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The internal resistance of supercapacitors

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Abstract
In this paper we study the transient behaviour of RC circuits with supercapacitors, varying R between 1 and 100 Ω. We demonstrate that supercapacitors behave as ideal capacitors in series with an internal resistance ($r \sim 8\ \Omega$ for $C = 0.2$ F, 5.5 V). This result is important to optimize the demonstration of RC circuits using a supercapacitor in series with a light bulb, because the r value is comparable with the effective resistance of the bulb. This means that the bulb brightness is significantly decreased by r.

Circuits using bulbs have been used as a qualitative indicator of electric current in the teaching of DC circuits [1]. These kinds of experiments are interesting for pedagogical research [2, 3] as well as teaching using enquiry based methods [4–7]. They are particularly useful for demonstrations, since the variation of bulb brightness can be noticed by a student sitting in a classroom. More recently, low cost capacitors with high capacitance (⩾0.1 F), so-called supercapacitors or ultracapacitors, have become available, enabling interesting applications in physics teaching [8]. Due to their very high capacitance, it is possible to observe the process of charging or discharging a capacitor using a bulb by observing the variation of its brightness, which takes tens of seconds. As shown in figure 1, this process is slow enough to be observed in qualitative experiments or demonstrations, which are ideal for enquiry-based methods. In a recent publication the problem of (dis)charging a capacitor through a bulb was quantitatively analysed, taking into account the resistance variation of the filament due to the temperature change [9]. From a qualitative point of view, the bulb can be considered as a quasi-resistor, and the transient behaviour of the supercapacitor–bulb circuit is similar to an RC circuit, as shown in figure 1. However, the transient behaviour of a supercapacitor in an RC circuit differs from the usual due to the capacitor voltage jump at $t = 0$, which can be observed in both the circuit with a bulb (figure 1) or with a regular resistor, $R = 8\ \Omega$, as shown figure 2. A supercapacitor can behave very differently from an ideal capacitor, even in a qualitative experiment. For instance, the initial brightness of the bulb in the RC circuit at $t = 0$ should be the same as for a circuit without the capacitor, since the initial current would be $I_0 = \varepsilon/R$. However, in a circuit with a supercapacitor the bulb is usually much dimmer.

In order to investigate this problem, we measured the (dis)charge curve of a supercapacitor (supercapacitor 0.22 F, NEC/Tokin Series FS 5.5 V) as a function of the resistor value, varying R from 1 to 100 Ω. We analysed the magnitude of the voltage jump, ΔV, as well as the exponential response time, τ, and showed that all data could be modelled considering the supercapacitor as an ideal capacitor, with capacitance C, in series with an internal resistance, r. We obtained the values of C and r from the analysis of the t dependence with

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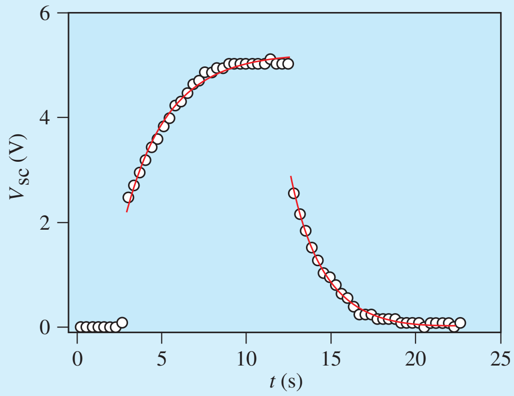

Figure 1. Transient behaviour of a charge–discharge supercapacitor-bulb circuit. The solid line represents the exponential fit with response time ~2 s.

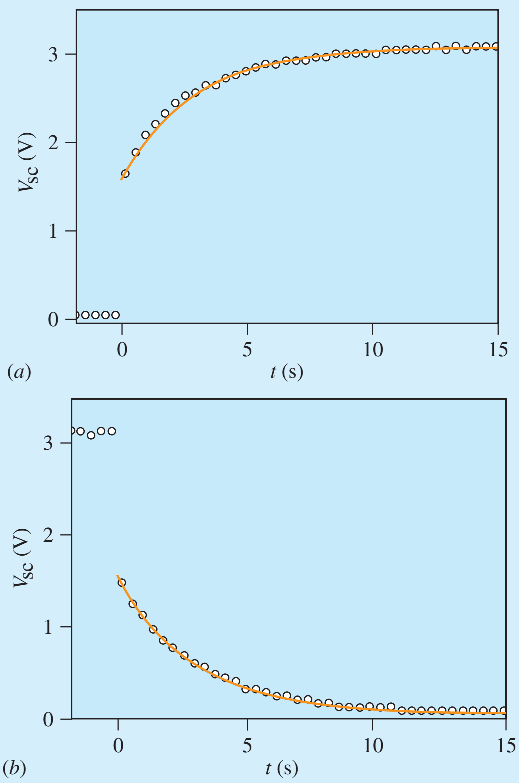

Figure 2. Transient signals of a supercapacitor-resistor (RC) circuit, with $C = 0.22$ F (nominal value) and $R = 8.1\Omega$, where $V_{SC}(t)$ represents the supercapacitor voltage. (*a*) Charging the capacitor and exponential fit with $\tau = 2.832$ s (equation (7)); (*b*) the discharge curve fit by equation (9) with $\tau = 2.992$ s. These transients were obtained using the circuit shown in figure 3, with an $\varepsilon = 3.0$ V power supply.

the external resistor, R. The r value can also be obtained from the analysis of dependence of ΔV with R. Finally, we discuss the effect of r in the circuit of a bulb with a supercapacitor and how to maximize the bulb brightness.

We will first consider the standard problem of charging a capacitor in an ideal series RC circuit connected to a DC voltage supply with constant emf, ε. Assuming that the capacitor discharged at $t = 0$, the instant when the switch is connected, the dependence of the charge and current are given by:

$$q(t) = Q_f\left[1 - \exp\left(-\frac{t}{\tau}\right)\right], \qquad (1)$$

$$I(t) = I_0 \exp\left(-\frac{t}{\tau}\right), \qquad (2)$$

where $\tau = RC$, $Q_\mathrm{f} = \varepsilon C$ and $I_0 = \varepsilon/R$. The current $I(t)$ actually has a discontinuity at $t = 0$, since it jumps from zero (for $t < 0$) to $I_0 = \varepsilon/R$ and is given by equation (2) only for $t \geqslant 0$. In the discharge of a capacitor, with initial charge $Q_0 = \varepsilon C$, we have:

$$q(t) = Q_0 \exp\left(-\frac{t}{\tau}\right). \qquad (3)$$

Like in the charging case, the current is given by equation (2) for $t \geqslant 0$, and is also discontinuous at $t = 0$. Note that the current direction is opposite in the charge and discharge of the capacitor. In both cases, charge and discharge, $q(t)$ and consequently the capacitor voltage, are continuous functions since variation of charge is proportional to the integral of the current.

Figure 2 shows that for the supercapacitor a voltage, and consequently the current, are discontinuous at $t = 0$. This behaviour can be explained assuming that the supercapacitor behaves like an ideal capacitor, of capacitance C, in series with its internal resistance value, r, as shown in figure 3. Therefore, the voltage across the supercapacitor, V_{SC}, is given by:

$$V_{SC}(t) = rI(t) + \frac{q(t)}{C}. \qquad (4)$$

Thus, even with null charge at $t = 0$, the initial voltage across the capacitor is not zero due to internal resistance, $V_{SC}(0) = rI_0$. Therefore, for a supercapacitor connected to a power supply (ε) and

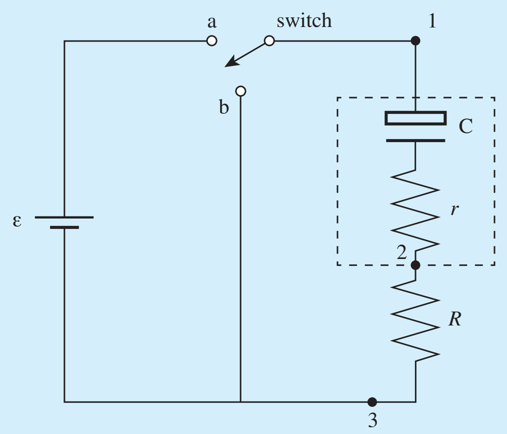

Figure 3. Circuit of the charging and discharging of a supercapacitor with a dc power supply, ε, connected to a resistor, *R*. The supercapacitor is represented with dashed lines, as an ideal capacitor, *C*, with a series resistance value, *r*.

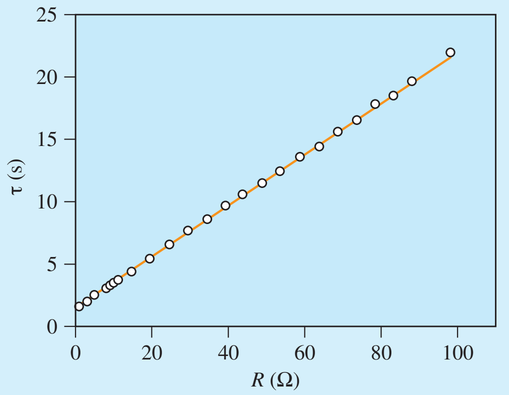

Figure 4. Discharging response time (τ) versus the external resistance (*R*). The linear fit results in τ (s) = 1.515 + 0.201*R*.

external resistor, R, the initial current is given by:

$$I_0 = \frac{\varepsilon}{(r + R)}. \tag{5}$$

Since the equivalent resistance of the circuit is $R_{eq} = (r + R)$, consequently, the charge $q(t)$ is given by equation (1) with $Q_f = \varepsilon C$ and

$$\tau = C(r + R). \tag{6}$$

Substituting equations (1) and (2) into (4), for the charging curve leads to:

$$V_{SC}(t) = \varepsilon + (\Delta V - \varepsilon) \exp\left(-\frac{t}{\tau}\right), \tag{7}$$

with:

$$\Delta V = \varepsilon \frac{r}{r + R}. \tag{8}$$

So $V_{SC}(t)$ is discontinuous at $t = 0$, jumping from zero to the value ΔV.

In the process of discharging, at $t = 0$ the current is given by equation (5) and the voltage jumps from $V_{SC}(t) = \varepsilon$ (for $t < 0$) to V_{SC} $(t = 0) = \varepsilon - \Delta V$. Therefore, for $t \geqslant 0$:

$$V_{SC}(t) = (\varepsilon - \Delta V) \exp\left(-\frac{t}{\tau}\right). \tag{9}$$

To test the validity of the model described above we measured the transient behaviour of the circuit illustrated (figure 3), using the same supercapacitor and varying R. The transient curves were obtained using a digital oscilloscope (Tektronix, TDS 2022B Two Channel Digital Storage Oscilloscope 200 MHz). A typical discharge curve is shown in figure 2(b), fitted by an exponential decay, according to equation (9), which provided the values of ΔV and τ. The same procedure was repeated for all values of R, varying between 1 and 100 Ω. Figure 4 shows that τ increases linearly with R but the straight line does not cross the origin, in accordance with equation (6). From the linear fit and equation (6), the values $C = 0.204$ F and $r = 7.42$ Ω were obtained. Figure 5 shows the behaviour of ΔV as a function of R. The fit of this curve by equation (9) provided the value $r = 8.3$ Ω, in reasonable agreement with the 7.4 Ω value obtained before. Therefore, we can take the average value $r = 7.9$ Ω.

The same procedure was applied to analyse the data obtained charging this supercapacitor. However, the values obtained, $C = 0.165$ F and $r = 13$ Ω, are lower by 20% and higher by 65%, respectively, than those obtained from the discharging data. Consequently, the τ values for the charging curves are ~32% higher. This is a general feature of this kind of supercapacitor; according to the manufacturer [10] the capacitance in the charging circuit is 50% higher than in the discharging one. We have also noticed that supercapacitors are very sensitive to ageing, with a significant increase of r and decrease of C. Finally, we measured the self-discharge of the supercapacitor, due to leakage current, in order

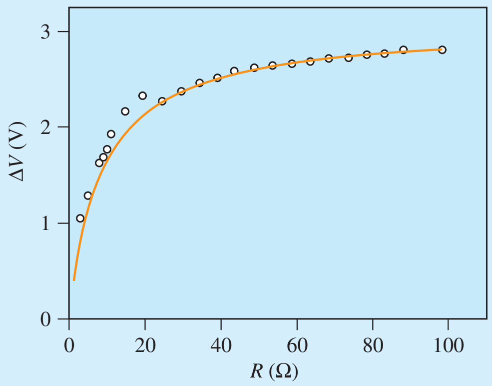

Figure 5. The behaviour of ΔV (V) as a function of R (Ω) and fit by equation (8).

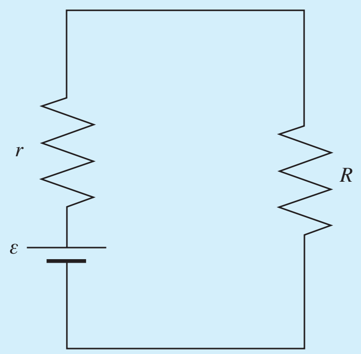

Figure 6. The effect of the supercapacitor internal resistance, r, is similar to the internal resistance of a power supply.

to estimate the capacitor parallel resistance. We observed that the self-discharge time constant is larger than one week (6×10^5 s), which is equivalent to a parallel resistance $\sim$3 MΩ for $C = 0.2$ F. Therefore, the leakage effect is negligible for time intervals shorter than a few hours, so it was not considered in this paper.

When the supercapacitor is used to demonstrate the RC properties using a bulb, its internal resistance has a very important effect on the bulb brightness. In order to analyse this effect we will first consider the power dissipated in the external resistor, R, shown in figure 3. Since the current is maximum at $t = 0$, the initial power is also maximum and given by:

$$P_{R0} = \varepsilon^2 \frac{R}{(r+R)^2} = \frac{\varepsilon^2}{R} \frac{1}{(1+r/R)^2}, \qquad (10)$$

where the term ε^2/R is the power dissipated in the circuit of a single resistor connected to an ideal power supply. Equation (10) shows that P_{R0} is lower than ε^2/R due to the supercapacitor internal resistance, r; only for the case of an ideal capacitor ($r = 0$), do we have $P_{R0} = \varepsilon^2/R = RI_0^2$, where I_0 is the initial current. Equation (10) can be rewritten using equation (8) as:

$$P_{R0} = \frac{\varepsilon^2}{R}(1 - \Delta V/\varepsilon)^2. \qquad (11)$$

So the term in brackets represents how much the initial power ($t = 0$) decreases compared to the ideal capacitor, where $r = 0$ and consequently $\Delta V = 0$. Equation (11) is useful, because the term $\Delta V/\varepsilon$ can be directly obtained from the decay curve. For instance, in figure 2, $\Delta V/\varepsilon \sim 0.5$ so $(1 - \Delta V/\varepsilon)^2 = 0.25$. Therefore, in this case P_{R0} is about four times lower than the value expected for an RC circuit with an ideal capacitor.

Supercapacitors have been widely used in series with a bulb where the bulb brightness indicates the magnitude of the current. The model studied in this paper allows us to reach important conclusions. First, it should be noted that the internal resistance of a supercapacitor (r) strongly decreases the bulb brightness. In the ideal case (where $r = 0$) the initial brightness (at $t = 0$) would be equal to the brightness of the circuit without the capacitor (the circuit with only the source and the bulb). For instance, this brightness comparison is an activity that is proposed in [4] but we showed that the effect of r can invalidate this comparison, unless $r \ll R$, where R can be the effective resistance of a bulb. So the factor r/R is an important parameter for comparison of the real with the ideal RC circuit. Note that this parameter appears in equations (8) and (10) and when $(r/R) \to 0$ then $\Delta V \to 0$. Moreover, we can infer that in order to maximize brightness (which can be important in a classroom demonstration, for instance) the effective resistance of the bulb should be close to the value of r. Therefore, the role played by r in a supercapacitor is similar to the effect of the internal resistance of a power supply or battery (figure 6).

We consider that the supercapacitor–bulb circuit is an excellent qualitative demonstration of RC circuits, as proposed by many authors. However, the instructor should be aware of the supercapacitor's nonideal behaviour, although this does not need to be mentioned to beginning students. Moreover, the problem of how to model the supercapacitor can be proposed as a challenge (black box circuit) to more advanced students.

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