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Anisotropic Flow in Event-by-Event Ideal Hydrodynamic Simulations of $\sqrt{s_{NN}} = 200$ GeV Au + Au Collisions

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We simulate top-energy Au + Au collisions using ideal hydrodynamics in order to make the first comparison to the complete set of midrapidity flow measurements made by the PHENIX Collaboration. A simultaneous calculation of v_2, v_3, v_4, and the first event-by-event calculation of quadrangular flow defined with respect to the v_2 event plane ($v_4\{\Psi_2\}$) gives good agreement with measured values, including the dependence on both transverse momentum and centrality. This provides confirmation that the collision system is indeed well described as a quark-gluon plasma with an extremely small viscosity and that correlations are dominantly generated from collective effects. In addition, we present a prediction for v_5.

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Introduction.—Evidence suggests that in a collision between ultrarelativistic heavy nuclei, a strongly interacting low-viscosity fluid—the quark-gluon plasma—is created. The clearest indication of this behavior is seen in the azimuthal anisotropy [1] among the bulk of emitted particles. In theory, one characterizes this anisotropy in terms of a single-particle probability distribution for each collision event. By writing this distribution as a Fourier series with respect to the azimuthal angle of outgoing particles ϕ, one can define flow coefficients v_n and event plane angles Ψ_n

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi - \Psi_n), \quad (1)$$

$$v_n e^{in\Psi_n} \equiv \langle e^{in\phi}\rangle, \quad (2)$$

where the brackets indicate an average over the single-particle probability, and the event plane angles Ψ_n are chosen such that v_n are the (positive) magnitudes of the complex Fourier coefficients.

Experimentally, one measures the azimuthal dependence of event-averaged correlations between detected particles. These measurements indicate the presence of a very large "elliptic flow" coefficient v_2 [2,3], which typically can only be reproduced in calculations where the system is modeled as a strongly interacting fluid. In this picture, the large momentum anisotropy is generated as a hydrodynamic response to the spatial anisotropy of the nuclear overlap region in collisions of nonzero impact parameter. It even appears that the created quark-gluon plasma must be an almost perfect (zero viscosity) fluid, with a ratio of shear viscosity to entropy density η/s that is at most a few times $1/(4\pi)$, a value that was famously conjectured to be a universal lower bound [4]. (The bound is now known to be violated in some theories [5–7], and it may even be possible to have an arbitrarily small value [8], though the *effective* viscosity may still have a finite bound [9].) However, the extraction of a precise finite value is hampered by poor knowledge of the earliest stages of the collision as well as other uncertainties [10].

An important recent development was the realization of the importance of quantum fluctuations, which in particular implies an event-by-event breaking of the symmetry naively implied by the collision of identical nuclei. Specifically, the coefficients v_n are generally nonzero also for odd n [11], the event plane angles do not necessarily point in the same direction as the impact parameter [12,13], and these quantities fluctuate significantly from one event to another, even at a fixed impact parameter [14].

These insights led to the possibility that *all* the measured long-range correlations may be generated solely from collective behavior [11,15].

Several new flow observables—specifically ones implied by the presence of event-by-event fluctuations—were recently measured for the first time [16–20]. Studies of these new observables indicate that, individually, they indeed appear to have properties that are consistent with a hydrodynamic origin [15,21–23]. However, they have not yet all been reproduced in a single calculation within one model using a single set of parameters. This has left some lingering doubt about whether the interpretation in terms of collective behavior is indeed correct [24]. In addition, each measurement provides an independent constraint on theory, so identifying models and sets of parameters that can simultaneously satisfy all the constraints is a necessary first step in reducing various theoretical uncertainties.

In this Letter, we perform state-of-the-art ideal hydrodynamic calculations and compare the results to the first measurements [16] of these new observables at the Relativistic Heavy-Ion Collider (RHIC) as well as previous measurements by the same collaboration [25]. Other groups have presented calculations from some of these observables using event-by-event ideal [26–29] or viscous [30] hydrodynamics or transport models [31]. The present study encompasses simultaneously, for the first time, all the measured flow observables at midrapidity.

Observables.—All the experimental results considered here were obtained using the event-plane method [32]. With this method, one first identifies an event plane Ψ_n in each event using a specific detector at forward rapidity and then calculates the correlation of particles near midrapidity with this event plane, e.g.,

$$v_n\{\Psi_n\} \equiv \langle \cos n(\phi - \Psi_n)\rangle, \tag{3}$$

where the brackets indicate an average over particles in a large number of events. A rapidity gap with the event-plane detector suppresses nonflow correlations [15,33]. At RHIC, "triangular flow" $v_3\{\Psi_3\}$ and "quadrangular flow" $v_4\{\Psi_4\}$ were measured for the first time, as a function of the particle transverse momentum p_t in various centrality classes by the PHENIX collaboration [16] (preliminary data from STAR have now also been presented [20]).

Previously, a different quadrangular flow observable has been measured, defined with respect to Ψ_2 [25,34]. We use a different notation for this quantity to avoid confusion,

$$v_4\{\Psi_2\} \equiv \langle \cos 4(\phi - \Psi_2)\rangle. \tag{4}$$

v_n is analyzed using a large sample of events, and its value fluctuates from one event to the other. These fluctuations (which were not appreciated when the method was developed), combined with the use of a finite sample of particles in the analysis, cause the measured value to deviate from the event average of the theoretical coefficients defined in Eq. (1). Generally, $v_n\{\Psi_n\}$ lies between the mean value and the root-mean-square (rms) value of v_n. One can parametrize the resulting measurement as [35]

$$v_n\{\Psi_n\} \simeq \langle v_n^\alpha\rangle^{1/\alpha}, \tag{5}$$

where here the brackets indicate an average over events. The value of α depends on the event plane resolution $\mathrm{Res}\{\Psi_n\} \sim v_n\sqrt{N}$ [36]: If the resolution is poor, $\alpha \simeq 2$, and the measured v_n is a rms value, while if the resolution is large, $\alpha \simeq 1$, and the result gets closer to the mean value.

The most recent data from PHENIX have a maximum event plane resolution of 0.74 (for v_2 around 30% centrality [25]) and much smaller value for v_3 and v_4 [16], which implies $\alpha > 1.81$ [36]. So in general, the results are very close to a rms value of v_n. Nevertheless, in the following we compute both limiting cases $\alpha = 2$ and $\alpha = 1$ in order to show the size of the effect of fluctuations on event-plane analyses.

Likewise, the measured value $v_4\{\Psi_2\}$ depends on the resolution [37] and is usually close to $\langle v_4 v_2^2 \cos(4\Psi_4 - 4\Psi_2)\rangle/\sqrt{\langle v_2^4\rangle}$ but with increasing resolution approaches $\langle v_4 \cos(4\Psi_4 - 4\Psi_2)\rangle$.

Results.—Using the hydrodynamic code NeXSPheRIO [38], we simulate top-energy Au-Au collisions at RHIC. This code solves the equations of ideal relativistic hydrodynamics using fluctuating initial conditions from the event generator NeXus [39].

NeXus aims at a realistic and consistent approach of the initial stage of nuclear collisions [39]. It is a Monte Carlo generator which takes into account not only the fluctuations of nucleon positions within nuclei [30] but also fluctuations at the partonic level: the momentum of each nucleon is shared between one or several "participants" and a "remnant", which implies nontrivial dynamical fluctuations in each nucleon-nucleon collision. The resulting full energy-momentum tensor is matched to a hydrodynamic form, resulting in a fluctuating flow field in addition to a fluctuating initial energy density, in all three spatial dimensions, with the transverse length scale of the fluctuations set mostly by the size of the incoming nucleons.

At the end of the hydrodynamic evolution, discrete particles are emitted using a Monte Carlo generator. (Freeze-out occurs at a constant temperature. Hadrons do not rescatter after freeze-out [40–42], but resonance decays are implemented.) NeXSPheRIO provides a good description of rapidity and transverse momentum spectra [43], elliptic flow v_2 [44], and the rapidity-even v_1 observable directed flow at midrapidity [45]. In addition, it is known to reproduce the long-range structures observed in two-particle correlations [46]. All parameters were fixed from these earlier investigations, before any of the new observables (v_3, v_4) were measured—nothing has been tuned here.

For this work, we generated 110 NeXus events each in 5% centrality classes up to 60% centrality, solving the hydrodynamic equations independently for each event. As in Ref. [47], at the end of each hydro event, we run the Monte Carlo generator many times, so that we can do the flow analysis using approximately 6×10^5 particles per event. This significantly reduces statistical noise and allows for an accurate determination of v_n and Ψ_n in every event. It also suppresses nonflow correlations from, e.g., particle decays. These quantities are then calculated by Eq. (2), with the average taken over all particles in the pseudorapidity interval $-1 < \eta < 1$. The procedure used to measure v_n in hydrodynamics thus mimics the experimental procedure, with two differences: (i) there is no need for a rapidity gap, because nonflow correlations are negligible; (ii) there is no need for a resolution correction, because the huge multiplicity per event ensures that the resolution is close to 1 for all events [47].

Figure 1 displays v_n as a function of the particle transverse momentum p_t, averaged over events in a centrality class. The average over events is estimated in two different ways to illustrate the effect of event-by-event flow fluctuations on the experimental analysis. The first estimate, labeled NeXSPheRIO −, is a plain mean value [corresponding to $\alpha = 1$ in Eq. (5)]. The second estimate, labeled NeXSPheRIO+, is a weighted average

$$v_n^+\{\Psi_n\} \equiv \frac{\langle v_n \cos n(\phi - \Psi_n)\rangle}{\sqrt{\langle v_n^2\rangle}}. \tag{6}$$

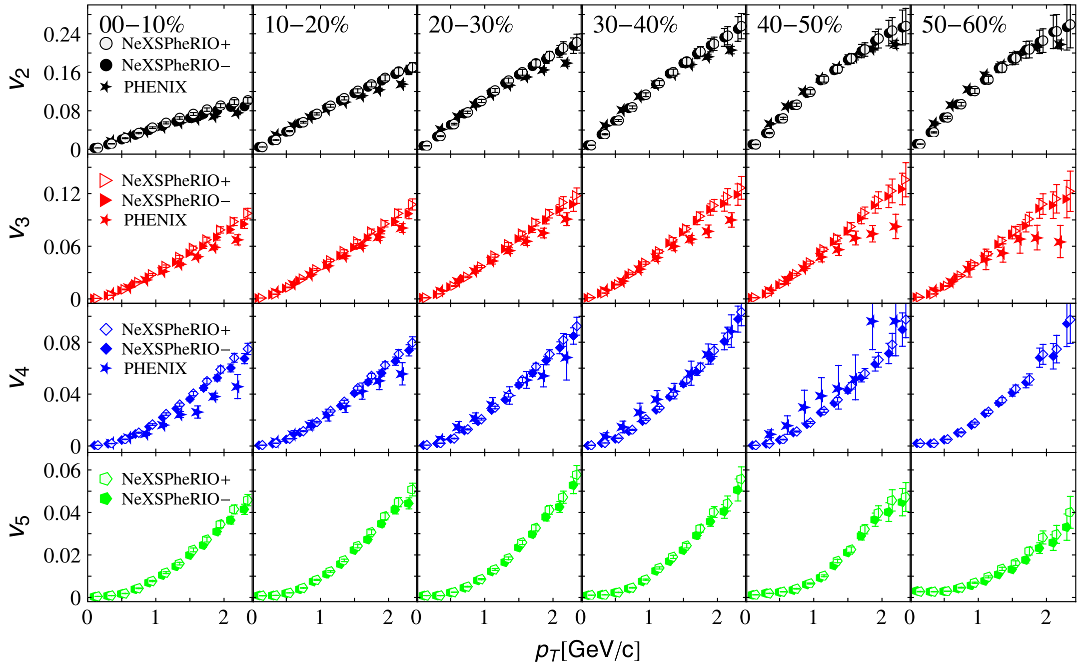

FIG. 1 (color online). Results for $v_n\{\Psi_n\}$ for $n = 2$–5, compared to published data from the PHENIX collaboration [16]. Closed and open symbols correspond to two different ways of averaging over events (mean and rms value, respectively). Error bars represent statistical uncertainty from the finite number of events. The left column (0–10%) represents the 10% most central collisions, with each column to the right increasingly peripheral.

The average of $v_n^+\{\Psi_n\}$ over p_t is the rms average of v_n [$\alpha = 2$ in Eq. (5)]. For Gaussian flow fluctuations [48], the ratio of the rms to the mean is $\sqrt{4/\pi} \simeq 1.13$ for v_3 and v_5 and closer to 1 for v_2 and v_4.

Figure 1 shows that our event-by-event ideal hydrodynamic calculation reproduces well the observed centrality and transverse momentum dependence of v_2, v_3, and v_4. The p_t dependence is a generic feature of ideal hydrodynamics [21]. The magnitude and centrality dependence of v_n, on the other hand, depend on the initial conditions: v_2 is mostly driven by the almond shape of the overlap area, which depends on the particular model used [49], while higher harmonics are mostly driven by initial fluctuations [11], which explains why they have a mild centrality dependence [50].

Originally, quadrangular flow v_4 had been measured with respect to the event plane of elliptic flow. Recent results show that $v_4\{\Psi_2\}$ [25] is smaller than $v_4\{\Psi_4\}$ [16], typically by a factor of 2 for peripheral collisions and by a factor of 5 for central collisions. Although $v_4\{\Psi_2\}$ is smaller, it is measured with a better relative accuracy than $v_4\{\Psi_4\}$, because of the better resolution on Ψ_2. This makes $v_4\{\Psi_2\}$ a useful quantity for detailed model comparisons [51]. As in the case of $v_n\{\Psi_n\}$, we perform the average over events in two different ways in order to illustrate the effect of event-by-event flow fluctuations. The first estimate, labeled NeXSPheRIO−, is a plain mean value, as in Eq. (4). The second estimate, labeled NeXSPheRIO+, is a weighted average

$$v_4^+\{\Psi_2\} \equiv \frac{\langle v_2^2 \cos 4(\phi - \Psi_2)\rangle}{\sqrt{\langle v_2^4\rangle}}. \tag{7}$$

The actual event-plane value is expected to lie between these two limits, depending on the resolution [37].

Since $v_4\{\Psi_2\}$ can be generated by elliptic flow as a second-order effect [52], we scale it by $v_2\{\Psi_2\}^2$ for each p_t. Hereafter, we denote $v_4\{\Psi_2\}$ and $v_2\{\Psi_2\}$ simply by v_4 and v_2. Figure 2 displays this first event-by-event hydrodynamic calculation of v_4/v_2^2 as a function of p_t for different centralities. The measured ratio is remarkably constant as a function of p_t and increases mildly for central collisions. Ideal hydrodynamics predicts $v_4/v_2^2 \simeq 1/2$ at high p_t for a single event [52]. For all centralities, the measured value of v_4/v_2^2 is greater than $1/2$, even at high p_t. This can be explained [37] by v_2 fluctuations, except for the two most central bins, where one expects $v_4/v_2^2 \simeq 1$ [37], smaller than the measured value, which is between 1.5 and 2 for the most central bin. For these two central bins, our results from event-by-event hydrodynamics are in good agreement with experiment (first two panels in Fig. 2). This shows that other sources of flow fluctuations, other than v_2 fluctuations alone, contribute to v_4/v_2^2. A similar finding has been reported in a transport calculation with v_4 and v_2 both defined with respect to the direction of the impact parameter [31]. Our calculated v_4/v_2^2 value is slightly higher than that of the data for the next two bins (10–20%). Above 20% centrality, data are within the range spanned by our calculations.

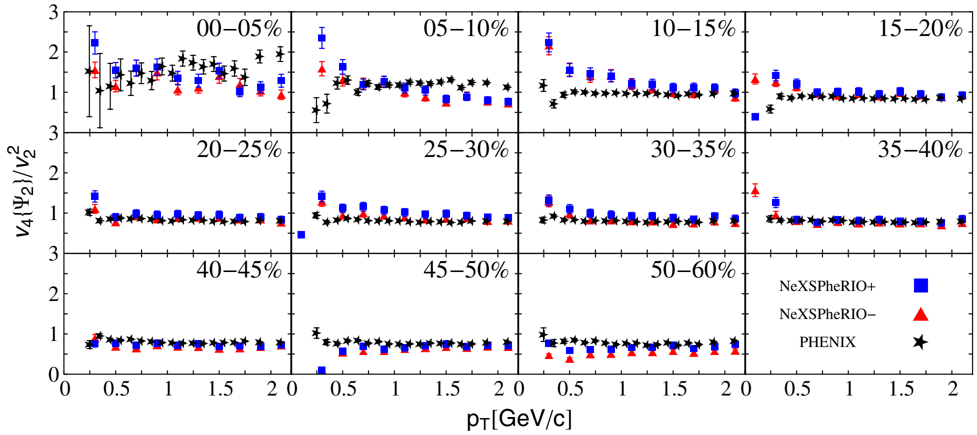

FIG. 2 (color online). Results for the first event-by-event hydrodynamic calculation of $v_4\{\Psi_2\}/v_2\{\Psi_2\}^2$, compared to published data from the PHENIX collaboration [25]. As in Fig. 1, closed and open symbols correspond to two different ways of averaging over events (see text), error bars represent statistical uncertainty from the finite number of events, and smaller percentile refers to more central collisions.

The calculations shown here simulate the system evolution using ideal hydrodynamics, i.e., with negligible viscosity. These results prove that no nonzero quark-gluon plasma viscosity is required to reproduce these data. In fact, this calculation *requires* a negligible viscosity—keeping everything else fixed, a viscosity the size of the conjectured bound $\eta/s = 1/4\pi$ would suppress v_n and destroy the remarkable fit to data. In addition, the ratio $v_4/v_2^2(p_T)$ depends strongly on η/s, and any nonzero value usually tends to destroy the flat curve that ideal hydrodynamics predicts [51,53].

However, this requirement of negligible viscosity depends crucially on aspects of the model which are not entirely constrained. In particular, although the NeXus model provides an honest effort at a reasonable description of the physics, with many realistic elements, there is considerable uncertainty about the early stages of a heavy-ion collision and the resulting initial conditions for hydrodynamic evolution. In principle, another model coupled to viscous hydrodynamics might well be able to fit these data. For example, Ref. [30] presents event-by-event viscous hydrodynamic calculations with Glauber initial conditions that require a value close to $\eta/s = 0.08$ to give reasonable agreement with the quantities in Fig. 1 at several centralities, though they underpredict v_3 for central collisions. Second, although v_4/v_2^2 is not very sensitive to the initial conditions, the effect of nonzero viscosity depends significantly on the way it is implemented at freeze-out [53], and the correct implementation is an open issue.

Thus, this work is only a first step in identifying models that are compatible with data, and strong conclusions cannot yet be drawn about, e.g., the precise value of η/s. Although the success of these calculations is an important milestone, proving that at this point no lower bound can yet be placed on η/s, we can not yet make a precise statement about an upper bound—only that it still appears unlikely that a value significantly larger than $1/4\pi$ will be possible.

Conclusions.—Using an ideal hydrodynamic model with fluctuating initial conditions, we have performed the first simultaneous calculation of $v_2\{\Psi_2\}$, $v_3\{\Psi_3\}$, $v_4\{\Psi_4\}$, and $v_4\{\Psi_2\}$ as a function of transverse momentum and centrality. Our results are in good agreement with the most recent experimental results for all the observables at RHIC, at all centralities and in a wide range of transverse momentum. This provides convincing confirmation of the current paradigm that collective effects alone can explain all long-range correlations in the soft sector. Further, since all such measured correlations are generated consistently in a single calculation, this provides a complete unified picture of the bulk evolution of a heavy-ion collision as an extremely low-viscosity fluid. Indeed, for our model of initial conditions, a negligible viscosity is required for a good fit to all midrapidity flow observables. Therefore, no lower bound can currently be placed on the shear viscosity of the quark-gluon plasma. Further study will be needed to determine a reliable upper bound, but finding models (such as this one) that are compatible with all measured data is a significant first step.

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