*J. Braz. Chem. Soc.*, Vol. 19, No. 2, 273-276, 2008.
Printed in Brazil - ©2008 Sociedade Brasileira de Química
0103 - 5053 $6.00+0.00

Article

# Three-nucleon Potentials

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Neste artigo descrevemos um efeito sutil na física nuclear associado com forças de três nucleons, o qual é, no entanto, fundamental na interpretação de resultados experimentais. É importante notar que os efeitos de três corpos são de origens não-pertubativos o que torna este problema mais envolvente teoricamente. O uso da Cromodinâmica Quântica é fundamental no entendimento do processo físico.

In this work we describe a subtle effect in nuclear physics, associated with three-nucleon forces, which is nevertheless fundamental in the interpretation of experimental results. It is important to notice that three-body effects are of non-pertubative origins, which makes this problem more involving theoretically. The use of Quantum Chromodynamics is fundamental in the understanding of the physics process.

**Keywords:** nuclear forces, three-nucleon forces

## Introduction

Science promotes access to a world which is both complex and wonderful, both rational and magic. Research in science is a task full of difficulties and intellectual tensions, but it is definitely worthwhile, for it entitles one to contemplate the unity of nature and opens the door to important applications. In Physics, theoretical constructions, which rely on both mathematics and empirical data, allow one to understand the working of matter in regions so small or so large that cannot be reached by our senses and, only with some effort, by our imagination. As a tribute to Professor Ricardo Ferreira for his lifelong devotion to science in Brazil, in this work we describe a subtle effect in nuclear physics, associated with *three-nucleon forces,* which is nevertheless fundamental in the interpretation of experimental results.

## Early Nuclear Forces

Rutherford proposed the existence of the atomic nucleus in 1910, but the modern understanding of the mechanisms that keep protons and neutrons together only began more than two decades later, with the work of Yukawa. His idea of the existence of mesons was set on a firm basis after the detection of the pion ($\sigma$), by Lattes, Occhialini and Powell, in 1947. This became the foundation of a powerful research program after 1951, when Taketani and collaborators[1] argued that nuclear interactions are due to pion exchanges, which are relatively simple at large distances and become gradually more complex as one moves inward. Strong interactions do not distinguish a proton ($p$) from a neutron ($n$) and hence both particles are denoted by the generic name nucleon ($N$). According to the early picture of nuclear forces, nucleons interact by exchanging a single pion when the relative distance is larger than 3 fm, two pions when this distance lies between 1.5 fm and 3 fm, and so on. This is the famous *range expansion* that has guided the construction of nuclear potentials ever since, in which one counts the number of pions exchanged.

The nucleus of the hydrogen atom is the simplest possible and contains just one proton. Adding a neutron, one gets the deuteron. Three-nucleon systems come next and two varieties are known, namely $^3$H ($pnn$) and $^3$He ($ppn$). Increasing the number of nucleons, hundreds of other observed combinations become possible. As the binding energy of each nucleus is unique, a main challenge of the research program on nuclear forces is to understand this

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enormous amount of data in terms of a few basic interaction mechanisms. The idea, in principle, is to transform the interaction into a potential to be used in a numerical solution of the Schrödinger equation.[8] However this task is not easy, because the number of degrees of freedom even in a relatively small nucleus is too large to be treated in detail. One is then forced to resort to simplifying approximations. In the case of the interaction, a usual procedure was to assume that the potential energy of a system containing several nucleons could be well described as the sum of the potential energies of the various pairs of nucleons that could be decomposed. This procedure, *known as the two-body approximation*, can be thought as the nuclear counterpart of the linear superposition principle used in electrostatics.

In the 1980's an anomaly began to appear in light nuclei. Progress in computing enormously increased the precision of theoretical calculations. The results produced by different groups using the best NN potentials available detected a rather visible under binding in three-nucleon systems. In the search for explanations, a 1957 work by Fujita and Miyazawa[2] came into evidence. They noted that, if the participating particles in the interaction could be excited, the superposition principle could not hold. And, indeed, nucleons can be excited into a ± particle, through the nuclear reaction $\sigma + N \Leftarrow \pm$. The existence of this process gives rise to an interaction which can only occur when three nucleons are present. Interactions of this kind are known as *three-nucleon potentials* (3NPs). Intermediate ± excitation is just a possibility amongst others that contribute to three-nucleon interactions. Three-body forces, based on a different mechanism, were first considered in 1939 by Primakoff and Holstein.[3] They have shown that their effect is very small in atomic physics, but conclude their abstract with the prophetic words: *The usual description of nuclei in terms of two-body potentials cannot [...] be considered satisfactory, except in the case of the deuteron*. A historical review can be found in Reference 4.

Since the early seventies, one knows that the most important 3NP the longest possible range and is due to the process in which the pion emitted by one of the nucleons is scattered before being absorbed by the third one. This process, known as two-pion exchange three-nucleon potential (TPE–3NP), is represented in Figure 1. The potential then depends on a reliable theoretical description of the intermediate σN amplitude.

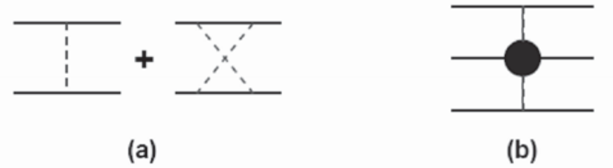

**Figure 1**. Nuclear interactions: (a) contributions to the two-body potential; (b) two-pion exchange three-nucleon potential.

## Strong Interactions: the Modern Framework

In the early fifties, it was realized that the perturbative methods employed in quantum electrodynamics could not be automatically transferred to strong interactions, because the coupling constants in the latter are rather large. On the other hand, the σ*N* is relatively small and this paradoxical situation puzzled the scientific community in that decade. The theoretical tool that was used to solve the problem began to appear in the sixties, but its full strength could only be realized after quantum chromo dynamics (*QCD*) became established as the theory of strong interactions. *QCD* the interactions of quarks (*q*) among themselves by means of gluon exchanges and had a very profound impact in our understanding of the behavior of matter. According to this theory, the vacuum is not empty but, rather, full of quark-antiquark ($q\bar{q}$) pairs that behave like Cooper pairs in a superconductor. In this framework the pion corresponds to the smallest possible disturbance of this vacuum, produced by a ($q\bar{q}$) pair with specific quantum numbers. Nucleons, on the other hand, are systems of three quarks, confined by the pressure of the vacuum. The presence of the confined quarks induces a phase transition when one approaches the center of the nucleon and its interior is empty. Moreover, as the quarks inside this hole bump against its wall, a pion cloud is produced in the outer region of the nucleon, as indicated in Figure 2.

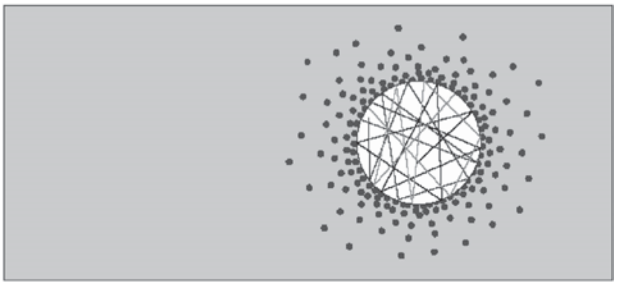

**Figure 2**. The nucleon system in *QCD*: the shaded region in the background corresponds to the vacuum, filled with $q\,\bar{q}$ pairs; in the white region, with radius ~ 0,6 fm, space is empty; quarks move within this volume and bump against the nucleon wall, giving rise to the pion cloud, represented by the dots in the outer region.

In this way, able to shed light into the basic mechanism operating inside nuclei. In spite of the clear picture produced, it is very difficult to use *QCD* in calculations of hadronic properties at low energy, owing to the fact that gluons, the objects analogous to photons quantum electrodynamics in the theory, are not neutral and carry charge. This makes *QCD*be non-Abelian. The strategy employed to overcome this difficulty is to use effective field theories (*EFTs*) in which pions and nucleons are the basic degrees of freedom. One ensures that the *EFT* as close as possible to *QCD* imposing both theories to share the same symmetries. One of these concerns invariance under transformations of the Poincaré group, which is exact.

In the typical scale of *QCD*, interactions in nuclear physics involve only relatively low energies processes, which are dominated by just the quarks π and *d*. The fact that their masses are very small allows us to consider an approximate symmetry of *QCD*, which has a very important dynamical role. It has been noted that, in the case these masses were set to zero, the *QCD* would possess an *exact chiral symmetry*. The name *chiral*, derived from the Greek word for *hand*, is associated with the *right-left symmetry* of massless fermions. One then works with a simplified version of *QCD*and treats the quark masses as perturbations in a chiral symmetric lagrangian. This idea allows the systematic incorporation of low-energy features of *QCD*into the nuclear force problem. In the early nineties, it gave rise to a high precision theoretical tool, which became rather well established, owing to the works of Weinberg who formulated a chiral perturbation theory (ChPT).[5] Today, the physical motivation for the use of chiral symmetry is both solid and appealing in the description of nuclear interactions.

## Chiral Three-body Forces

The application of chiral symmetry to the construction of three-nucleon potentials precedes by two decades[6] the formulation of chiral perturbation theory, and the modern version of the force was already produced in the early eighties. In 1979, the so called Tucson-Melbourne (TM) 3NP was produced,[7] which incorporated chiral symmetry into the intermediate σ*N* amplitude in an abstract way. Somewhat later, in 1983, the authors of this paper developed a model dependent 3NP, based on resonance excitations.[8] This version of the 3NP became known in the literature as *Brazil potential* and it generalized and unified previous approaches. From the dynamical point of view, the essential difference with the Tucson-Melbourne potential concerns a term describing the S-wave component of the intermediate σ*N* interaction. We were able to argue that the form adopted in the TM paper had an improper contact interaction that should be removed.[9] This term was indeed removed in a new version produced not long ago, known as TM.[10] A pedagogical review of three-nucleon forces can be found in Reference 11.

In a recent work,[12] the *1983 Brazil potential* was extended, in the light of developments in chiral perturbation theory. The new version of the TPE-3NP contains both corrections to numerical coefficients and new structures, representing loop integrals and non-local operators. However, the influence of these new features over observables has been estimated and found to be at least one order of magnitude smaller than results obtained from the previous version.

## Three-body Forces In Nuclei

The most comprehensive calculation of the effects of three-body forces in nuclei was performed by Pieper, Pandharipande, Wiringa and Carlson[13] and here we rely on their results in order to produce a feeling for the importance of these interactions in light systems. In 2001, using a powerful computational machinery, these authors performed essentially exact calculations of the static properties of all nuclei in the range 3 ℑ A ℑ 8 and found out that, in all cases, the two-body interaction produces underbidding. In their study, the fraction of the experimental binding energy produced by the two-body potential lies between 76% and 92%, the larger value corresponding to the lightest $^3$H and $^3$He nuclei. The inclusion of three-body forces, which are attractive, brings the discrepancy to 2%, in the worst cases.

**Table 1**. Partial contributions to the $^8$Be binding energy; all values are in MeV

| | <*K*> | < $V_2$ > | < $V_3$ > | *BE* | experi-mental |
|---|---|---|---|---|---|
| no 3BFs | 241 | -287 | - | -48 | -57 |
| with 3BFs | 268 | -303 | -23 | -57 | -57 |

It is important to note that three-body effects are generated by non-perturbative processes. Indeed, besides shifting the binding energies by the correct amount, they also produce visible modifications in the wave function. For instance, in the case of $^8$Be, we denote respectively by <K>, <$V_2$> and <$V_3$> the partial contributions of the kinetic energy and potential energies due to two-body and three-body interactions to the binding energy BE, and find the values given in Table 1.[13] Inspecting the values quoted, one notes that the introduction of three-body forces increases both <K> and <$V_2$>, indicating modifications of the wave function. This, in turn may affect observables such as r.m.s. and electromagnetic form factors. The pattern for other nuclei is quite similar, indicating that three-body forces are indeed essential for the precise description of nuclear properties.

## Conclusions

One can summarize the conclusions as follows: subtle effects in nuclear physics caused by three-body forces are fundamental for the precise description of nuclear properties; three-body effects are generated by non-pertubative processes which make the theoretical work more involving; *QCD* inspired models are considered in this paper since it became the theory of strong interactions.

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*Received: August 10, 2007*
*Published on the web: February 29, 2008*

**FAPESP helped in meeting the publication costs of this article.**