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Entropy, complexity and disequilibrium in compact stars

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ABSTRACT

We used the statistical measurements of information entropy, disequilibrium and complexity to infer a hierarchy of equations of state for two types of compact stars from the broad class of neutron stars, namely, with hadronic composition and with strange quark composition. Our results show that, since order costs energy, Nature would favor the exotic strange stars even though the question of how to form the strange stars cannot be answered within this approach.

1. Introduction

In the recent past, scientists from different areas have looked at information theory to characterize physical and biological systems, their patterns and correlations. The idea is that a statistical measure of *complexity* (to be defined precisely below) encodes the self-organization of a system, and links the information stored in it (or the logic/information entropy) to its "distance" to the state of equilibrium probability distribution [1]. Recently, Sañudo and Pacheco [2] first related such measures to an astrophysical object, a white dwarf star, while Chatzisavvas et al. [3] applied these same concepts to another type of compact stars, collectively known as "neutron stars", where matter is in the densest form known in Universe and is under even more extreme physical conditions, namely supra-nuclear densities.

The importance of performing information theory studies on compact astrophysical objects results from the fact that the very nature of the matter in such extreme physical conditions is still uncertain, and these studies can shed a new light on this subject from a different point of view. In this Letter, we address "neutron stars" made of nuclear hadronic matter and made of free quarks (the self-bound strange stars, modeled in the context of MIT Bag Model).

The extension of these information concepts to astrophysical macroscopic objects is not straightforward, since the forces involved in the respective equilibrium configurations are very different from that ones in an atomic system. In atomic systems, the factor that determines the self-organization is mainly the Coulomb interaction and the Fermi exclusion principle. On the other hand, in a (ordinary) neutron star, gravity, strong and weak interactions all contribute. Finally, in a quark star the nuclear structure and the nucleons themselves have been bypassed, and the truly fundamental degrees of freedom show up to form a self-gravitating ball which is nevertheless bound by strong interactions, not gravity, although the latter is still very important for the overall structure. It is an open question whether the information quantities can be used for a gross description of these equilibrium configurations.

Motivated by these considerations, we compared the information and complexity stored in these two "neutron" stars of different microscopic composition, and found that these quantities are comparable in general, but sensitive to the composition, because the latter determines the behavior of the *radii* of the stars for the same mass. Since the value of the radius is also an important feature for an observational identification [4,5], information theory may link the formation and structure aspects.

2. Calculations and models

We used the statistical measure of complexity as defined by López-Ruiz, Mancini and Calbet [1], as modified by Catalán et al. [6]:

$$C = H \times D,$$

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where $H = \exp(S)$ and S is the information entropy (or the information content of the system) in natural logarithmic units, D is the disequilibrium (identified with the distance of the system to its state of equiprobable probability distribution). In its original definition, the expressions for S and D are the following

$$S = -\int \rho(\mathbf{r}) \ln\left[\rho(\mathbf{r})\right] d\mathbf{r},$$
$$D = \int \rho^2(\mathbf{r})\, d\mathbf{r}.$$

The quantity $\rho(\mathbf{r})$ is the normalized probability distribution that describes the state of the system. S describes the uncertainty associated to that probability distribution while D stands for the information energy (as defined by Onicescu [7]), or the quadratic distance to the equiprobability.

In order to study our two types of compact objects in this way, we need an analogue to the probability distribution. Because the energy density distribution, $\epsilon(r)\,[\mathrm{erg/cm^3}]$, is related to the probability of finding a number of particles in a given location inside the star, we use the energy density profile as the quantity to enter in the integrals. However, in the case of the structure of our stars, the gravitation is non-Newtonian and we must solve the Tolman–Oppenheimer–Volkoff equations, or the equation of relativistic hydrostatic equilibrium of the star plus the mass integral, both complemented by the equation of state which describes the micro-physics (composition) of the stellar matter, to finally find $\epsilon(r) = c^2\rho(\mathbf{r})$, where $c = 3\times 10^{10}$ cm/s is the velocity of light and $\rho(r)$ $[\mathrm{g/cm^3}]$ is the matter density.

In this work we use the same approach of [3] to solve the TOV equation: first we define the barred quantities as the dimensionless variables scaling as follows

$$M(r) = \bar{M}(r)M_\odot \qquad \epsilon(r) = \bar{\epsilon}(r)\epsilon_0,$$
$$P(r) = \bar{P}(r)\epsilon_0 \quad \epsilon_0 = 1\ \mathrm{MeV/fm^3},$$

where $M(r)$ is the mass of the star in solar units ($M_\odot$), $P(r)$ is the pressure and ϵ_0 is an energy density scale, which in turn provide us the following form of the TOV equation and the mass equation

$$\frac{\bar{P}(r)}{dr} = -1.474\frac{\bar{M}(r)\bar{\epsilon}(r)}{r^2}\left(1+\frac{\bar{P}(r)}{\bar{\epsilon}(r)}\right)$$
$$\times\left(1+11.2\times 10^{-6}r^3\frac{\bar{P}(r)}{\bar{M}(r)}\right)\left(1-2.948\frac{\bar{M}(r)}{r}\right)^{-1},$$
$$\frac{\bar{M}(r)}{dr} = 11.2\times 10^{-6}r^2\bar{\epsilon}(r).$$

Thus, the integrals to be evaluated are

$$S = -b_0\int \bar{\epsilon}(\mathbf{r})\ln\left[\bar{\epsilon}(\mathbf{r})\right]d\mathbf{r}, \tag{1}$$
$$D = b_0\int \bar{\epsilon}^2(\mathbf{r})\,d\mathbf{r}, \tag{2}$$

where $\bar{\epsilon}$ is the dimensionless energy density (which is just $c^2\rho/\epsilon_0$) and obtained from the solution of the TOV equation. The parameter $b_0 = 8.89\times 10^{-7}\ \mathrm{km^{-3}}$ is just a properly chosen quantity that makes S and D dimensionless. The integration is performed from 0 to the radius R [km]. We now refer separately to the specific cases of the hadronic star and the (strange) quark star, defined by different micro-physical descriptions.

A first treatment of the pure hadronic case has been given by [3], using a theoretically-motivated model equation of state. We instead use the so-called SLy4 equation of state in its analytic form [8] directly in the above form of the TOV equations, to obtain the energy density profiles for each initial value of the central pressure. Analytical representations of the equation of state are preferred over the tabulated ones, because they avoid two major problems of the latter: the ambiguity of the interpolation and impossibility of calculating the derivatives precisely. Furthermore, the analytical form is constructed obeying all the thermodynamic relations [8]. A suitable form of the equation of state SLy4 is

Table 1
Parameters of the fit.

i	a_i (SLy)	i	a_i (SLy)
1	6.22	10	11.4950
2	6.121	11	−22.775
3	0.005925	12	1.5707
4	0.16326	13	4.3
5	6.48	14	14.08
6	11.4971	15	27.80
7	19.105	16	−1.653
8	0.8938	17	1.50
9	6.54	18	14.67

$$\zeta = \frac{a_1 + a_2\xi + a_3\xi^3}{1+a_4\xi}f_0\big(a_5(\xi - a_6)\big)$$
$$+ (a_7 + a_8\xi)f_0\big(a_9(a_{10}-\xi)\big)$$
$$+ (a_{11} + a_{12}\xi)f_0\big(a_{13}(a_{14}-\xi)\big)$$
$$+ (a_{15} + a_{16}\xi)f_0\big(a_{17}(a_{18}-\xi)\big), \tag{3}$$

where the coefficients are given in Table 1 and $\xi = \log(\rho/\mathrm{g\,cm^{-3}})$, $\zeta = \log(P/\mathrm{dyn\,cm^{-2}})$.

This is currently a popular choice for detailed studies of dense matter and has all nuclear features of interest already built-in [9]. Another reason for choosing the *SLy*4 is that it allows maximum mass around $2M_\odot$, a minimum value similar to the quark equation of state discussed below.

The strange star models also need an equation of state describing the (self-bound) quark particles and their interactions. This is notoriously more involved than in the nuclear phase, since deconfinement is not yet properly understood. To calculate the information entropy, the disequilibrium and the complexity for our model of strange quark stars, we used one of the few analytical exact solution of the Einstein equations (which is of course a solution of the static TOV equation) for a spherically symmetric non-rotating perfect fluid. This solution is the anisotropic expression first obtained by Sharma and Maharaj [10] and studied by us in [11]. The advantage of this very accurate model is that in this way we have an analytical expression for the energy density that can be integrated easily, namely

$$\bar{\epsilon}(r) = \frac{1}{3}\frac{\rho_c c^2}{\epsilon_0}\frac{3+r^2/r_o^2}{(1+r^2/r_o^2)^2}. \tag{4}$$

In that expression, ρ_c is the central density and $r_o = r_o(\rho_c)$ is a parameter that controls the decay of the density profile. This analytical solution is obtained imposing the MIT Bag model for strange quark matter equation of state

$$p = \frac{1}{3}\left(c^2\rho - 4B\right), \tag{5}$$

where $B \simeq 57.5\ \mathrm{MeV/fm^3}$ is the energy density of vacuum. This simple expression (3) has been widely used because it readily captures the essential features of the deconfined phase. Crucial to our considerations of self-boundness (that is, a bound star even in the absence of gravitation [12]) is the existence and numerical value of the parametric vacuum energy density B, representing non-perturbative confining interactions. It is easily shown that for this massless quarks case

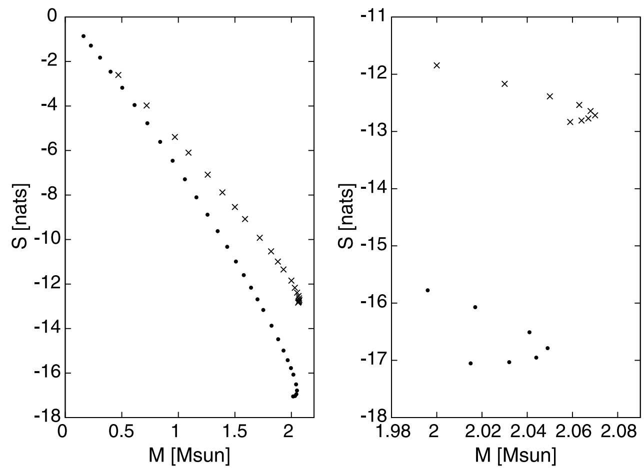

Fig. 1. Information entropy versus mass (left). The inflexion (zoomed in the right inset) marks the maximum mass of the sequence. The dots correspond to SLy4 (Eq. (3)) and the crosses to the strange quark matter (Eq. (5)).

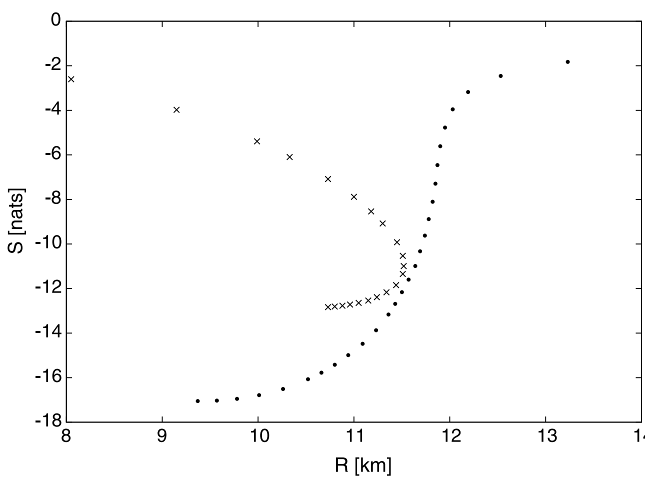

Fig. 2. Information entropy versus radius. The strong forces bounding strange quark matter change the behavior of entropy with the radius, from monotonically growing in the hadronic case (most massive models featuring the smallest radii) to the opposite (less massive models featuring the smallest radii). The symbols are the same as in Fig. 1.

$$r = r_\star \sqrt{\frac{c^4}{4BG}} \qquad m = m_\star \sqrt{\frac{c^8}{4BG^3}}, \tag{6}$$

where $r_\star$ and $m_\star$ are the dimensionless radius and mass, respectively, and G is the gravitational constant. The units of r and m come out from the unit system chosen for the constants inside the square roots.

3. Results and discussion

We now present our results showing the quantities of interest and how they depend on the mass of the star and the respective radius:

We see immediately from Figs. 1, 3 and 5 that both types of stars show pretty much the same behavior of the quantities with the mass, and also that this behavior is consistent with the one obtained in [3]. However, the behavior of the same quantities as a function of the radius (Figs. 2, 4 and 6) differ enormously. While, for instance, we find that S is a *decreasing* function of the mass for both cases in the stable region, we can state that for the hadronic star this is due to the fact that when the mass increases, the radius decreases (see Fig. 1 and Fig. 2) and the energy density become

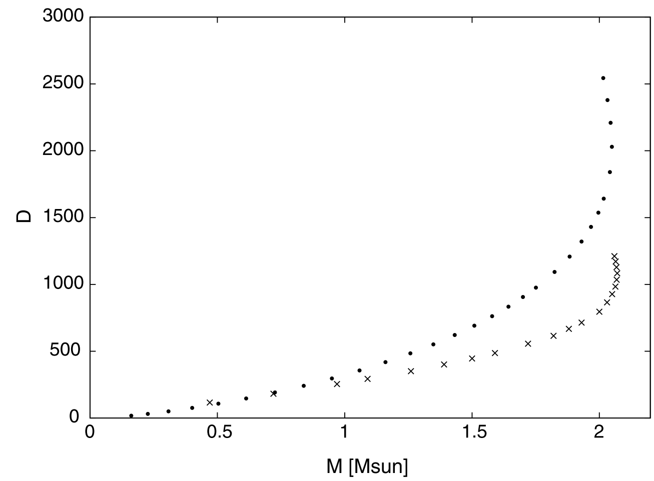

Fig. 3. Disequilibrium versus mass. The symbols are the same as in Fig. 1.

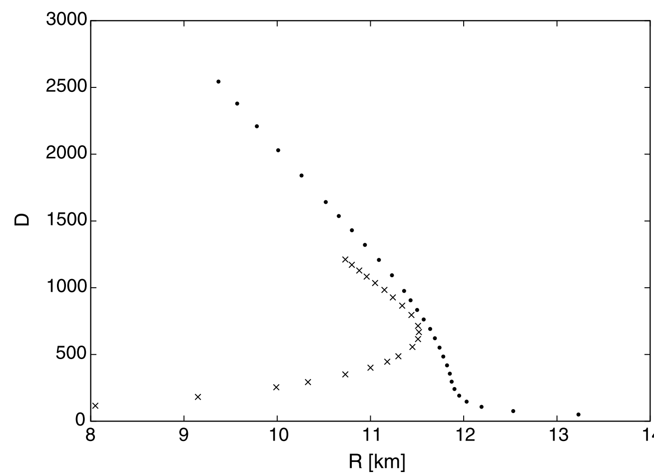

Fig. 4. Disequilibrium versus radius. A feature similar to the one in Fig. 2 is present. The symbols are the same as in Fig. 1.

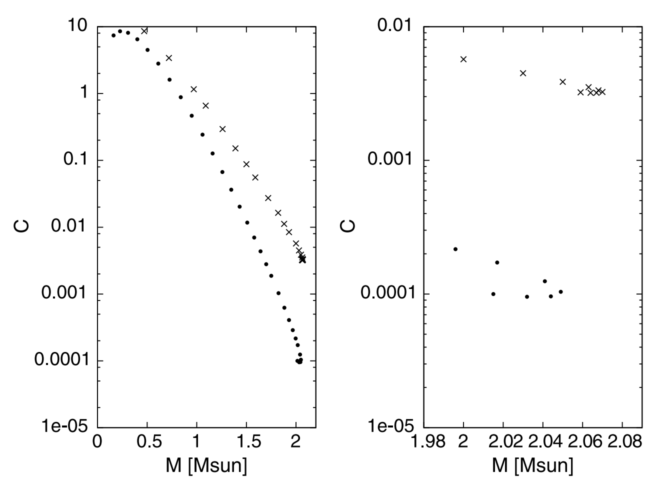

Fig. 5. Complexity versus mass (left). Notice that the log scale of the vertical axis causes a behavior similar to the entropy. The points around the maximum mass are zoomed in the right inset. The symbols are the same as in Fig. 1.

more localized (smaller radius, smaller S). On the other hand, for the self-bound quark stars, the behavior of the entropy with radius is quite different: the larger the radius, the smaller the entropy S until a certain value of the former, from where we recover a behavior quite analogous to the hadronic star. This is due to the

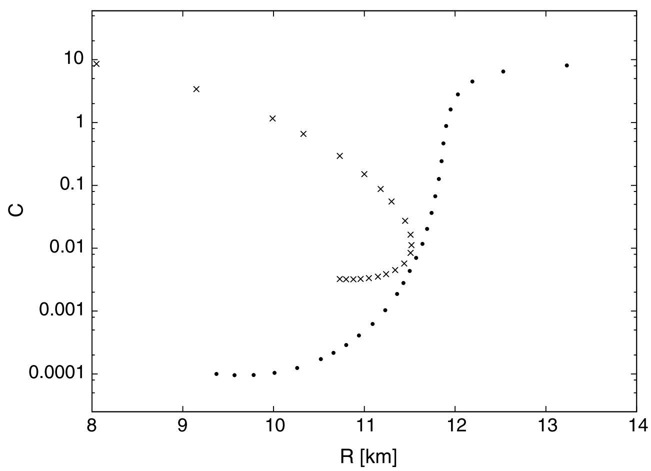

Fig. 6. Complexity versus radius. Notice that the log scale of the vertical axis causes a behavior similar to the entropy. The symbols are the same as in Fig. 1.

very different nature of the quark stars, made of free quarks and bound together by the strong interactions, in which $R \to 0$ when $M \to 0$. Therefore, in spite that the entropy carried by the gravitational field decreases its value when the mass decreases, the admixture more than compensates this and the *total* entropy increases.

We can now discuss our results in terms of the concepts of information content, distance to the equilibrium probability distribution and complexity or self-organization. It is expected that the complexity vanishes for two ideal cases, in the opposite extremes of the concept of order and disorder: the perfect crystal (a perfectly ordered system) and the ideal gas (a totally disordered system) [1]. If we compare both sequence of models for compact stars, the complexity is very low in the two cases, being smaller for the hadronic star. The authors of [3] concluded that neutron stars (however, as pointed out above, they assumed a different hadronic equation of state) are low-complexity, ordered systems because the complexity parameter is low and the disequilibrium is higher for the stars with the smaller radii, that is, for the higher masses. From these considerations one can conclude that dense matter hadronic stars systems behave similarly to the perfect crystal: higher mass $\to$ smaller radii, implies in turn a more localized energy density, higher disequilibrium and lower complexity.

On the other hand, we have shown that strange quark stars display the opposite behavior with the radius: the larger the radius (until the one corresponding to the maximum mass point), the higher the mass, implying a more *de*localized energy density, higher disequilibrium and lower complexity. We conclude that the strange quark stars are "less ordered" (i.e. more entropic) than the twin hadronic stars, pending to the side of the ideal gas, but still far from it, since the latter shows a low disequilibrium.

It is interesting to compare these cases with the other type of "low-density" compact star, the white dwarfs, studied in [2]. They found for these quasi-Newtonian objects that the complexity *grows* with increasing mass, reaching a maximum finite value at the Chandrasekhar limit. This behavior is consistent with the ones that have been reported for atomic systems [13–15], if the mass is replaced by the atomic number, and it can be related to the degenerate electron gas features. In contrast, both hadronic and strange quark stars are the result of the interplay between strong interactions and (strong) gravity, the latter being progressively less important for low masses in the second case.

It is also of interest to see the composite dependence of the statistical quantities simultaneously with mass and radius. We show in Appendix A the 3D plots (Figs. A.7, A.8, A.9).

4. Conclusions

In the preceding section we have calculated the information entropy, disequilibrium and complexity for two kinds of compact star sequences: hadronic stars and a quark stars. We observe a similar behavior of these quantities with mass, but a very different one with radius. The question that arises regarding the true composition of the neutron stars is, then, which state would be realized in nature. One could guess that, since order costs energy, then nature should favor exotic strange quark stars, but we are aware that the *path* followed to reach the state (hadronic or quark) is also very important: the parameter that controls the formation of such objects is thought to be the central density attained just after the events that lead to the huge contraction of the iron core.

Let us imagine that a hadronic star has been formed out of the iron core and settles into a equilibrium state. The strangeness barrier mentioned before would preclude the formation of a strange quark star, in spite of being the preferred state, unless the conditions for a path towards the latter can be found. A couple of possibilities arise: the first is the conversion through fluctuations [16], possibly delaying the conversion by an astronomical timescale, being a strong function of the mass. The other is the formation of a two-flavor (strangeness ~ 0), followed by the decay towards strange matter on weak-interactions time-scale ($\sim 10^{-8}$ s). The difference of free energy per particle, which is by the strange matter hypothesis, released in the process, would come out in neutrinos [17] and a structural adjustment [18,19].

Our calculations show that the complexity of these two types compact stars with different compositions is very low, i.e., there is a trend for these stars to be at a state of minimum complexity. Calbet and López-Ruiz [21,20] have shown that for a system out of equilibrium, or extremum, there is, in fact, a tendency of the complexity to reach an extremum. Thus, if there is a transition from a hadronic star to a strange star, the system as a whole would, in general, be out of equilibrium. In this way it become clear the parallelism between the systems treated here and those addressed by Calbet and López-Ruiz, a fact which calls for further studies of this subject.

Finally we must remind that the transition will *not* conserve the star mass (the binding energy, which is negative, will be larger after the process), but possibly just the total baryon number, and therefore care should be taken in the comparison of the hadronic and strange stars, in spite that the actual difference is not large and has been ignored above. Therefore, the static analysis of equilibrium configurations cannot reveal the actual situation of matter inside compact stars, but suggests that strange quark stars should be preferred.

The question of the entropy trend as a function of the compactness for a given value of the mass also points towards the self-bound quark stars, but one should say that there is an entropy barrier between the two states that has to be overcome (a full analysis from the stellar evolution point of view is in progress and will be published elsewhere). This analysis provides a complementary look to the question of compact star composition and, in particular, to the accessibility of a (still hypothetic) self-bound quark state, somewhat reminiscent of Carathéodory view of irreversible processes [22].

Acknowledgements

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Appendix A. 3D plots of information entropy, disequilibrium and complexity

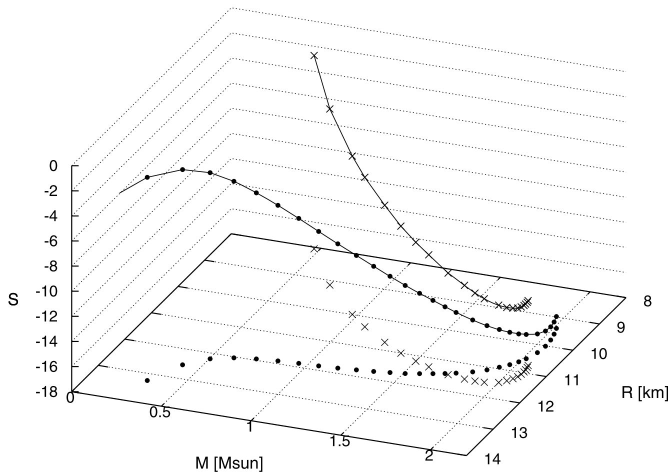

Fig. A.7. 3D plot of entropy. Notice the mass-radius relation in the M–R plane. The symbols are the same as in Fig. 1.

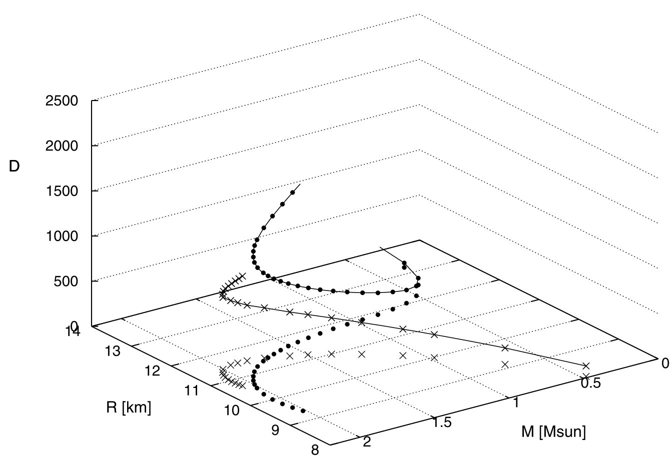

Fig. A.8. 3D plot of disequilibrium. Notice the mass-radius relation in the M–R plane. The symbols are the same as in Fig. 1.

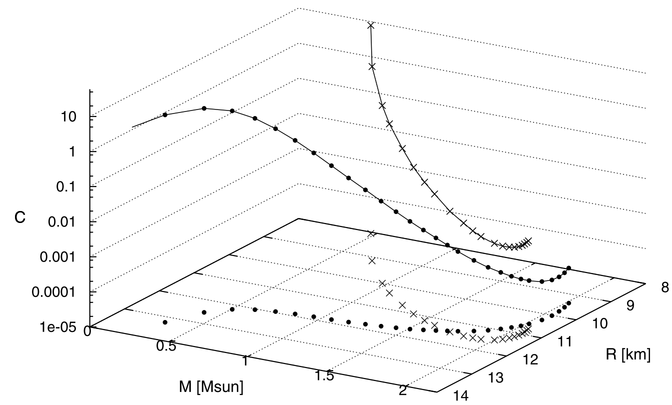

Fig. A.9. 3D plot of complexity. Notice the mass-radius relation in the M–R plane. The symbols are the same as in Fig. 1.

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