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Black hole formation with an interacting vacuum energy densityM. Campos^{1,2,*} and J. A. S. Lima^{1,†}¹*Departamento de Astronomia, Universidade de São Paulo, 05508-900 São Paulo, São Paulo, Brazil*²*Departamento de Física, Universidade Federal de Roraima, 69304-000 Boa Vista, Roraima, Brazil*

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We discuss the gravitational collapse of a spherically symmetric massive core of a star in which the fluid component is interacting with a growing vacuum energy density. The influence of the variable vacuum in the collapsing core is quantified by a phenomenological β parameter as predicted by dimensional arguments and the renormalization group approach. For all reasonable values of this free parameter, we find that the vacuum energy density increases the collapsing time, but it cannot prevent the formation of a singular point. However, the nature of the singularity depends on the value of β . In the radiation case, a trapped surface is formed for $\beta \leq 1/2$, whereas for $\beta \geq 1/2$, a naked singularity is developed. In general, the critical value is $\beta = 1 - 2/3(1 + \omega)$, where ω is the parameter describing the equation of state of the fluid component.

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I. INTRODUCTION

It is widely known that the observed Universe is undergoing an expanding accelerating stage [1,2]. The simplest explanation in the context of Einstein's general relativistic theory (GRT) is the existence of a new dark component (in addition to cold dark matter) whose energy density remains constant or slowly varies in the spacetime. The most theoretically appealing possibility for the so-called dark energy is the energy density stored on the true vacuum state of all existing fields in the Universe; i.e., $\rho_v = \Lambda_0/8\pi G$, where Λ_0 is the cosmological constant. At the level of general relativistic theory, the Λ term is usually interpreted as a relativistic simple fluid with equation of state (EoS) $p_v = -\rho_v$ [3,4].

The so-called cosmic concordance model (Λ CDM), a flat cosmology with baryons and cold dark matter plus a relic Λ term, seems to be in agreement with all cosmological observations available. From the theoretical viewpoint, however, the well-known cosmological constant problem; i.e., the unsettled situation in the particle physics/cosmology interface in which the cosmological upper bound ($\rho_v \leq 10^{47}$ GeV⁴) differs from theoretical expectations ($\rho_v \sim 10^{71}$ GeV⁴) by more than 100 orders of magnitude, originates an extreme fine-tuning problem [5].

A natural attempt at alleviating the so-called cosmological constant problem is to allow a time dependence of Λ or equivalently, of the vacuum energy density. Historically, the idea of a time-varying $\Lambda(t)$ term was first advanced in the paper of Bronstein [6]. Different from Einstein's cosmological constant, such a possibility was somewhat missed in the literature for many decades, and, probably, it was not important to the recent development initiated by Ozer and Taha [7] in the late 1980s. After their papers, a

number of models with different decay laws for $\Lambda(t)$ were proposed by many authors, and their predictions were confronted with the available observational data [8–12]. It is worth mentioning that the most usual critique to these $\Lambda(t)$ CDM scenarios is that in order to establish a model and study their observational and theoretical predictions, one needs first to specify a phenomenological time dependence for Λ . However, there are some attempts to represent out-of-equilibrium dynamical Λ models by a scalar field [13,14], as well as some based on a Lagrangian description [15].

Besides the evolution of the Universe, an important process in gravitational physics that can also be affected by a dynamical $\Lambda(t)$ term is the formation of black holes. A basic difference here is that the interacting vacuum energy density decreases in the expanding Universe, whereas during black hole formation (a collapsing process), it grows over the course of time. Therefore, it is natural to ask about the influence of a growing vacuum energy density during the gravitational collapsing process of a star. In principle, since a time-varying $\Lambda(t)$ exerts an increasing repulsive force on its surrounding medium, it might also prevent the ultimate formation of a spacetime singularity.

Another closely related issue is the possible influence of $\Lambda(t)$ on the cosmic censorship hypothesis (CCH), as well as on the nature of the singularity. In its weak form, this conjecture eliminates the occurrence of naked singularities in the spherical gravitational collapse, whereas its strong version states that all singularities in any realistic spacetime are never visible to a distant observer because they are hidden behind an event horizon [16]. Since the earlier counterexample to the CCH discussed by Papapetrou [17], the emergence of naked singularities or black holes has been intensively investigated in the literature, including the effect of different material components [18]. However, as far as we know, the possible influence of a time-varying Λ term in the last stages of a collapsing

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system (including the formation of a trapped surface and naked singularities) has not been analyzed in the literature. In principle, this is an important issue due to two combined effects: (i) unlike what happens in an expanding Universe, the energy density of a coupled vacuum component grows in the course of the gravitational contraction, and (ii) since the vacuum pressure is negative and generates repulsive gravity, potentially, it might alter significantly the late stages of any collapsing matter distribution.

In this paper, we discuss the formation of black holes (and naked singularities) during the gravitational collapse of a fluid interacting with a time-varying vacuum energy density. For given initial conditions, the equations describing the evolution of the two-fluid interacting mixture are analytically solved, but in order to study the different roles played by the matter EoS during the collapse of the core, special attention is dedicated to the dust and radiation cases. The formation of the black holes here is simply identified with the development of apparent horizons before the formation of the singularity. As we shall see, due to its repulsive gravitation, a time-varying vacuum energy density as modeled here increases the collapsing time, but under certain conditions it cannot prevent the formation of black holes. Our results also suggest that the CCH conjecture (at least in its weak form) is generically violated in the presence of a time-varying vacuum due to the formation of naked singularities.

II. COLLAPSING SPHERICAL STAR WITH A GROWING VACUUM COMPONENT

A. Star medium: Composition and geometry

Let us now consider the gravitational collapse of a spherically symmetric massive core of a star with finite thickness. The massive core medium is formed by a two-fluid interacting mixture: a material component plus a growing vacuum energy density represented by a $\Lambda(t)$ term.

To begin with, let us divide the spacetime into three different regions, Σ and V^\pm , where Σ denotes the surface of the star, and V^- (V^+) the interior (exterior) of the massive core. For the sake of simplicity, we also assume that the spacetime inside the massive core is homogeneous and isotropic, a particular case of the inhomogeneous Oppenheimer-Snyder model [19]. This means that the spacetime inside the core is described by the homogeneous and isotropic flat Friedmann-Robertson-Walker (FRW) geometry:

$$ds_-^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2), \quad (1)$$

where $a(t)$ is the scale factor and $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$ is the area element on the unit sphere. Although this is a very ideal case, we do believe that it captures the main features of gravitational collapse in the presence of a growing vacuum energy density. We recall that for expanding Universe models the curvature effects are not important at the early stages of evolution [20]. Similarly, it will be

assumed here that the same happens for the late stages of the collapsing core.

Following standard lines, in this paper we shall focus our attention mainly on the spacetime inside the star [21]. If the collapsing massive core finally forms a black hole (BH), an apparent horizon must develop inside it, and, therefore, there exists a moment at which the whole core collapses inside the apparent horizon.

The apparent horizon is a trapped surface lying in a boundary of a particular surface S , and can differ from the intersection of the event horizon with the surface S , where the event horizon is the boundary of the region S that is not possible to escape to infinity [22]. Using more technical language, consider a two-sphere S embedded in a slice Σ of spacetime M , and let s^μ be the outward-pointing spacelike unit normal to Σ , and n^μ the future-pointing timelike unity normal to Σ . Hence, the vector $\kappa^\mu = s^\mu + n^\mu$ is a null vector, and S is a marginally trapped surface if $\kappa^\mu_{;\mu} = 0$ holds everywhere on S [23].

However, considering that the matching conditions and the spacetime outside the star must affect the total mass and the global structure of the black hole, the key aspect of black hole formation is to know whether apparent horizons develop inside the core. In other words, the ultimate formation of the black hole singularity does not depend on either the matching or the choice of the spacetime outside the star. Here we are mainly interested in discussing under which conditions a BH is formed during the collapse of dust and radiation fluids when such components are interacting with a growing vacuum energy density.

Let us first consider the Einstein field equations within the star:

$$G^{\mu\nu} = \chi \left[T^{\mu\nu} + \frac{\Lambda}{\chi} g^{\mu\nu} \right], \quad (2)$$

where $G^{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu}$ is the energy-momentum tensor, and $\chi = 8\pi G$ ($c = 1$) is Einstein's constant. According to Bianchi identities, the above equations imply that Λ is constant only if $T^{\mu\nu} \equiv 0$ or separately conserved; i.e., $T^{\mu\nu}_{;\nu} = 0$. This means that a time-varying vacuum is possible only by assuming the previous existence of some sort of nonvanishing interacting fluid which is changing energy with the vacuum component.

Before proceeding further, it should be stressed that the basic discussion here is related to black holes and naked singularities formed from collapsing star cores. In particular, this means that supermassive black holes like the ones found in galactic centers are not part of our investigation, and the same applies to the recently discovered quasar at redshift $z = 7.085$ and estimated mass $M = 2 \cdot 10^9 M_\odot$ [24]. When such an object was formed, the Universe had existed for less than 10^9 years after the big bang. Therefore, it will be assumed here that such supermassive structures were seeded by massive collapsing star cores of population III, but their extremely large masses are the result of

cosmological accretion mechanisms and mergers in the course of their evolution.

B. Basic equations and solutions

In what follows, it will also be explicitly assumed that the vacuum and fluid components are coupled and that $T^{\mu\nu}$ has the form of a perfect fluid:

$$T^{\mu\nu} = (\rho_f + p_f)u^\mu u^\nu - p_f g^{\mu\nu}, \quad (3)$$

where ρ_f , p_f and u^μ , are the energy density, the pressure and the four-velocity of the fluid component, respectively.

In this case, by taking the divergence of Eq. (2) and projecting the result in the direction of the four-velocity, one finds

$$u_\mu T^{\mu\nu}{}_{;\nu} = -u_\mu \left(\frac{\Lambda g^{\mu\nu}}{\chi} \right)_{;\nu}. \quad (4)$$

In the background [Eq. (1)], the above energy conservation law for the comoving observer reads

$$\dot{\rho}_f + 3\frac{\dot{a}}{a}(\rho_f + p_f) = -\dot{\rho}_v, \quad (5)$$

where a dot indicates the time derivative and $\rho_v = \Lambda(t)/8\pi G$ is the time-dependent vacuum energy density.

As in the standard case [without a $\Lambda(t)$ term], the energy conservation law [Eq. (5)] is also contained in the Einstein field equations:

$$8\pi G\rho_f + \Lambda(t) = 3H^2, \quad (6)$$

$$8\pi Gp_f - \Lambda(t) = -2\dot{H} - 3H^2, \quad (7)$$

where $H = \dot{a}/a < 0$ is the ‘‘Hubble function’’ for the collapsing mixture.

In order to solve the above equations, we need to specify the fluid EoS and the time-varying vacuum energy density. For the sake of definiteness, it will be assumed that the matter component of the star satisfies a barotropic EoS

$$p_f = \omega\rho_f, \quad (8)$$

where $0 \leq \omega \leq 1$ is a positive parameter (here we are not particularly interested in the case of a pure or interacting dark energy fluid). Note that if the collapsing fluid is itself a mixture (for instance, matter and radiation), we are implicitly assuming that the variable vacuum is interacting only with the dominant fluid component, and that such a mixture determines the overall evolution of the collapsing medium.

Many phenomenological functional forms have been proposed in the literature for describing a time-varying $\Lambda(t)$. Based on dimensional arguments, Carvalho *et al.* [9] have shown that a natural dependence is $\Lambda \propto H^2$. Later on, this functional dependence was derived within a renormalization group approach (including a bare cosmological constant Λ_0) by Shapiro and Solà [25].

Following these authors, we consider here that the $\Lambda(t)$ term is given by Refs. [9,25]:

$$\Lambda = \Lambda_0 + 3\beta H^2, \quad (9)$$

where β is a dimensionless constant parameter and the factor 3 was added for mathematical convenience.

Using Eqs. (8) and (9), a simple manipulation shows that the scale factor satisfies

$$\frac{\ddot{a}}{a} + \left[\frac{3}{2}(1+\omega)(1-\beta) - 1 \right] \frac{\dot{a}^2}{a^2} - (1+\omega)\frac{\Lambda_0}{2} = 0. \quad (10)$$

Now, by integrating the above equation, we obtain

$$a(t) = a_i \left[\left(\frac{1 - \tilde{\Omega}_{\Lambda_{0i}}}{\tilde{\Omega}_{\Lambda_{0i}}} \right)^{1/2} \sinh \xi \sqrt{\tilde{\Omega}_{\Lambda_{0i}}} (t_c - t) \right]^{2/3(1+\omega)(1-\beta)}, \quad (11)$$

where $\xi = \frac{3}{2}(1-\beta)(1+\omega)H_i$; $\tilde{\Omega}_{\Lambda_{0i}} = \Omega_{\Lambda_{0i}}/(1-\beta)$; and the positive quantities a_i , H_i define the initial conditions of the collapsing core [$a(0) = a_i$, $H(0) = -H_i$]. We have also defined the initial vacuum (bare) density parameter by $\Omega_{\Lambda_{0i}} = \Lambda_0/3H_i^2$; whereas the collapsing time, t_c , is fully determined by the initial condition $a(0) = a_i$ and, as such, it depends only on the physically meaningful parameters (H_i , β , ω and $\Omega_{\Lambda_{0i}}$).

The main aim here is to understand the influence of the growing vacuum energy density on the final stages of the collapsing process. In this way, one may conclude that the contribution of the bare cosmological constant Λ_0 becomes rapidly negligible in comparison to the variable $\Lambda(t)$ term ($\propto H^2$). Therefore, even considering that the problem can be analytically solved in its full generality, we now focus our attention on the behavior of the solutions derived by taking the limit $\Omega_{\Lambda_{0i}} \rightarrow 0$. In this case, the scale factor given by Eq. (11) reduces to

$$a(t) = a_i \left[\frac{3}{2}(1+\omega)(1-\beta)H_i(t_c - t) \right]^{2/3(1+\omega)(1-\beta)}. \quad (12)$$

It is worth mentioning that apart from the physical choice of constants, the above solution for $\beta = 0$ reduces to the one derived by Cai and Wang [21] in their study of a collapsing one-fluid component. From Eq. (12), we also see that the collapsing time is given by

$$t_c = \frac{2H_i^{-1}}{3(1+\omega)(1-\beta)}. \quad (13)$$

As it appears, the modulus of the initial collapsing Hubble function, H_i , sets the time scale to reach the singularity. Note that in the limiting case $\beta \rightarrow 1$, the collapsing time $t_c \rightarrow \infty$ and the model is nonsingular (pure de Sitter vacuum). From now on, it will be assumed that $0 \leq \beta < 1$.

The following equivalent forms for Eq. (12) are also useful:

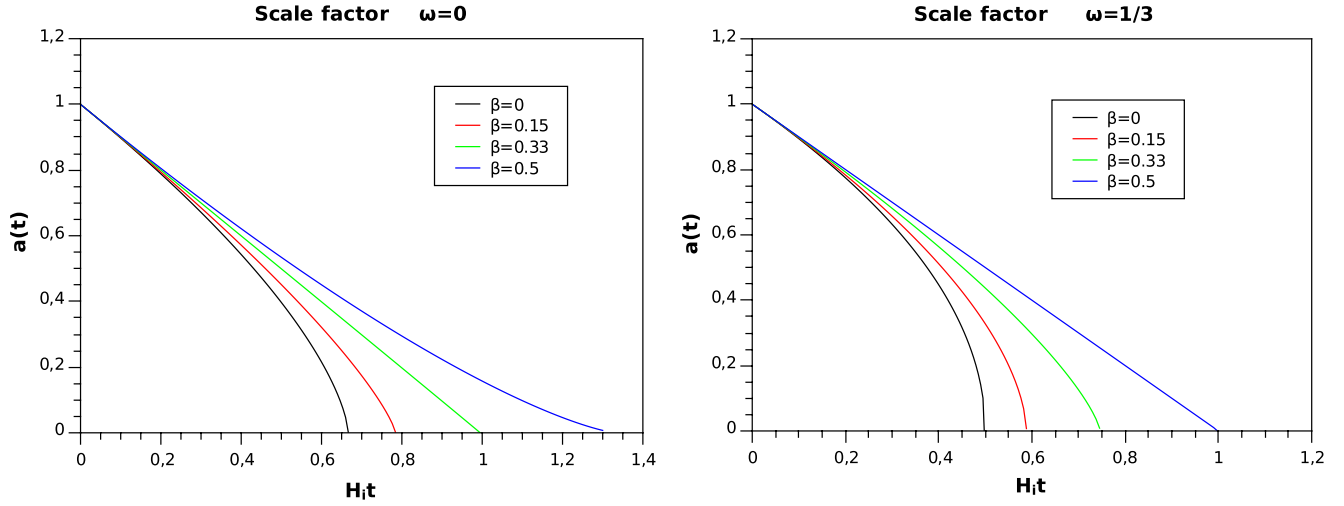


FIG. 1 (color online). Evolution of the scale factor for the collapsing mixture. The right and left panels correspond to dust ($\omega = 0$) and radiation ($\omega = 1/3$) coupled to a growing vacuum component, respectively. The overall effect of the vacuum component is to increase the time of collapse with respect to a pure fluid medium. However, the total collapse is not avoided for the selected values of β depicted in the panels. Similarly, the same happens for any value of $\beta < 1$ [cf. Eq. (12) and comments in the body of the text].

$$a(t) = a_i \left(1 - \frac{t}{t_c}\right)^{2/3(1+\omega)(1-\beta)} \quad (14)$$

and

$$a(t) = a_i \left[1 - \frac{3}{2}(1+\omega)(1-\beta)H_i t\right]^{2/3(1+\omega)(1-\beta)}. \quad (15)$$

Note also that the condition $\dot{a}(t) < 0$ characterizing the collapse process has been taken into account in the present formulation, since the collapsing Hubble function reads

$$H(t) = \frac{-H_i}{1 - \frac{3}{2}(1+\omega)(1-\beta)H_i t}, \quad (16)$$

so that $H(0) = -H_i$ as should be expected [see comment below Eq. (11)].

Without loss of generality, from now on we consider the initial scale factor $a_i = 1$. By neglecting Λ_0 , Eq. (10) describing the acceleration reads

$$\frac{\ddot{a}}{a} = \left[1 - \frac{3}{2}(1+\omega)(1-\beta)\right] \frac{\dot{a}^2}{a^2}. \quad (17)$$

We see that in the limiting case $\beta = 1/3$ and $\omega = 0$ (dust case), we have $\ddot{a} = 0$; whereas for $\beta > 1/3$ the gravitational collapsing process happens in an accelerating way ($\ddot{a} > 0$). On the other hand, considering $\beta = 1/2$

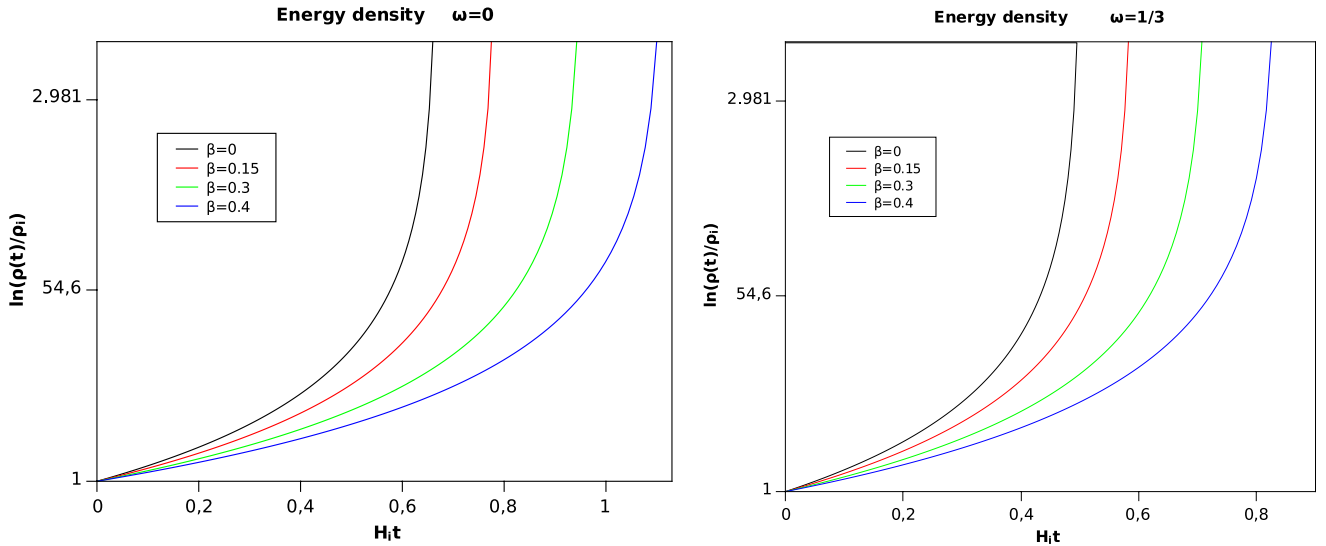


FIG. 2 (color online). Evolution of the total energy density. In the left panel we display the total density for a mixture of dust ($\omega = 0$) plus a growing vacuum component as a function of the dimensionless time. In the right panel we show the same plot, but now for a radiation fluid ($\omega = 1/3$) plus vacuum. For both cases, the evolution is heavily dependent on the value of the β parameter.

and $\omega = 1/3$ (radiation case), we have an identical behavior. This is the first vacuum effect. Only for $\beta = 0$ is the standard result recovered (see Ref. [21]).

In Fig. 1, we display the behavior of the scale factor as a function of the dimensionless time, $T = H_i t$, and some selected values of the vacuum β parameter. Two different scenarios are considered: (i) a dust-filled core ($\omega = 0$) coupled to a growing vacuum component (left), and (ii) radiation ($\omega = 1/3$) plus a growing vacuum (right). Note that for fixed values of H_i and ω , the collapsing time grows for greater values of β . This is the second vacuum effect.

As mentioned above, in the case of dust ($\omega = 0$), the evolution of the scale factor is altered when $\beta = 1/3$ (see green line in the left panel of Fig. 1). Similarly, for $\omega = 1/3$, the scale factor modifies its evolution at $\beta = 1/2$ (blue line in the right panel of Fig. 1).

To close this section of exact results, we write below the energy density for the vacuum and fluid components:

$$\rho_v(t) = \frac{3\beta\rho_i}{\left[1 - \frac{3}{2}(1+\omega)(1-\beta)H_i t\right]^2}, \quad (18)$$

$$\rho_f(t) = \frac{3(1-\beta)\rho_i}{\left[1 - \frac{3}{2}(1+\omega)(1-\beta)H_i t\right]^2}, \quad (19)$$

where $\rho_i = H_i^2/8\pi G$.

In Fig. 2, we show the time behavior for the total energy density. For all values of $\beta < 1$, we see that the energy density diverges at the collapse time (t_c), which is strongly dependent on the value of β . As should be expected, for a given value of β , the collapsing time for a coupled radiation component ($\omega = 1/3$) is also reduced in comparison to the dust case. In a more realistic treatment, a transition from radiation ($\omega = 1/3$) to the limit case described by Zeldovich's stiff-matter medium ($\omega = 1$) may occur at the late stages. Naturally, such a final state is also included in the general solutions for $a(t)$ and $\rho(t)$, with similar plots appearing in Figs. 1 and 2.

III. APPARENT HORIZON AND COLLAPSE

In the FRW spacetime, the observers describing the behavior of the matter fields are comoving with the fluid

volume elements. This means that we can define a constant geometrical radius for the surface dividing the star interior from the exterior, namely r_Σ . For such a surface, the metric can be written as

$$ds_\Sigma^2 = d\tau^2 - R(\tau)^2 d\Omega^2, \quad (20)$$

where $t = \tau$ and $R(t) = r_\Sigma a(\tau)$.

The decision about the final stage of the collapse process is closely related to the emerging apparent horizon which must be formed before the collapsing time ($t = t_c$); that is, when the real singular point is attained. Apparent horizons are spacelike surfaces with future point converging null geodesics on both sides of the surface [26]. For an initially untrapped star, when the apparent horizon appears before the singularity, one may say that a BH is formed; otherwise, a naked singularity is the final stage of the collapse.

The formation of the apparent horizon is driven by the condition [21,27,28]

$$R_{,\alpha} R_{,\beta} g^{\alpha\beta} = (r\dot{a})^2 - 1 = 0, \quad (21)$$

where $()_{,x} = \frac{\partial}{\partial x}$ and $R(t, r) = ra(t)$.

In this work, we assume that the star is initially not trapped, and, as such, the comoving surface is spacelike. Hence

$$R_{,\alpha} R_{,\beta} g^{\alpha\beta} = [r_\Sigma \dot{a}(t_i)]^2 - 1 < 0, \quad (22)$$

which implies that $0 < R_i H_i < 1$. Such a domain for the product $R_i H_i$ will be useful when we discuss the criteria for BH formation below. Likewise, another important physical quantity is the mass function that furnishes the total mass inside the surface with radius r at time t . Originally, Cahill and McVittie [28] wrote such a function for a particular reference system that here takes the form [21]

$$m(t, r) = \frac{1}{2} R(1 + R_{,\alpha} R_{,\beta} g^{\alpha\beta}) = \frac{1}{2} R \dot{R}^2, \quad (23)$$

that appears in the literature more frequently [29].

In our study, the condition to bring into being the apparent horizon assumes the form

$$\dot{R} = R_i H_i \left[1 - \frac{3}{2}(1+\omega)(1-\beta)H_i t_{AH} \right]^{2-3(1+\omega)(1-\beta)/3(1+\omega)(1-\beta)} = 1, \quad (24)$$

where t_{AH} is the time marking the apparent horizon formation,

$$t_{AH} = \frac{2H_i^{-1}}{3(1+\omega)(1-\beta)} \times \left[1 - (R_i H_i)^{3(1+\omega)(1-\beta)/3(1+\omega)(1-\beta)-2} \right], \quad (25)$$

or equivalently [see Eq. (13)],

$$\frac{t_{AH}}{t_c} = \left[1 - (R_i H_i)^{3(1+\omega)(1-\beta)/3(1+\omega)(1-\beta)-2} \right]. \quad (26)$$

In particular, for $\omega = \beta = 0$, $H_i \sim 30\text{sec}^{-1}$, and $H_i R_i \sim 1/2$, we find that $t_{AH} \sim 2 \times 10^{-2}$ sec, while for $\omega = 0$, $\beta = 0.2$, we obtain $t_{AH} \sim 3 \times 10^{-2}$ sec. The formation time of the apparent horizon is strongly correlated with β .

In Fig. 3, we display the behavior of the dimensionless ratio t_{AH}/t_c as a function of the β parameter for some selected values of the product $R_i H_i$. Since the product $R_i H_i$ is positive and smaller than unity [see the discussion below Eq. (22)], as long as the quotient in the exponent of Eq. (26) is greater than zero, the formation of the apparent horizon will occur before the collapsing time t_c ($t_{AH} < t_c$). As one may check, this condition is defined by

$$\beta < 1 - \frac{2}{3(1+w)}. \quad (27)$$

In the present context, the above relation describes a kind of compromise between the dynamics of the collapsing system and the formation of the apparent horizon. Naturally, when such a condition is violated within the accelerating collapsing mixture (fluid plus vacuum), the formation of the apparent horizon will be avoided, thereby giving rise to a pure naked singularity.

At this point, one may ask about the specific signatures of black holes and naked singularities. In other words, how does one observationally discriminate between these two singular structures? As far as we know, there is no experimental suggestion aiming to distinguish them based uniquely on the physics of the collapsing process. It is also not clear whether the very energetic events named gamma-ray bursts are somewhat related to the formation of spacetime singularities. Some aspects of gravitational collapse and spacetime singularities containing a discussion about such a possibility were recently published by Joshi and Malafarina [30]. Nevertheless, some authors have claimed that observations involving strong gravitational

lensing [31,32] and accretion disks [33] are able to discriminate black holes from naked singularities.

In the strong lensing regime, for instance, it was found that the number of relativistic images and Einstein rings formed in the case of naked singularities are more separated from each other than in the case of black holes [32]. More recently, Kovács and Harko [33] also argued that the thermodynamic and electromagnetic features of accretion disks are different for these two classes of objects, thereby giving a clear-cut signature that could distinguish such spacetime singularities. In particular, they show that the conversion efficiency of the accretion mass into radiation in the case of rotating naked singularities is always higher than that of black holes (see their Table I).

On the other hand, there are some useful constraints on the β parameter given by observations coming from the cosmic dark sector. For instance, Birkel and Sarkar [34] derived the upper limit $\beta < 0.13$ by using big bang nucleosynthesis in the presence of a decaying vacuum. Later on, Lima *et al.* [35] rediscussed this bound inferred from big bang nucleosynthesis, thereby obtaining $\beta \leq 0.16$. It is also known that a possible effect related to an adiabatic decaying vacuum into thermalized photons is to alter the redshift temperature law of the CMB to $T = T_0(1+z)^{1-\beta}$ [36]. In this case, several authors have constrained β by using different cosmological probes [37]. Such results were obtained by assuming that the decaying vacuum is coupled only with photons, but that other decay channels are not forbidden from first principles. For instance, in the context of interacting dark energy models, it may decay into dark matter ($\omega = 0$). In this case, Basilakos [38] obtained $\beta \sim 0.004$ (γ in his notation) for a coupling with dust. All the results mentioned above are in agreement

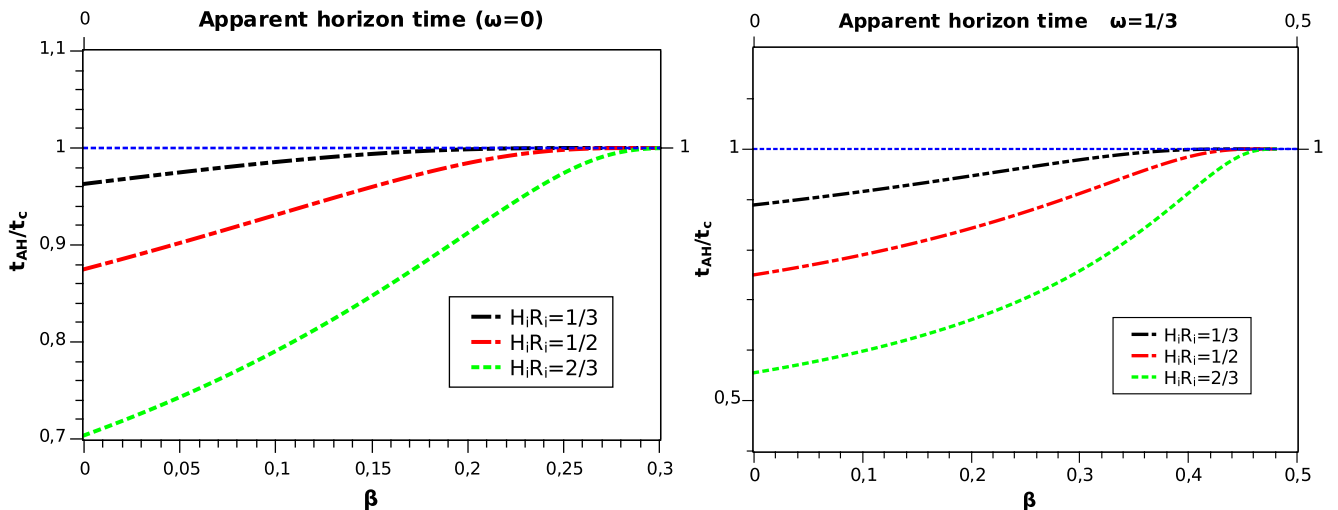


FIG. 3 (color online). The dimensionless ratio t_{AH}/t_c as a function of the β parameter [see Eq. (26)]. As in the previous figure, we display the behavior for dust (left panel) and radiation (right panel) cases. For a fixed initial condition ($H_i R_i$), the formation of the horizon is delayed for higher values of β as long as the condition in Eq. (27) is satisfied. When Eq. (27) becomes an equality; that is, for $\beta = 1/3$ (dust) and $1/2$ (radiation), we see that $t_{AH} = t_c$ and the corresponding effective mass goes to zero (see Fig. 4). For greater values of β , the effective mass becomes negative, and the corresponding singularities are naked.

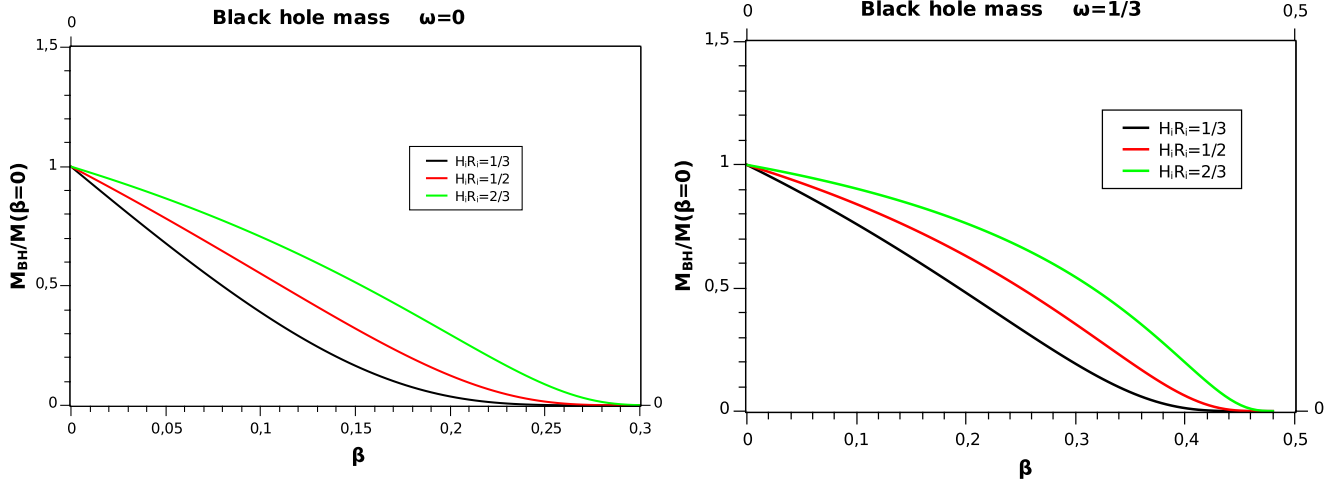


FIG. 4 (color online). The ratio between the mass of the BH formed and the mass of a pure fluid versus the β parameter. As in the other plots, we also consider separately the dust case plus the interacting vacuum (left panel), and the radiation case plus vacuum (right panel). For a fixed value of the product $H_i R_i$, we see that the BH mass not only is heavily dependent on the β parameter, but also decreases for higher values of β satisfying the constraint given by Eq. (27). All these results are based on the Cahill and McVittie mass definition [28].

with our condition in Eq. (27); however, higher values of β for collapsing systems are not forbidden from first principles.

Now, in order to complete our description, let us discuss the collapsed mass inside the radius r at time t [see Eq. (23)]. It can be written as

$$m(r, t) = \frac{1}{2} R_i^3 H_i^2 \left\{ 1 - \frac{3}{2} (1 + \omega)(1 - \beta) H_i t \right\}^{2[\beta(1+\omega) - \omega]/(1+\omega)(1-\beta)}. \quad (28)$$

This quantity yields the total mass of the collapsing star at time τ within the surface r_Σ ; that is, $M(\tau) = m(r_\Sigma, \tau)$. With the help of Eqs. (25) and (28), we find the expression for the total collapsed mass inside the apparent horizon, namely

$$M(\tau_{AH}) = \frac{1}{2} R_i^3 H_i^2 (R_i H_i)^{3(1+\omega)(1-\beta) - 3/1 - \frac{3}{2}(1+\omega)(1-\beta)}, \quad (29)$$

which reduces to $M(\tau_{AH}) = \frac{1}{2} R_i^3 H_i^2$ in the case of a pure dust fluid ($\omega = \beta = 0$). In physical units, the above mass can be rewritten as

$$M(\tau_{AH}) = \frac{R_i M_\odot}{r_{S\odot}} (R_i H_i)^{2/3(1+\omega)(1-\beta) - 2}, \quad (30)$$

where M_\odot and $r_{S\odot}$ are the solar mass and the Schwarzschild radius of the Sun, respectively. In the case of a pure radiation fluid ($\omega = 1/3$, $\beta = 0$), assuming $R_i = 10$ km and $R_i H_i \sim 1/2$, we find that $M \sim 13.3 M_\odot$, while for $\beta = 0.25$ and the remaining quantities as given above, a smaller total mass is obtained, $M \sim 6.6 M_\odot$. As expected, the total mass depends explicitly on the

decaying vacuum β parameter, which is constrained by Eq. (27) when a BH is formed.

In Fig. 4, we show the decaying vacuum effect on the total BH mass. For each fluid component, we have plotted the ratio $M_{BH}/M(\beta=0)$ as a function of the β parameter. For fixed initial conditions, we see that the BH mass is indeed heavily dependent on the β parameter. Interestingly, when $\beta = 1 - 2/3(1 + \omega)$, the total mass of the BH goes to zero. For each kind of fluid (ω), this critical value of β is the lower limit of the decaying vacuum parameter, signaling the formation of a naked singularity. The plots for the formation of the apparent horizon (Fig. 3) and the mass ratio of the formed BHs suggest that the growing of the repulsive gravitational energy of the coupled vacuum alters considerably the collapse process. As remarked before, for all cases violating the condition given by Eq. (27), the collapsing time t_c remains finite, but $t_{AH} > t_c$ and, as such, one must conclude again that the spacetime singularity is naked.

IV. FINAL COMMENTS

In this paper we have studied the gravitational collapse of a spherically symmetric star with finite radius filled by a homogeneous and isotropic fluid obeying the EoS $p = \omega \rho$, plus an interacting growing vacuum energy density. We stress that all the curvature effects were neglected and that the dynamical $\Lambda(t)$ term was assumed to obey the relation [9,25] $\Lambda = \Lambda_0 + 3\beta H^2$.

In our approach, all physical constants appearing in the solutions were properly identified. After highlighting some of their consequences, we have focused our study on the influence of a growing vacuum energy density on the BH mass enclosed by the trapped surface and the formation of a

naked singularity. In the applications, only positive values of the EoS parameter (ω) were studied, with special attention to dust ($\omega = 0$) and radiation ($\omega = 1/3$). However, the general solutions hold even for negative values of the ω parameter. Therefore, we have described a two-fluid collapsing mixture whose nature is characterized by a two-parametric (ω, β) phenomenological approach.

It was found that the final stages of the gravitational collapse depend heavily on the values assumed by the β parameter. For any positive value of the ω parameter, the collapsing process is delayed by the vacuum energy component. However, the condition constraining the formation of the event horizon changes for distinct values of ω [see Eq. (27)]. In particular, for a dust fluid the condition is $\beta < \frac{1}{3}$, while for radiation the formation of a BH is allowed for $\beta < \frac{1}{2}$. When such conditions are not obeyed, a naked singularity is formed. The critical value $\beta = 1 - 2/3(1 + \omega)$ defining the boundary between black holes and naked singularities implies that the total mass is zero (see Fig. 4). It should be stressed, however, that such an effect is not related to any dynamical mass evaporation process, like Hawking radiation. It is closely related with the vacuum pressure and, generically, must appear when sufficiently large negative pressures take place in the matter content.

Naturally, all the results derived here are heavily dependent on the form assumed for $\Lambda(t)$, and even the avoidance of the singularity may occur whether or not a more realistic description of the decaying vacuum is adopted. Nonetheless, the results obtained here suggest that a growing vacuum energy density may lead at the late stages to naked singularities even when inhomogeneities are taken into account. The physical effects of a growing time-varying vacuum energy density on the inhomogeneous collapse will be discussed in a forthcoming communication.

Finally, it is also clear that the hypothesis of homogeneity and isotropy of the spacetime of the collapsing star is the main caveat of the proposed model, since the pressure should be made to vanish at the boundary, thereby obtaining a smooth matching with the Schwarzschild-de Sitter vacuum solution outside the star.

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