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A SIMPLIFIED MODEL FOR EVALUATING TIRE WEAR DURING CONCEPTUAL DESIGN

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ABSTRACT−In the field of vehicle dynamics, commercial software can aid the designer during the conceptual and detailed design phases. Simulations using these tools can quickly provide specific design metrics, such as yaw and lateral velocity, for standard maneuvers. However, it remains challenging to correlate these metrics with empirical quantities that depend on many external parameters and design specifications. This scenario is the case with tire wear, which depends on the frictional work developed by the tire-road contact. In this study, an approach is proposed to estimate the tire-road friction during steady-state longitudinal and cornering maneuvers. Using this approach, a qualitative formula for tire wear evaluation is developed, and conceptual design analyses of cornering maneuvers are performed using simplified vehicle models. The influence of some design parameters such as cornering stiffness, the distance between the axles, and the steer angle ratio between the steering axles for vehicles with two steering axles is evaluated. The proposed methodology allows the designer to predict tire wear using simplified vehicle models during the conceptual design phase.

KEY WORDS : Tire wear, Qualitative analysis, Lateral vehicle dynamics, Simplified models

1. INTRODUCTION

Appropriate design decisions must be made early in the design process to shorten development cycles, reduce design costs and improve product performance (Van der Auweraer *et al.*, 2007). Virtual prototyping can be employed by designers during conceptual and detailed design to support their decisions. This practice is responsible for the major progress that has been witnessed in several product design fields in recent years. In the field of vehicle dynamics, commercial software can aid designers before the first physical prototypes are available. For instance, some metrics for standard cornering maneuvers, such as yaw and lateral velocities, are produced using simulations early in the design process (da Silva and Costa Neto, 2007; Kim, 2008; Yang and Kim, 2011).

However, it remains challenging to correlate these metrics with empirical quantities that depend on many external parameters and design specifications, such as tire wear.

Tire wear is unavoidable, but it can be delayed. Therefore, the designer should consider this problem in an early design phase and try to minimize it. Additionally, tire wear may become an important issue for vehicles with two steering axles. For this type of vehicle, an improper steering geometry design can cause rapid deterioration of the tires. Accordingly, the aim of this work is to show that engineers can evaluate tire wear during conceptual design despite the fact that a more complete vehicle model is unavailable.

Tire wear depends on frictional work developed by tire-road contact (Knisley, 2002). Therefore, an approach is proposed to estimate the tire-road frictional work during longitudinal and cornering maneuvers using wheel slip quantities, which are the relative differences in wheel velocities. A similar approach is also used to include tread wear in advanced tire models available in commercial multi-body software, such as FTire (Gipser, 2005). Properties of an FTire model, such as stiffness and damping in compression and shear, heat capacity and tread mass distribution are updated depending on tread depth. FTire models are based on frictional power and its relationship to tire wear rate (Gipser, 2005). A tool that is capable of numerically predicting global tire wear and qualitatively determining the wear distribution has been proposed by Braghin *et al.* (2006). Their methodology combines a mathematical model of the tire with an empirical local friction and wear correlation. While these approaches are usually employed during the detailed design phase, the approach for evaluating tire wear proposed in this paper can be employed during the conceptual design phase because it is based on simplified dynamic models.

The relationship between wheel slip ratio and friction

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coefficient is well known and can be described by the 'magic formula' (Bakker *et al.*, 1989). This relationship depends on road condition (dry, wet, etc.), road type (asphalt, concrete, etc.), tire type (carcass or radial), tread type and tread depth. Several works have been developed within this framework. For example, the real-time prediction of friction coefficient based on slip estimation has been studied by Lee *et al.* (2004), tire dynamics have been examined by Kang (2009), and the parameter identification of a Lugre friction model has been performed by Wu *et al.* (2011).

Experimental tire wear data can be found in many references. Among these references, it is important to highlight the tire wear model validation of Braghin *et al.* (2006), the correlation between tire wear and pavement texture of Gunaratne *et al*. (2000) and various patents on tire wear measuring devices.

The methodology proposed in this paper is based on qualitative analysis of tire wear considering some vehicle dynamic responses. Tire wear can be correlated with frictional work by the abradability or the abrasion coefficient (Veith, 1986; Knisley, 2002). Additionally, frictional work can be described in terms of the dynamic responses during steady-state longitudinal and lateral maneuvers. This approach is described in Section 2. Steady-state cornering maneuvers are examined using the qualitative formula obtained and the single-track model, which is a simplified model for the evaluation of lateral vehicle dynamics. A short review of the single-track model for two steering axles is described in Section 3. Using the qualitative formula and the single-track model, the evaluation of some design parameters, such as cornering stiffness, the distance from the axles to the vehicle's center-of-gravity and the steer angle ratio between the axles, is presented in Section 4. Additionally, some expressions found in the literature are noted that allow the designer to predict tire wear behavior using simplified models and these standard expressions. Finally, conclusions are drawn in Section 5.

2. TIRE WEAR: QUALITATIVE ANALYSIS

A qualitative formulation for tire wear is derived for steady-state longitudinal and lateral dynamic responses in the next subsections. First, the tire slip quantities that define the frictional work are introduced. Then, the frictional work is used to qualitatively estimate tire wear during the conceptual design phase.

2.1. Definition of the Tire Slip Quantities

The longitudinal and lateral tire slip quantities are introduced in the SAE axis system (Adams Manual, 2011). Figure 1 shows a diagram of the longitudinal slip velocity, V_{sx}; the longitudinal velocity, V_x; the lateral slip velocity, V_{sy}; the vehicle velocity, V; and the slip angle, α, for a wheel during a maneuver that combines cornering and breaking/traction.

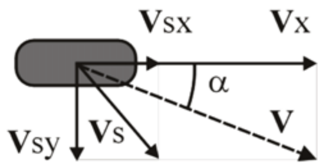


Figure 1. Diagram of the tire slip quantities (Adams Manual, 2011).

The longitudinal slip velocity is defined by:

$$V_{sx} = V_x - \omega_w R \tag{1}$$

where ω_w is the wheel's rotational velocity, and R is the wheel's radius.

The lateral slip velocity is equal to the lateral speed at the point of contact with the road plane; that is, $V_{sy} = V_y$.

The longitudinal slip ratio, β, and α are calculated according to the slip velocities at the point of contact (Winkler, 1998):

$$\beta = \frac{V_{sx}}{V_x} \qquad -1 < \beta < 1 \tag{2}$$

and

$$\tan(\alpha) = \frac{V_{sy}}{|V_x|} \qquad \alpha < 90^{\circ} \tag{3}$$

2.2. Definition of the Frictional Work

Tire wear is proportional to the frictional work, which can be defined as (Knisley, 2002):

$$L = \int_0^S F_t \cdot dl \tag{4}$$

where L represents the frictional work, F_t is the tangential force magnitude, and S is the distance travelled by the vehicle during a maneuver. For both steady-state longitudinal and lateral maneuvers, Equation (4) can be rewritten as follows.

For a steady-state longitudinal maneuver, Equation (4) can be described in terms of the tangential (or longitudinal) force, F_x, and the relative displacement, dx (*i.e.*, the relative displacement in the longitudinal direction). In these terms, the frictional work related to a steady-state longitudinal maneuver, L_O, can be described by:

$$L_o = \int_0^s F_x \cdot dx \tag{5}$$

The relative longitudinal displacement can be expressed in terms of the longitudinal slip velocity as $dx = V_{sx} \cdot dt$. Therefore, L_O can be rewritten as:

$$L_o = \int_0^T F_x \cdot V_{sx} \cdot dt \tag{6}$$

where T is the total time.

Using the definition of β stated in Equation (2), L_O can be described by:

$$L_o = \int_0^T F_x \cdot \beta \cdot V_x \cdot dt \tag{7}$$

In steady-state longitudinal maneuvers, not only are F_x and β constants, but $V_x = V$ because $V_y = V_{sy} = 0$, thus, Equation (7) becomes:

$$L_o = F_x \cdot \beta \cdot \int_0^T V \cdot dt = F_x \cdot \beta \cdot S \tag{8}$$

Equation (8) shows that, for steady-state longitudinal maneuvers, the frictional work is proportional to the longitudinal force, the longitudinal slip ratio and the distance travelled by the vehicle.

Similarly, for a steady-state lateral maneuver, Equation (4) can be described in terms of the tangential (or lateral) force, F_y, and the relative displacement, dy (*i.e.*, the relative displacement developed in the lateral direction). Therefore, the frictional work for a steady-state lateral maneuver, L_a, can be expressed as:

$$L_o = \int_0^S F_y \cdot dy \tag{9}$$

The relative lateral displacement can be expressed in terms of the lateral slip velocity using $dy = V_{sy} \cdot dt$. Therefore, L_a can be rewritten as:

$$L_a = \int_0^T F_y \cdot V_{sy} \cdot dt = \int_0^T F_y \cdot V_y \cdot dt \tag{10}$$

For small side slip angles, the tangent of the side slip angle can be approximated by the side slip angle, and the longitudinal velocity can be approximated by the velocity of the vehicle:

$$\tan(\alpha) \approx \alpha \approx \frac{V_{sy}}{|V_x|} \approx \frac{V_y}{|V|} \tag{11}$$

Combined with the fact that the lateral force and the side slip angle are constants, the frictional work for steady-state lateral maneuvers can be rewritten as:

$$L_a = \int_0^T F_y \cdot \alpha \cdot V \cdot dt = F_y \cdot \alpha \cdot S \tag{12}$$

Equation (12) shows that, for steady-state lateral maneuvers, the frictional work is proportional to the lateral force, the side slip angle and the distance travelled by the vehicle.

2.3. Qualitative Formulation for Tire Wear during Steady-state Lateral Maneuvers

Tire wear can be quantified by the rate of wear, W, which is the amount of rubber lost from a unit surface per tire revolution. A common assumption is that tire wear is proportional to the amount of frictional work performed by the tire (Veith, 1986; Knisley, 2002):

$$W = Ab \cdot L \tag{13}$$

where Ab is the abradability, which can be defined as the amount of rubber lost per unit area per unit frictional work under specified interface conditions (Veith, 1986). This correlation may seem simplistic and, indeed, a more complex correlation may be applied. However, most of the complexity is carried by Ab. Abradability is not a material constant; it depends on (Veith, 1986):

- tire characteristics, such as hardness, molecular structure, elongation at break, wear resistance, degree of vulcanization, quantity of carbon black, etc.;
- pavement characteristics, such as smoothness, grading zone, flakiness, etc.;
- air, road and tire temperature;
- and interfacial contaminants, such as water, dust, mud, etc.

The lack of a specific constant value for Ab may jeopardize the use of Equation (13) as a general equation for solving tire wear problems. However, it can be employed in qualitative analysis such as the one proposed in this manuscript.

At this point, it is necessary to highlight that the normal force also plays an important role in tire wear. In this work, a linear correlation between the tire wear and the normal force is considered. Therefore, the proposed qualitative formula for tire wear for steady-state lateral maneuvers can be described as:

$$W \propto Ab \cdot F_y \cdot \alpha \cdot S \cdot \left(\frac{F_N}{F_{N0}}\right) = Ab \cdot C_\alpha \cdot \alpha^2 \cdot S \cdot \left(\frac{F_N}{F_{N0}}\right) \tag{14}$$

where F_N represents the normal force and F_{N0} is a reference normal force. The lateral force F_y is proportional to the cornering stiffness, C_α, and α (Winkler, 1998).

To decrease tire wear, the product of lateral force and side slip angle should be reduced. These parameters can be extracted from simple models, such as single-track models, that are available during the conceptual design phase.

3. SIMPLIFIED VEHICLE MODEL FOR EVALUATION OF LATERAL DYNAMICS

The aim of this manuscript is to evaluate tire wear during the conceptual design phase by considering steady-state cornering maneuvers. Because little information is available during this design phase, the selection of the simplest vehicle model capable of predicting the lateral vehicle dynamics is advantageous. Therefore, the single-track model, also known as the bicycle model, has been selected (Gillespie, 1992). This two degree-of-freedom model considers the lateral load transfer to be negligible; thus, vehicle suspension is ignored. This assumption is only valid for low lateral accelerations. This model has been used to predict steady-state responses (which are necessary for qualitative analysis) for vehicles with one

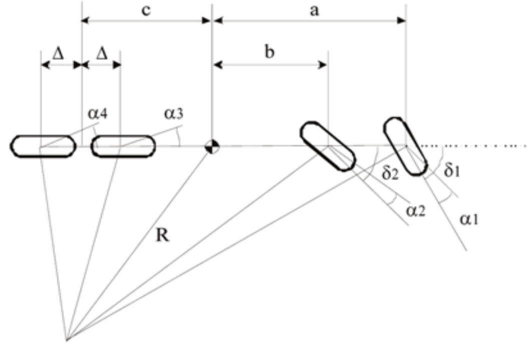


Figure 2. Single-track model for a four-axle vehicle with two steering axles.

and two steering axles. The single-track model for vehicles with two axles (one steering axle) is widely described in the literature (Winkler, 1998; Gillespie, 1992).

An extension of this model for vehicles with four axles (two steering axles) is derived here. A state-space model can be derived from the summation of forces and moments in the bicycle model shown in Figure 2. The state-space model is described by Equation (15), where the matrices **A** and **B** are defined by Equations (16) and (17), respectively. The matricies depend on the vehicle mass, M, and inertia, I, the cornering stiffness, C_α, of the different axles, the vehicle velocity, V, and the distances from the center-of-gravity to the axles, a, b, c and Δ (see Figure 2). The inputs to the system are the steer angles, δ_1 and δ_2, and the state variables are the angular velocity, ω_z, and the slip ratio, β.

$$\begin{bmatrix}\beta\\ \omega_z\end{bmatrix} = \mathrm{A}\begin{bmatrix}\beta\\ \omega_z\end{bmatrix} + \mathbf{B}\begin{bmatrix}\partial_1\\ \partial_2\end{bmatrix} \tag{15}$$

where

$$\mathbf{A} = \begin{bmatrix} -\frac{(C_{\alpha1}+C_{\alpha2}+C_{\alpha3}+C_{\alpha4})}{MV} & -1-\frac{(aC_{\alpha1}+bC_{\alpha2}-(c-\Delta)C_{\alpha3}-(c+\Delta)C_{\alpha4})}{MV^2} \\ -\frac{(aC_{\alpha1}+bC_{\alpha2}-(c-\Delta)C_{\alpha3}-(c+\Delta)C_{\alpha4})}{I} & -\frac{(a^2C_{\alpha1}+b^2C_{\alpha2}+(c-\Delta)^2C_{\alpha3}+(c+\Delta)^2C_{\alpha4})}{IV} \end{bmatrix} \tag{16}$$

$$\mathbf{B} = \begin{bmatrix} \frac{C_{\alpha1}}{MV} & \frac{C_{\alpha2}}{MV} \\ \frac{aC_{\alpha1}}{I} & \frac{bC_{\alpha2}}{I} \end{bmatrix} \tag{17}$$

From these equations, ω_z and β can be calculated. As demonstrated in Section 2, α is required for tire wear estimation during steady-state cornering maneuvers. The side slip angle for each axle, α_i (i = 1,2,3 or 4, numbering according to Figure 2), is given by:

$$\alpha_1 = \beta + \frac{a\omega_z}{V} - \delta_1 \tag{18}$$

$$\alpha_2 = \beta + \frac{b\omega_z}{V} - \delta_2 \tag{19}$$

$$\alpha_3 = \beta - \frac{(c-\Delta)\omega_z}{V} \tag{20}$$

$$\alpha_4 = \beta - \frac{(c+\Delta)\omega_z}{V} \tag{21}$$

From Equations 18-21, it follows that when the velocity is zero (*i.e.*, $\beta = 0$, $\omega_z = 0$), the side slip angles α_1 and α_2 are equal to the steer angles δ_1 and δ_2, respectively, and the side slip angles α_3 and α_4 are zero. This phenomenon can be observed in Figures 3 and 7.

4. QUALITATIVE ANALYSES

The qualitative formula for evaluation of tire wear during steady-state lateral maneuvers (Equation (14)) can be used to compare different case studies during the conceptual design phase of a vehicle.

To perform such comparisons, Ab, S and F_{N0} are assumed to be constant for the different alternatives and, therefore, no explicit values are assigned to these terms. Thus, the $C_\alpha \cdot \alpha^2 \cdot F_N$ term, which Equation (14) shows to be proportional to tire wear, is used to evaluate the different case studies considered here.

Different configurations have been evaluated for two different vehicles: one with two axles and one with four axles. Typical values of mass, inertia, mass distribution, distance from the axles to the center-of-gravity, etc. are the inputs for the simplified model that supplies the desired dynamic responses. Analyses of the vehicle with two axles are performed using typical parameters for a light commercial vehicle, which are shown in Table 1. For these case studies, the first and the second axles contain 2 and 4

Table 1. Model parameters for a light commercial vehicle.

Parameter	Description	Value
M	mass	9000 kg
I	inertia	43000 kg.m2
l	length	3.70 m
a	Distance from the CG to the first axle	2.35 m
C_α	cornering stiffness	95000 N/rad

Table 2. Model parameters for a heavy passenger vehicle.

Parameter	Value
M	23600 kg
I	419000 kg.m^2
a	4.60 m
b	3.30 m
c	3.00 m
Δ	0.74 m
C_α	170000 N/rad

tires, respectively. For the vehicle with two steering axles, typical parameters of a heavy passenger vehicle, as shown in Table 2, are employed. For these case studies, the first, second and fourth axles contain 2 tires, while the third axle contains 4 tires. The cornering stiffnesses in Tables 1 and 2 are for a single tire.

The main objective in this paper is to evaluate tire wear during the conceptual design phase. During this stage, only simple vehicle models are available. Therefore, only the parameters that are taken into account in these models could be evaluated. Considering the single-track model, the parameters that are readily available are the cornering stiffness, the location of the axles and the steer angle ratio.

4.1. Ackerman Geometry

According to Miller and Reed (1991), deviations from Ackerman angles for right or left steer angles can significantly affect tire wear. However, these deviations do not significantly affect directional response for low speed turns. Consequently, the methodology proposed here is not useful for evaluation of tire wear caused by deviations from Ackerman angles.

Although the methodology proposed in this paper does not evaluate the Ackerman angles, it is important to note that proper specification of the steer angles avoids high values of friction forces during cornering maneuvers, which can prevent early tire wear phenomena.

4.2. One Steering Axle Vehicle

The dynamic vehicle responses at different velocities for a step steering input have been determined. Based on these responses, the lateral accelerations for a single-track model of a light commercial vehicle have been calculated for each velocity. The quantities that are proportional to tire wear are shown in Figure 3.

Because the proposed methodology is formulated for steady-state maneuvers, the transient response should be ignored. Therefore, only the steady-state lateral accelerations are considered. The steady-state values for different velocities are shown in Figure 4. It can be observed that the tire wear of the rear axle is more severe.

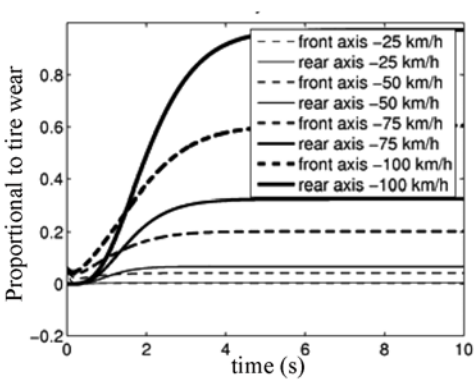


Figure 3. Qualitative tire wear values for a step steering input at different velocities.

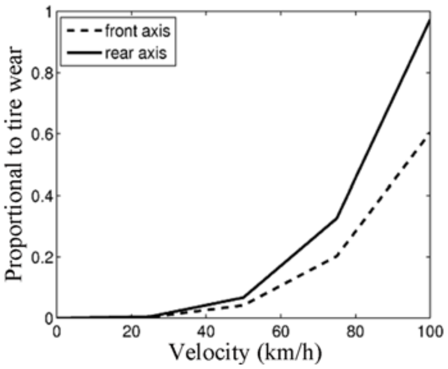


Figure 4. Steady-state qualitative tire wear values at different velocities for a vehicle with one steering axle.

To evaluate the impact of modifications to the design of a single steering axle vehicle, calculations modifying the stiffness coefficient and the location of the axles are described in the next subsections.

4.2.1. Cornering stiffness coefficient

Two alternatives have been evaluated for the initial cornering stiffness coefficient: a reduction of 50% and an increase of 100%. Comparisons between these cases and the original design are shown in Figure 5. In both cases, the

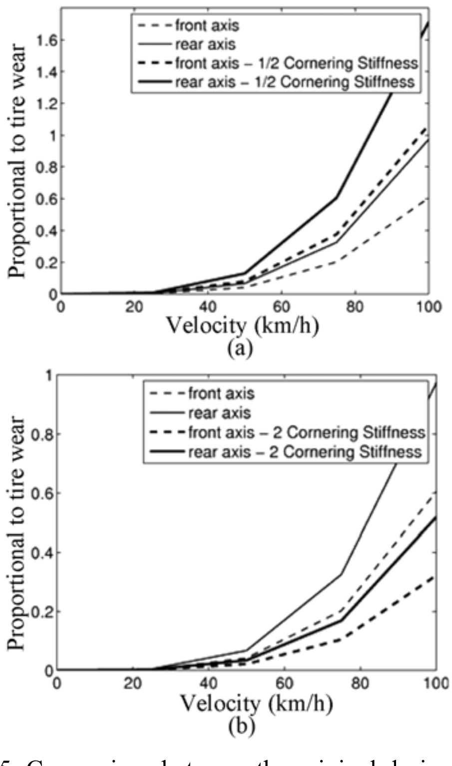


Figure 5. Comparison between the original design and the design modified by: (a) a reduction in the initial cornering stiffness coefficient of 50% and (b) an increase in the initial cornering stiffness of 100%.

rear axle still suffers more severe tire wear than the front. Additionally, the results demonstrate that the lateral force is proportional to the cornering stiffness coefficient.

4.2.2. Location of the axles
Two situations have been evaluated: a 10% longer vehicle and a vehicle 10% shorter than the original. Comparisons between these two configurations and the original are shown in Figure 6. There was no significant change in rear axle tire wear. The 10% longer configuration displayed approximately 15% improved tire wear compared to the original configuration.

4.3. Vehicles with Two Steering Axles
The approach described in Section 4.2 has been employed to evaluate a heavy commercial passenger vehicle with two steering axles. A step steering input has been applied to the vehicle model at different velocities. The results for 75 km/h are shown in Figure 7. The steady-state values for different velocities are shown in Figure 8. In this case, the tire wear of the second axle is more severe than the other axles. To change this condition, modifications to the steer angle ratio and the location of the axles have been assessed.

4.3.1. Steer angle ratio between the axles
Two changes in the steer angle ratio have been evaluated:

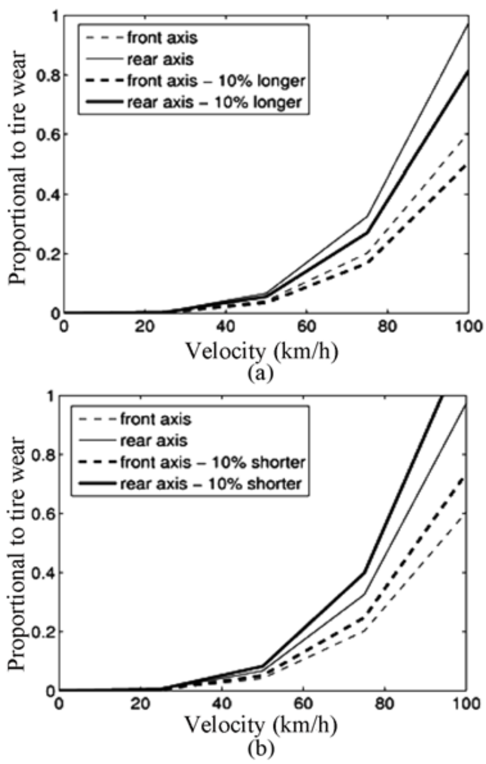


Figure 6. Comparison between the original configuration and one with: (a) a 10% longer vehicle and (b) a 10% shorter vehicle.

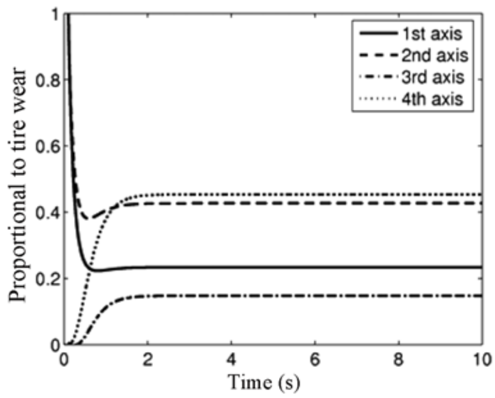


Figure 7. Qualitative tire wear values for a step steering input at 75 km/h.

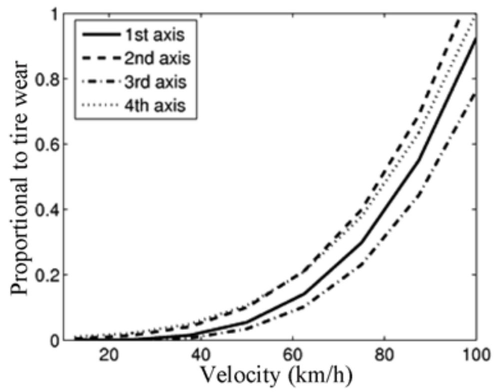
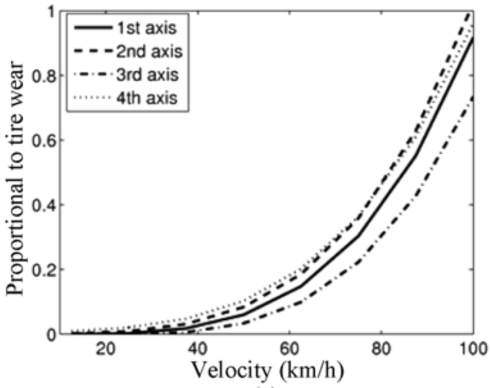


Figure 8. Steady-state qualitative tire wear values at different velocities for a vehicle with two steering axles.

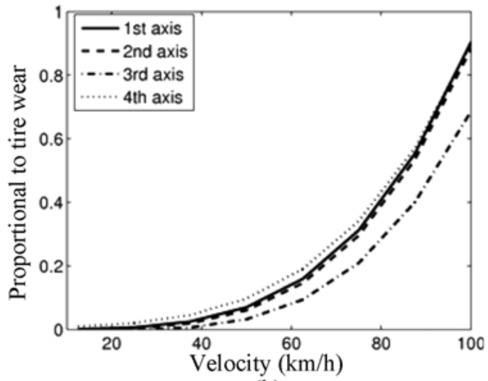
5% and 14% smaller value than the original ratio (0.992). These configurations are shown in Figure 9. Both modifications reduce the tire wear of the second, the third and fourth axles. The 14% reduced ratio configuration is the best steer angle ratio found in an extensive search of the parameter space.

4.3.2. Location of the axles
Eight cases have been created by individually increasing and decreasing the distances introduced in Figure 2 (a, b, c and Δ) to values 20% longer and 20% shorter than the original values. The steady-state qualitative tire wear values for these modified configurations are shown in Figures 10, 11, 12 and 13. From these figures, it can be concluded that modifications to b and c can effectivly reduce tire wear and that modifications to a and Δ are ineffective.

Table 3 summarizes the results illustrated in Figures 10, 11, 12 and 13. The symbols $\uparrow$, $\downarrow$ and – represent improvement (*i.e.*, tire wear reduction), greater wear and no significant change in wear, respectively. These results are case dependent, and these conclusions cannot be



(a)



(b)

Figure 9. Steady-state qualitative tire wear values for a steer angle ratio (a) 5% smaller and (b) 14% smaller than the baseline case.

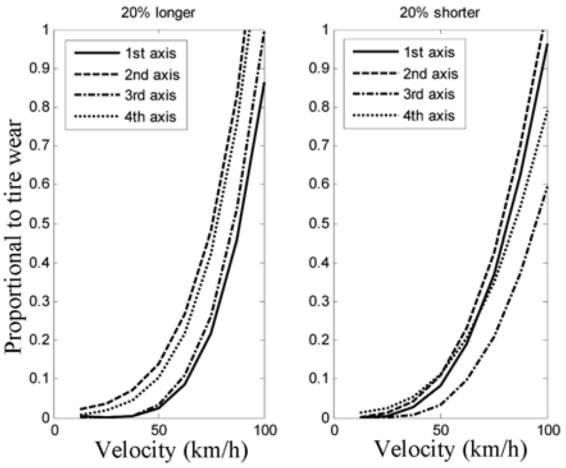


Figure 10. Steady-state qualitative tire wear values for a 20% increase in *a* (left) and a 20% decrease in *a* (right).

generalized for any vehicle with two steering axles.

5. CONCLUSION

A new method is derived for tire wear evaluation during

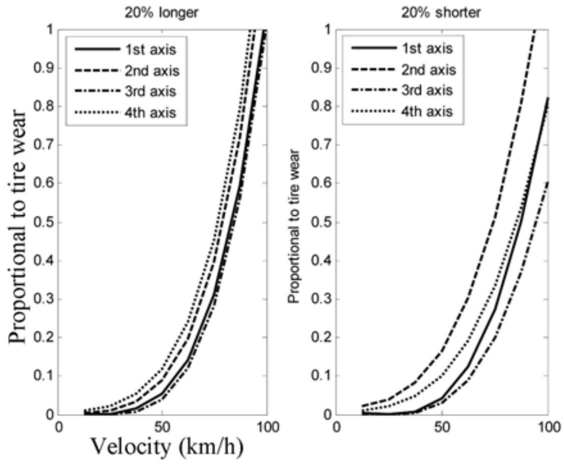


Figure 11. Steady-state qualitative tire wear values for a 20% increase in *b* (left) and a 20% decrease in *b* (right).

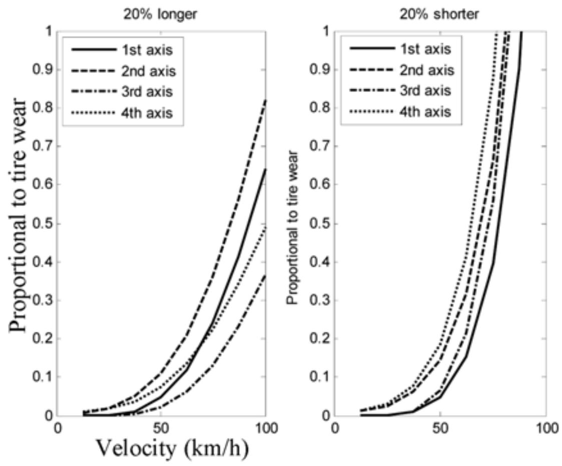


Figure 12. Steady-state qualitative tire wear values for a 20% increase in *c* (left) and a 20% decrease in *c* (right).

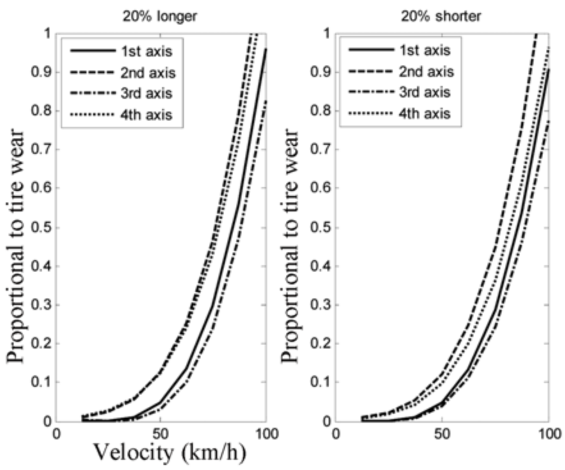


Figure 13. Steady-state qualitative tire wear values for a 20% increase in Δ (left) and a 20% decrease in Δ (right).

Table 3. Summary of the results of axle distance modification.

Cases	1st axle	2nd axle	3rd axle	4th axle
a 20% longer	↓	↑	↑	↑
a 20% shorter	–	–	↓	↓
b 20% longer	↑	↑	↑	↑
b 20% shorter	↑	↓	↑	↑
c 20% longer	↑	↑	↑	↑
c 20% shorter	↓	↓	↓	↓
Δ 20% longer	–	–	–	–
Δ 20% shorter	–	–	–	–

steady-state maneuvers. Because tire wear depends on frictional work developed by tire-road contact, an approach can be derived to estimate wear during steady-state longitudinal and cornering maneuvers using the wheel slip, which is the relative difference in wheel velocities. Using the qualitative formula obtained, cornering maneuvers employing a simplified model for lateral vehicle dynamics (the single-track model) have been evaluated for different conditions. This approach is also employed for a vehicle with two steering axles, where an extension of the single-track model is used. Some results are shown, and some statements found in the literature are highlighted which allows the designer to infer about the vehicle tire wear behavior using simple models and standard statements.

Conclusions can only be drawn for the specific vehicle cases examined. However, two clear conclusions can be observed. First, there is an optimal location for the axles on vehicles with one steering axle. Second, for vehicles with two steering axles, there is an optimal steer angle ratio.

Despite the fact that this approach does not include evaluation of the Ackerman angles, it is important to note that proper specification of the steer angles can prevent early tire wear phenomena because it avoids high values of friction forces during cornering maneuvers.

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