Lea (1997), Hsu and Chung (1998) proposed an algorithm based on a rule of dominance of the bottlenecks on each other, without making a comparison with the heuristic of

Fredendall and Lea (1997). With the goal of proposing a constructive heuristic that is more efficient than that of Fredendall and Lea (1997), Aryanezhad and Komijan (2004) presented a constructive heuristic that defines a production mix for each bottleneck and, based on this, identify the best production mix.

Finch and Luebbe (2000) affirm that the failure of the TOC technique is a valid concern and has been recognised for some time. They recall that, in a previous study, they identified difficulties with the TOC methodology when there are multiple constraints. Criticising the comparison of the TOC with LP carried out by Balakrishnan and Cheng (2000), Finch and Luebbe (2000) affirm that they are clearly different and accomplish different things. The key advantages of TOC, as a process, remain. It provides a well-developed framework for improving throughput that has broad implications for product mix determination, scheduling, inventory buffer placement, inspection placement, etc. Along the same lines, Mabin and Davies (2003) studied the product mix dilemma according to a variety of TOC approaches that complement the traditional treatments such as LP. The paper evaluated some TOC approaches, besides the product mix algorithm, such as the Five Steps, Evaporating Cloud and reality trees. The authors concluded that, due to the complexities inherent in the real world, the search for a 'best-fit' frame should be recognised and abandoned.

Souren, Ahn, and Schmitz (2005) exposed the premises, in the form of a checklist, on which the TOC-based product mix decision depends to generate an optimal solution. The premises are: (i) there should be only one binding constraint; (ii) the solutions should not be integers; (iii) all direct costs should be integrated in the throughput; (iv) joint material costs should not be integrated in the throughput; and (v) the goal function should be linear. Linhares (2009) reinforces the fragility of the TOC heuristic, pointing out some facts that go against the TOC literature, including some particular cases where the application of TOC to define the production mix fails even in small problems and with a single bottleneck.

By analysing how different types of management accounting systems and product mix methods interact in different conditions of planning horizons and product structures, Lea and Fredendall (2002) concluded that different configurations of the variables lead to different recommendations, with no same and best product mix method for every situation. The authors mention that the TOC heuristic, in combination with any accounting system, reduces bottleneck shiftiness, perhaps because it tries to fully utilise the bottleneck and allows idle time on non-bottlenecks to ensure – due to the variability in operations – that the bottleneck is fully utilised. This idle time on non-bottlenecks, or protective capacity, decreases shiftiness. The importance of protective capacity for the feasibility of the product mix solution is a central point of the present article.

3. The *P&Q* problem

The *P&Q* problem (Table 2) was presented by Goldratt (1990). It aims to permit discussion – by means of a simple and easy example – of the differences between the assumptions made by the TOC and other management approaches, especially those that use some form of cost allocation. In the present article, the goal is to make use of this example and of scenarios derived from it to discuss the feasibility of optimal solutions in light of the TOC. The ILP technique will be used here to provide optimal solutions to the problems suggested.

In this problem, a specific enterprise manufactures and sells two types of products, *P* and *Q*, and for that four resources are required: *A*, *B*, *C* and *D*. Sales prices, costs of raw materials – assumed here as the only *TVC* – and the demand for each product, as well as the unit processing time, in minutes, required for each resource are shown in Table 2.

These resources are paid to operate for 2400 minutes/week, and all the enterprise's costs and expenses, except for those associated with raw materials, correspond to $6000/week. The question is how many *P* and *Q* must be produced and sold so that the enterprise's profit is maximised. One can readily see that, in contrast to the other resources, resource *B* does not present enough capacity to meet the weekly *P* and *Q* demand. For this, 3000 minutes would be necessary, or 125% of available capacity. It is necessary to decide on a production mix that would maximise the profit of this enterprise.

Table 2. The *P&Q* problem.

Product	Processing time in resource				Market demand	Sales price ($)	RM cost ($)	Ti
	A	B	C	D				
P	15	15	15	20	100	90	45	45
Q	10	30	5	5	50	100	40	60

From this particular perspective, both TOC and LP (or ILP) use different approaches in order to find a better mix. According to TOC, the product that contributes the most to the enterprise's profit is P, because it produces the best (Ti/UC) relation. Each product P has a Ti of \$45 (90 − 45) and consumes 15 minutes of resource B, that is, generates \$3.00 (45 / 15) per minute used of the bottleneck resource. Product Q, on the other hand, allows only \$2.00 (60 / 30) per minute of the bottleneck. As resource B is best exploited when the production of P is prioritised, the TOC solution suggests that the maximum amount of P should be produced, that is, 100 units, and by using the remaining time from resource B, 30 units of Q must be produced. The maximum profit achieved with this mix is \$300$[(100 \times 45) + (30 \times 60) - 6000]$.

The same result is achieved when ILP is used, however for this to happen, the contribution that each product has to the enterprise's profit in the objective function must correspond to the Ti. The ILP does not suggest a specific criterion to determine the contributions that each decision variable must present in the objective function. The following ILP model can be elaborated for this type of problem:

$$\begin{array}{llcccccl}
\text{Maximise} & Z = & 45X_p & + & 60X_q & & & \\
 & & 15X_p & + & 10X_q & \leq & 2400 & (\text{Resource } A) \\
 & & 15X_p & + & 10X_q & \leq & 2400 & (\text{Resource } B) \\
 & & 15X_p & + & 10X_q & \leq & 2400 & (\text{Resource } C) \\
 & & 10X_p & + & 10X_q & \leq & 2400 & (\text{Resource } D), \\
 & & X_p & & & \leq & 100 & \\
 & & X_p & & & \geq & 0 & \\
 & & & & X_q & \leq & 50 & \\
 & & & & X_q & \geq & 0 &
\end{array}$$

where

X_p is the amount produced and sold of product P, and X_q is the amount produced and sold of product Q.

The solution of this model leads to $X_p = 100$, $X_q = 30$ and $Z = \$\ 6300$ (*Profit* = \$300). Even though both approaches yield the same results – and both of them optimal – the paths are different. Such differences become clearer when a modification is inserted into the original problem. Suppose the processing time for resource D is increased, in the final operation for product P, from 20 to 25 min. This situation is denoted here as a Reviewed $P\&Q$ Problem. In addition to resource B, this alteration results in a new bottleneck resource, resource D. While resource B still carries a 125% load, resource D assumes a 115% load. In fact, under these new conditions, the bottleneck resource can change from B to D, or *vice versa*, depending on what is being produced at the time.

Even under these conditions, when the previously suggested TOC approach is followed, resource B would be regarded as the main bottleneck, since it is more overloaded than D. Therefore, the TOC solution would maintain the same Ti/UC values for each product and would continue to suggest product P as the most profitable for the enterprise. However, only 96 units of P can now be produced, due to the lack of capacity in D, and it is not possible to manufacture a single unit of Q. In this case, there would be a loss of \$1680, and resource B would be operating at a 60% load and D at a 100% load.

The reader can verify that, if resource D is selected as the bottleneck to be exploited, product Q would become the most profitable, and the loss would decrease to \$300, with the load in resource B at 100% and at 73% in D. A different solution is obtained when ILP is applied to the reviewed problem. In this case, the enterprise would obtain a profit of \$120, and the optimal production mix would be 88 units of P and 36 units of Q. In this solution, resource B would present a load at 100% and D at 99%.

When analysing these results, the superiority of ILP over TOC seems evident, given that ILP allows for establishing better results and also better use of constrained resources. However, from the TOC point of view, other aspects would need to be taken into consideration in situations and decisions of this kind, suggesting that the solution provided by ILP is not necessarily the best. The following section will discuss these aspects.

4. The feasibility of the solutions according to TOC

The previous example shows that the Ti/UC approach – suggested by TOC in the identification of a better production mix – does not achieve satisfactory results when there is more than one bottleneck resource. Is TOC a limited approach or applicable only to specific situations? Why not always choose the solutions provided by ILP or any other optimal solutions aimed at finding optimal solutions? This section intends to answer these questions based on a critical analysis of the assumptions on which the TOC is founded.

4.1 *The interactive constraint concept and a new TOC perspective*

The interactive constraint concept is fundamental for understanding TOC analyses situations such as that shown in Section 3. When observing the depicted ILP model, one notes a large number of constraints considered for the problem solution. Such constraints, however, are not necessarily considered interactive, according to the TOC language.

According to Goldratt (2008), the complexity of a system – as in the *P&Q* problem under analysis – is determined by the number of interactive constraints, which present a reciprocal impact. For the author, the interactive constraints significantly increase system complexity, thus greatly reducing its performance. Therefore, the best way to improve the system is to determine its inherent simplicity, avoiding any kind of interactive constraints (Barnard 2010).

The inability of systems to tolerate interactive constrained resources comes directly from the coexistence of two phenomena: (i) statistical fluctuations – internal variations such as absenteeism, unplanned shutdowns of equipment, longer than expected setup times, unforeseen quality problems, layoffs, power failures, or external variations such as changes in market demand, and supplier reliability in terms of time, quantity and quality – and (ii) dependent events, which means that the work to be performed by a resource depends on the performance of one or more resources (Schragenheim and Dettmer 2001, Goldratt and Cox 2004, Kendall 2004, Goldratt and Goldratt 2006).

Although such phenomena are absent in the original *P&Q* problem, as well as in its modified version, they are present in any organisational or production system. Thus, this problem, like many others used to evaluate the efficiencies of TOC solutions for the production mix, fails by not incorporating these phenomena, making them unrepresentative of real environments. If this phenomenon was incorporated in the modified *P&Q* problem, in which the loads were above 100% in resources *B* and *D*, and considering the fact that the more overloaded resource changes according to the production mix, it would configure the existence of interactive constraints, sharply increasing the complexity of the situation and making impracticable the profit of $120 found by the ILP solution.

The existence of interactive constraints is related to the concept of protective capacity, which is defined by Blackstone (2010) as the capacity, over the productive capacity, needed to restore the buffers to their ideal state after a disruption. The protective capacity is vital because if there is insufficient protective capacity, then the buffer – the constraint's protection – cannot be refilled quickly enough and thus system capacity is vulnerable to possible starvation by upstream stations or blocking by downstream stations.

The protective capacity should be understood as the excess capacity all non-constraint resources have with regard to the constraint of the system in which they are part (Goldratt and Goldratt 2006). If the performance of a system is determined by the performance of its constraint, the protective capacity in all non-constraint resources becomes fundamental. According to Goldratt and Goldratt (2006), a constraint can only be truly exploited – a constrained resource does not stop due to the lack of parts (in the event the system constraint is a bottleneck resource) or that the demand is fully met (in the event the system constraint is in the market) – when all other resources present significant protective capacity.

According to TOC, interactive constrained resources, as illustrated in the reviewed *P&Q* problem example, must be avoided, at any price (Schragenheim and Dettmer 2001). This can be explained by the second step of the TOC process. 'Deciding how to exploit the constraint' does not necessarily mean using the simple arithmetic solution taught by Goldratt (1990) in the *P&Q* example. The author used the problem to illustrate the five steps, which must be applied to real situations. According to this point of view, when ILP or another method, heuristic or not, does not prevent interaction between constraints, its solution is infeasible.

Goldratt (1990) asserts that, in interactive constraint situations, a company should strategically 'decide' where best to have the constraint. Therefore, the company should buy additional capacity for the other constraint resources – elevate them – or the demand should be reduced, thus preventing interactive constraints presenting their effect. This view suggests that, for the entire system to remain stable, effective and in control, there must always be a single constraint. TOC offers some techniques that support decisions regarding capacity or demand management, such as buffer management, capacity buffer management or bottleneck workload control, aimed at avoiding interactive constraints. More details can be found in Schragenheim, Dettmer, and Patterson (2009).

Schragenheim and Dettmer (2001) indeed recognise that the *Ti/UC* rule has no value in interactive constraint situations. They report the conditions for when that rule could be applied.

(1) The bottleneck must be a true constraint all the time.
(2) There is no more than one bottleneck in the system, meaning that there are no interactive constraints.
(3) The decision to be made cannot cause a change in the constraint for another resource.
(4) When facing major sales contracts, the decision maker must always consider the global impact of this decision on the *T*, *I* and *OE* indicators, as a general rule.

While TOC considers the absence of interactive constraints in the system desirable, there may be situations where it is not possible, since increasing the capacity of one of the bottlenecks is time-consuming and, sometimes, money-consuming. Therefore, what and how much should be produced up to the moment the effect of the decision to eliminate the interactive constraints appears? The following topic will address this issue.

4.2 *The rule of Barnard (2006) and a new proposal*

Using once more the reviewed *P&Q* problem, the solution proposed by the ILP approach suggests that resource *B* must operate at 100%, while resource *D* must operate at 99%, or 1% (100% – 99%) of protective capacity. According to Barnard (2006), this solution would be unachievable, because the excessively low protective capacity of *D* compared with *B* would make the result infeasible. The author applied simulation experiments in order to substantiate his solutions.

If the Ti/UC rule is not valid for interactive constraint situations, and the solution proposed by the ILP is not recommended because it does not preserve the protective capacity, what can be done in such situations?

The TOC recognises (Barnard 2006, Goldratt, and Goldratt 2006, Schragenheim and Dettmer 2001, Goldratt, Schragenheim, and Ptak 2000) that, due to the elevated levels of uncertainty and variation inherent in organisations, the noise level in systems becomes so high that there is no reason to search for optimal solutions, but rather to search for solutions that are good enough. Therefore, there would be a range of possibilities that would comprise a safe zone around those solutions that are good enough. If solutions that are different from the optimal solutions could produce unsatisfactory results when seen from the ILP perspective, there would be a safe zone of good enough solutions under the TOC perspective. Figure 1 illustrates this idea.

In this sense, decisions must be made concerning stocks, capacity, time buffers, production policies, price establishment and even production mix – among others – by considering this zone as an area of solutions that are good enough. Barnard (2006) suggests a rule in order to determine the production mix for interactive constraint situations that would position the solution inside the safe zone. At the same time, it would be feasible in maintaining protective capacity. The suggested rule would be defined as: produce an amount of each product equivalent to the percentage obtained from the ratio between the available capacity and the required capacity of the most overloaded resource. For the author, such a solution attempts to ensure good utilisation of the interactive constraint resources by placing the solution in the security zone to (from this point on, and observing the impacts of such decisions on the *T*, *I* and *OE* measurements) make improvement decisions in order to actually eliminate the interactive constraints of the system.

By applying this rule to the reviewed *P&Q* problem, the most overloaded resource is *B*, which needs 3000 minutes to meet the entire demand. As 2400 minutes is the weekly time available, a percentage equivalent to 2400 divided by 3000 – or 80% of each product – must be produced. Here, this percentage will be called Barnard's factor. The suggested mix would be, therefore, 80 units of $P(0.8 \times 100)$ and 40 units of $Q(0.8 \times 50)$.

With this mix, resource *B* would carry a workload equivalent to 100%, and resource *D* with 92%, or 8% of protective capacity. The reader will be able to verify that, with this mix, the enterprise obtains a maximum profit of zero (null profit). This result – although lower than the maximum theoretical of $120 – is higher than the negative $1680 reached by means of the Ti/UC rule. Barnard (2006) emphasises that, although this simple rule seems to provide promising results, a great deal of research is still necessary. Taking this into account, a heuristic method is developed to identify the production mix in accordance with Barnard's rule. The heuristic method is shown in the next section.

4.3 *Heuristic method*

Throughout the paper, the following notation and decision variables are used.

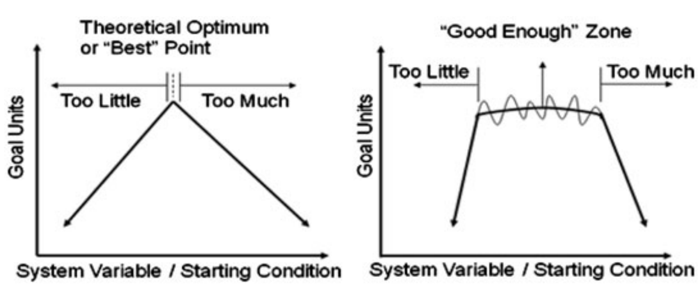

Figure 1. Differences among the approaches based on the 'optimal theoretical' and 'zone of good enough'.
Source: Adapted from Barnard (2006).

4.3.1 *Notation*

Indices

$i = 1, 2, \ldots, n$	products
$j = 1, 2, \ldots, m$	productive resources
$q = 1, 2, \ldots, m$	constraint resources

Decision variables

BN_q	constraint resource q
CP_j	production capacity of resource j
CM_i	contribution margin of product i or sales price of product i less raw material cost of product i
CR	vector with constraint resource
d_j	the difference between a resource's capacity and its demand
D_i	the forecast market demand for product i
q	number of constraints resources
t_{ij}	the processing time of product i in productive resource j
x_i	quantity of product i
Bf	Barnard's factor
DB	dominant bottleneck
CP_q	protective capacity in bottleneck q

4.3.2 *Proposed heuristic*

The purpose of the heuristic is to develop an initial solution considering the heuristic TOC-h presented by Fredendall and Lea (1997) and verified if the initial solution has protective capacity in $m-1$ resources. If the initial solution does not have protective capacity in $m-1$ resources, the proposed heuristic calculates and applies Barnard's factor to define the production mix.

Before proceeding, some points should be made clear: the percentage of protective capacity that non-constraint resources will have after applying Barnard's factor depends on the initial load imposed on these resources and on Barnard's factor itself. In the revised *P&Q* problem, the load on *D* was 115% and Barnard's factor was 0.8. Therefore, the ultimate load on *D* was 115 times 0.8 or 92% (8% protective capacity). Since the initial loads and Barnard's factor are not under control of the manager, one cannot apply the rule of Barnard in order to obtain a certain predetermined minimum level of protective capacity. Thus, an adjustment is made to the rule so that one can predefine the minimum level of protective capacity that $m-1$ resources must have to meet the production mix. The proposed heuristic is shown in the next section.

4.3.2.1 *Proposed heuristic.*

Step 1: Verify if there are system bottlenecks.

(a) Calculate the difference (d_j) between a resource's capacity and its demands:

$$d_j = CP_j - \sum_{i=1}^{n} t_{ij} D_i \quad \begin{array}{l} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, m. \end{array}$$

(b) Determine $CR = \{BN_1, BN_2, \ldots BN_q\}$, $q \leq m$, where CR is the set of constrained resources with $d_q \leq 0$ and $d_1 \leq d_2 \leq \ldots d_q$.

(c) If $CR = \emptyset$, set the product mix considering the current market demand and go to Step 5.

(d) If $q = 1$, apply TOC-h to define the product mix and go to Step 3. Otherwise, $q > 1$, go to Step 2.

Step 2: Set initial solution.
Check if TOC-h identifies the dominant bottleneck and defines the product mix. Otherwise, determine the product mix considering BN_q as the dominant bottleneck. According to Fredendall and Lea (1997), the dominant bottleneck is the first resource that presents a lack of capacity.

Step 3: Check the protective capacity.

(a) Calculate the protective capacity for each resource as

$$PC_j = \frac{(CP_j - \sum_{i=1}^{n} t_{ij} \times x_i)}{CP_j} \times 100.$$

(b) Verify if the product mix provides protective capacity in $m - 1$ resources. The ideal value of the protective capacity should be chosen by the manager. If there is protective capacity in $m - 1$ resources, then go to Step 5. Otherwise, go to Step 4.

Step 4: Apply Barnard's factor.

(a) Calculate Barnard's factor as

$$Bf = \frac{CP_{DB}}{\sum_{i=1}^{n} t_{iDB} \times D_i}.$$

(b) Determine the product mix by multiplying the current market demand (D_i) for each product by Barnard's factor.
(c) Verify if there is protective capacity in $m - 1$ resources. If $m - 1$ has protective capacity, then go to Step 5. Otherwise, do $Bf = Bf - 0.01$, then go to Step 4(b).

Step 5: Determine throughput.
Calculate the throughput of the product mix as

$$Throughput = \sum_{i=1}^{n} x_i \times CM_i.$$

The proposed heuristic is straightforward and is best summarised by the flow chart shown in Figure 2.

4.3.2.2 *Numerical examples.* In order to test the heuristic presented in Section 4.3.2, an extensive computational experiment is carried out. The proposed heuristic is applied to solve a range of 100 problems, 12 of them based on problems proposed by Goldratt (1990), Luebbe and Finch (1992), Patterson (1992), Lee and Plenert (1993), Fredendall and Lea (1997) and Aryanezhad and Komijan (2004). The remaining 88 problems are randomly generated problems, including scenarios that blend manufacturing environments with two to eight products and with four to eight resources. These situations expose problems in which met demand gives rise to one to four bottlenecks. Several scenarios present interactive bottlenecks. The proposed heuristic was developed in VBA from Microsoft Excel and tested using an Intel Core i5 2.67 GHz computer with 6 GHz of RAM memory.

Table 3 and Figure 3 summarise the results obtained by the proposed heuristic applied to problems where the TOC-h does not maintain $X\%$ protective capacity in $m - 1$ resources and compare them with the optimal result obtained from applying the ILP and from an adapted ILP method that aims to ensure a minimum level of protective capacity in $m - 1$ resources. The proposed heuristic is tested for 24 levels of protective capacity, as shown in Table 3 and partly in Figure 3. It should be noted that, in the various problems, the application of the TOC-h heuristic obtained the expected protective capacity and therefore did not require the application of Barnard's rule.

The results show that, up to 10% of protective capacity, the performance achieved by applying the proposed heuristic is above 80% of the theoretical maximum allowed by the application of ILP, or over about 84% of the application of ILP with protective capacity. With more than 10% of protective capacity, the performance is below 80% of the application of ILP – or even below 70% when the protective capacity approaches 30%. However, if compared with the ILP

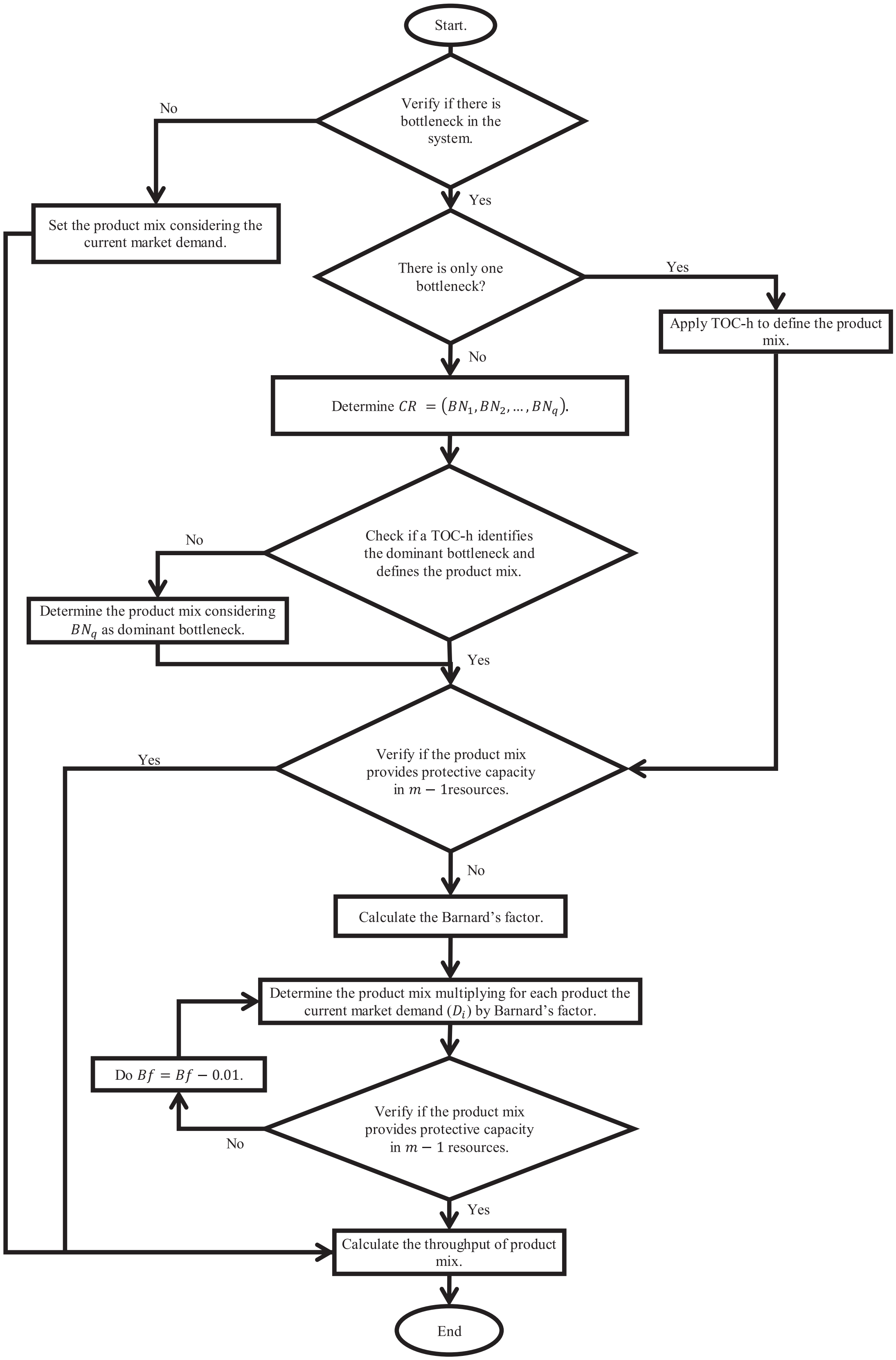

Figure 2. Flow chart of the proposed heuristic.

Table 3. Results obtained by the proposed heuristic.

X% protective capacity	Number of problems in which the proposed heuristic was applied	% Average performance	
		Regarding ILP	Regarding ILP with protective capacity
0.50	16	86.66	86.86
1.00	22	86.75	87.10
1.50	23	86.89	87.39
2.00	25	86.78	87.41
2.50	31	86.78	87.49
3.00	34	85.52	86.33
3.50	37	85.04	85.98
4.00	38	84.93	86.03
4.50	42	83.25	84.43
5.00	42	83.10	84.47
5.50	45	83.11	84.66
6.00	45	82.94	84.70
6.50	47	82.75	84.71
7.00	48	82.05	84.18
7.50	48	81.84	84.17
8.00	48	81.57	84.12
8.50	50	81.64	84.39
9.00	52	81.65	84.53
9.50	52	81.35	84.45
10.00	54	80.67	83.99
15.00	65	77.73	83.01
20.00	74	74.48	82.04
25.00	79	71.53	81.40
30.00	85	67.29	79.41

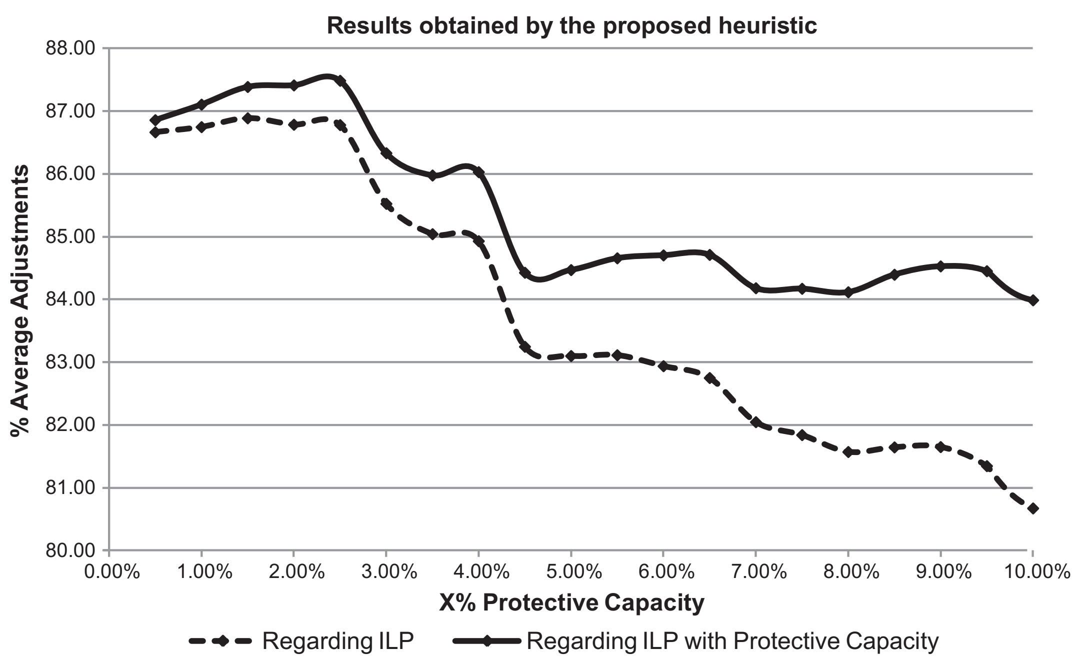

Figure 3. Results obtained by the proposed heuristic.
Note: For reasons of scale, the graphic is limited to the 10% level of protective capacity.

method adapted to take the protective capacity into account, the performance falls only marginally below 80% with levels of protective capacity close to 30%. More specifically,

- from zero to about 2.5% of protective capacity, the proposed heuristic performance seems to behave in a non-decreasing manner;
- from about 2.5% of protective capacity, when compared with the optimal solutions obtained with the ILP method, the performance tends to fall indefinitely with increased levels of protective capacity;
- from about 5 to 10% of protective capacity, the heuristic appears to have a tendency to stabilise the performance when compared with the ILP method with protective capacity. From this point, the relative performance seems to fall, but smoothly.

An important aspect must be registered. As already predicted by Barnard (2006), the classic TOC solution based on the T_i/UC rule does not meet the need for keeping protective capacity – since the TOC, in principle, deliberately seeks to ensure protective capacity of its non-constraints resources. Although the ratio Ti/UC allows throughputs superior to the proposed heuristic, the production mix achieved with the Ti/UC ratio does not preserve, for example, 10% of protective capacity in 54 of the problems tested.

The average performance of the proposed heuristic, however, disguises the significantly poor results for some of the tested problems. As an illustration, at a level of at least 10% of protective capacity, the performance of the heuristic is less than 70% in six problems, and less than 40% in one problem. With the intention of finding an explanation for such behaviour, a correlation coefficient of 0.846 is observed between the performance of the heuristic and Barnard's factor, i.e. the lower the Barnard factor, the worse the performance. If, for example, the problems that presented a Barnard's factor of less than 0.65 were excluded from the calculation of the average performance, the average performance of the heuristic increases to 88.01%. One possible reason for this could be the fact that lower values for Barnard's factor reflect a sharp linear cut in the total product mix. In these situations, resources with a relatively low load should have their production significantly reduced. Although tests have shown no further explanation for the loss of performance in some situations, it is worth noting that the proposal made by Barnard (2006) does not distinguish different throughputs between products. Thus, by applying Barnard's factor, products with high or low throughputs are treated in the same way.

5. Conclusions

Since the studies of Lee and Plenert (1993) and Plenert (1993), there has been a vast production-mix literature demonstrating the fragility of applying the Ti/UC rule in specific situations, especially when multiple constraint resources exist. New solutions – heuristics, genetic algorithms, tabu search and even fuzzified linear programming – have been proposed to overcome this weakness of TOC (Fredendall and Lea 1997, Hsu and Chung 1998, Balakrishnan and Cheng 2000, Finch and Luebbe 2000, Onwubolu and Mutingi 2001a,b, Onwubolu 2001, Aryanezhad and Komijan 2004, Wang, Sun, and Yang 2009, Sobreiro and Nagano 2012). However, as Linhares (2009) points out, "there is still a clear need for further research of cases with imperfect and non-deterministic information".

This paper used a numeric example called the *P&Q* problem in order to assess the feasibility of optimal production mix solutions in non-deterministic – and therefore real – environments. Accordingly, a feasible production plan should take into account the presence of protective capacity in the system, an aspect usually ignored by the exact and heuristic techniques, targeted to maximise the theoretical system throughput. Therefore, the first important conclusion of this paper is that any method that preserves the protective capacity in their solutions is intentionally suboptimal, or should allow results that are just good enough and thus any kind of comparison with optimisation methods does not make sense.

The TOC's focus on the constraint, the assumption that every system must always present a few constraints, and the emphasis on recognising variability in production systems, are indicative of the need for using protective capacity. In this sense, as described by Schragenheim and Dettmer (2001), the TOC approach is based on simple and effective rules, distancing itself from optimising solutions. From the perspective of TOC, besides protective capacity, real environments should also have buffers to protect system constraints, reflecting not only the perception that the constraint must be protected from system variability, but also that, due to this variability, any attempt to find optimal values during the planning phase, including the production mix, is inappropriate. In TOC applications, the search for optimal or accurate values of these planning elements is seen as a fruitless attempt to optimise within the noise (Goldratt and Goldratt 2006, Schragenheim, Dettmer, and Patterson 2009).

Mabin (2001) affirms that, as a 'heuristic', the Ti/UC rule can lead to good solutions, but can also lead to erroneous solutions in some situations. To the author, the TOC recognises the fragility of the product mix problem and TOC solution and deals with it by means of complementary methods, such as the drum-buffer-rope scheduling approach, which allows adapting the initial solution to "a robust operational plan to cope with sequential and variable operations".

From the TOC point of view, the lack of protective capacity, resulting from the presence of interactive constraints, is something that should be avoided because it increases system complexity. To this end, a strategy to eliminate this situation should be defined as soon as possible. However, since this strategy does not present an effect, a simple and effective solution must be created with the purpose of achieving good (but not optimal) results and which, at the same time, maintains certain levels of protective capacity for its feasibility. In summary, it can be concluded that from the TOC point of view:

(1) the existence of multiple bottleneck resources should be seen as undesirable and a control system must be used – such as buffer and capacity management – to avoid it;
(2) in the case of failure, a strategy to eliminate this situation should be defined as soon as possible; and
(3) while this situation persists, a feasible production mix should be found. To this end, this paper proposes a heuristic based on Barnard (2006). Even though less simple and comprehensible than the Ti/UC rule, such a proposed heuristic requires relatively little effort for modelling to be used and does not rely on prior knowledge in operational research.

The proposed heuristic should be used with caution. The calculations showed some situations in which the performance of the heuristic significantly worsens. Although the reasons for this deterioration of performance are not entirely clear, there seems to be evidence pointing to the existence of a positive correlation between the Barnard's factor and the throughput generated. Levels of protective capacity above 10% also seem to lead to the most significant decreases in performance.

It is hoped that this paper has successfully demonstrated its intentions, thereby contributing to enrich discussions on the actual objectives of the TOC. Current and future research efforts on TOC should concentrate on evaluating the proposal presented – including new research aimed at better understanding the reasons underlying the results achieved by the heuristic – or proposing other heuristics to find viable solutions that maintain some level of protective capacity of the chosen company's strategic constraint. The behaviour of the proposed heuristic should be checked, also applying it in more complex environments, involving more resources and products. Finally, it is recommended to simulate and test the feasibility of the proposed solution in several scenarios where the data effectively incorporate dependencies and some degree of variability or adapt the proposed heuristic to different production systems.

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