Radar channel waveform design in an active communications channel

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RADAR CHANNEL WAVEFORM DESIGN IN AN ACTIVE COMMUNICATIONS CHANNEL

by

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September 2013

Thesis Advisor: Ric Romero
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# RADAR CHANNEL WAVEFORM DESIGN IN AN ACTIVE COMMUNICATIONS CHANNEL

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In this thesis adaptive radar transmit waveform design and its effects on active communications systems are explored. Specifically, design for point targets (i.e., impulse response targets) is investigated. The transmit waveform is optimized in the frequency domain by accounting for the modulation spectrum of the communication system while trying to efficiently use the remaining spectrum. Design aspects of the adaptive waveform are investigated, and an upper bound for the waterfilling variable is presented.

With the use of adaptive radar waveform, it is shown that the symbol error rate (SER) performance of the communication system is minimally affected compared to the SER performance when the system is interfered with by a classical non-adaptive pulsed-radar waveform where severe degradation is evident. Moreover, the detection performance of the adaptive waveform is less impacted by the active communication compared to that of the pulsed radar waveform design. In other words, the radar is able to coexist with a friendly communication system.

waveform design, spectrum sharing, cognitive radar, electronic warfare, spectrum management.
RADAR CHANNEL WAVEFORM DESIGN IN AN ACTIVE COMMUNICATIONS CHANNEL

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ABSTRACT

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<th>Acronym</th>
<th>Definition</th>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>$P_D$</td>
<td>Probability of Detection</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>$P_{FA}$</td>
<td>Probability of False Alarm</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Probability of Symbol Error</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<tr>
<td>Radar</td>
<td>RAdio Detecting And Ranging</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Ratio</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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The arduous task of spectrum management and allocation is an ever increasing problem. The need for systems that are able to coexist on the modern battlefield and share the same portion of the radio frequency (RF) spectrum has grown very rapidly in recent years. On the battlefield today there are many instances where operators experience momentary loss of a communications link due to interference from both friendly and enemy systems.

The traditional radar field has dealt with this problem in the past. Most of the work has been applied to receiver processing where the radar attempts to recognize interference and then subtracts it from the received signal. Some methods such as polarization filters, chirp signals, and even spectrum avoidance make an effort at adjusting the transmit signal to mitigate interference. The problem with these types of systems and their method of noise mitigation is that they do not in any way help a communication signal that operates in the working band of the radar.

To address this problem the use of adaptive radar waveform is proposed. This type of radar recognizes that communications signals are present within the frequency band of the radar and attempts to minimize the use of the communication’s frequency band in a very specific manner. This technique allows for the communications system to continue operating without knowledge that the radar is present in the surrounding spectrum.

The waveform design proposed in this work uses a signal-to-noise ratio (SNR) based method of creating a transmit signal that is referred to as the waterfilling technique. This technique uses the interference spectrum to shape the spectrum of the transmit pulse. The radar’s transmitter waterfills the spectrum as dictated by

\[ \varepsilon_s(f) = T|S(f)|^2 = \max\left(\frac{\sqrt{P_s(f)/R-P_x(f)}}{P_h(f)}, 0\right), \]

where \( \varepsilon_s(f) \) is the final energy spectral density of the radar, \( |S_x(f)|^2 \) is the effective power spectrum of the transmit signal, \( P_s(f) \) is the received thermal noise plus...
interference signal and $P_h(f)$ is the clutter response of the radar that is sometimes present, which are all defined in the frequency domain.

In order for this equation to be valid, the waterfilling variable $1/\lambda$ must be greater than the minimum value of $P_x(f)$ or else the resulting power is zero for all frequencies. The maximum value of $1/\lambda$ found in this thesis is $4\min(P_x(f))$ which is the bound needed for the waterfilling equation to output a spectrum that appropriately places more energy where there is less interference. An example of the waterfilling technique is shown in Figure 1.

![Figure 1. An example of the waterfiling technique.](image-url)

It can be seen in Figure 1 that the radar only places energy in those portions of the spectrum where the interference signal is low. Since the resulting transmit signal is phase
tolerant, it is seen that there are many signals that may satisfy this spectral response. It is sufficient for this research to simply choose one.

This signal is transmitted, and the radar receiver uses a suboptimum detector that has two detection hypotheses given by

\[ H_0: \bar{x} = \bar{q} + \bar{n} \]  
\[ H_1: \bar{x} = \bar{s} + \bar{q} + \bar{n} \]

where \( \bar{x} \) is the received signal vector, \( \bar{s} \) is the transmit signal and is eventually the deterministic radar response of a unit amplitude point target, \( \bar{q} \) is the quadrature phase shift keyed (QPSK) random communication signal, and \( \bar{n} \) is AWGN noise. \( H_0 \) represents the return when no target is present, and \( H_1 \) represents the return when a point target is present.

The probability density functions (PDFs) under the two hypotheses assuming the total interference to be a correlated Gaussian process are given by

\[ p(\bar{x}|H_0) = \frac{1}{\pi^{N} \text{det}(C)} \exp[-\bar{x}^H C^{-1} \bar{x}] \]
\[ p(\bar{x}|H_1) = \frac{1}{\pi^{N} \text{det}(C)} \exp[-(\bar{x} - \bar{s})^H C^{-1} (\bar{x} - \bar{s})] \]

where \( C \) is the correlation matrix of QPSK signal plus thermal noise. We decide \( H_1 \), i.e., a target is present, if

\[ \frac{p(\bar{x}|H_1)}{p(\bar{x}|H_0)} > \gamma \]

which is easily reduced to

\[ \text{Re}(\bar{s}^H C^{-1} \bar{x}) > \gamma'. \]

It can be shown that the approximate theoretical probability of detection \( P_D \) and probability of false alarm \( P_{FA} \) are given by

\[ P_D = Q \left( \frac{\gamma' - \bar{s}^H C^{-1} \bar{s}}{\sqrt{\bar{s}^H C^{-1} \bar{s}^H C^{-1} \bar{s}/2}} \right) \]
\[ P_{FA} = Q \left( \frac{\gamma'}{\sqrt{\bar{s}^H C^{-1} \bar{s}/2}} \right). \]
Solving (9) for $\gamma'$, we get

$$\gamma' = \sqrt{\frac{\hat{s}^H C^{-1} \hat{s}}{2} Q^{-1}(P_{FA})},$$

(10)

and substituting (10) back into (8), we get

$$P_D = Q \left( Q^{-1}(P_{FA}) - \sqrt{d^2} \right)$$

(11)

where the deflection coefficient is given by

$$d^2 = 2\hat{s}^H C^{-1} \hat{s}.$$  

(12)

Performance curves are plotted using (11) for multiple scenarios. These theoretical performance curves are compared against extensive Monte Carlo simulations for the adaptive waveform and also compared against the performance of a legacy pulsed radar waveform. It is shown that the performance is actually better than that predicted by (11) due to the fact that the equations assumed correlated Gaussian interference in calculating $\gamma$ (the detection threshold) when in fact our QPSK interference is a non-Gaussian random information signal.

A clear advantage of the optimum transmit waveform design is that it minimizes the disruptive effect to the friendly communication system. In the case of a system employing QPSK modulation, our goal is to minimize the effect on the symbol error ratio (SER) or probability of correct symbol detection $P_s$. Using Monte Carlo simulations, we produced performance curves for a selected QPSK receiver interfered with by the adaptive radar waveform.

Multiple scenarios are investigated, and the corresponding performance curves are presented. Baseline performance curves of the receiver with only AWGN present as well as the performance with a legacy pulse radar waveform present are presented for comparison. When the adaptive radar’s bandwidth is much larger than the bandwidths of the communications signals, the radar and communication systems suffer no appreciable performance degradation. The more bandwidth that is available to the radar when creating its adaptive waveform, the less performance impact is experienced by the radar and the communications systems. Since the radar potentially does not interfere with the
communications systems, the radar designers are allowed to spread the signal over more of the spectrum to almost eliminate the negative effect on its detection probability. Of course, with each interference signal that is present, the radar loses bandwidth and, therefore, radar range resolution. This is a reason that a radar should be allocated with a large bandwidth which usually occurs in practice.

This study helps to demonstrate the benefits of adaptive radar transmit signal design on friendly communication systems. This work should lead to new system designs that assist in the arduous task of managing the RF spectrum on the modern battlefield. This work may also assist future radar designers in understanding the limitations of the adaptive radar waveform design.
ACKNOWLEDGMENTS

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I. INTRODUCTION

A. BACKGROUND

Radio detecting and ranging (RADAR) systems were first developed in the 1930’s. Their military utility quickly became evident, and the evolution of radar systems was extremely rapid during World War II. Today, the modern battlefield is filled with systems that use this technology. Radar systems are used for the detection and tracking of enemy ships, aircraft, missiles, and mines as well as the tracking and control of friendly ships, aircraft, missiles and various weapons systems. These systems often congest the radio frequency (RF) spectrum when combined with the multiple communication systems on the battlefield.

In addition to these many friendly interferences, there is also interference from systems designed to deny us or our enemy the use of the RF spectrum. The use of these jammers has become even more prevalent with the recent widespread use of improvised explosive devices (IEDs), many of which are activated via RF signal. Any operator on the modern battlefield today can attest that communications may go down when a convoy with turned-on jamming devices is in relative proximity to their antenna. This loss of communications underscores the need for systems to be able to adapt and share portions of the spectrum.

B. OBJECTIVE

The primary difference between traditional radar, one with a pre-decided transmit pulse shape, and a cognitive or adaptive radar is that the adaptive radar looks at the preceding returns and use the information present to make a decision as to what the next transmit pulse should look like. In this manner the adaptive radar tries to make an efficient use of the spectrum at its disposal. This is not exactly the same as the concept of frequency hopping, where a receiver monitors the spectrum and places a pulse in the least noisy segment of the spectrum but rather a continuous reshaping of the pulse within that least noisy band.
It is indeed possible that on the modern battlefield, with numerous transmitters and jammers present, the radar may find a need to use the portion of the spectrum that is assigned to friendly systems. It is not only important that the radar be able to detect targets in the presence of the legacy friendly communication signal but also that it not be disruptive to that same communications signal. In practice total disruption to friendly radio systems is usually encountered when a high-powered radar system is turned on. The design of transmit radar waveform for point targets such that it mitigates the interference effects of an active friendly communication system and vice versa is considered in this thesis.

C. THESIS OUTLINE

This thesis is organized into five chapters as follows. In Chapter II the traditional radar is examined. The radar equation is derived, and the effects of receiver thermal noise are reviewed. Traditional pulse detection is investigated, and the use of a decision statistic to declare targets is discussed. Interference in the form of a quadrature phase-shift keying (QPSK) modulated signal is introduced and an explanation of its negative effects on radar performance is presented. Finally, the effects of the traditional radar on an existing communication system are discussed and a closed form solution of the symbol error ratio (SER) is presented to detail these negative effects.

In Chapter III adaptive radar waveform design is investigated. Waveform design using the signal-to-noise ratio (SNR) metric in the frequency domain is presented and upper and lower bounds for the waterfilling variable are derived. A sub-optimum match filter receiver is presented, and a decision variable based on the probability of false alarm is discussed. A set of performance curves are presented with a QPSK signal present in the center of the radar’s frequency band. Next, the effects of the adaptive radar signal on the existing QPSK communication system are demonstrated. Finally, performance curves for the QPSK receiver are shown.

In Chapter IV multiple radar and communication scenarios are demonstrated. Two categories of radar signal are defined based on the available bandwidth compared to a standard communications signal bandwidth. In each category of radar transmit signal,
two communication scenarios are presented, and performance curves for both the radar and the communication systems are derived via Monte Carlo simulations.

In Chapter V conclusions are made and a summary of the completed work and recommendations for further work in the field of waveform design for adaptive radar in communications environment are presented.

D. POTENTIAL APPLICATIONS/BENEFITS

This study will help to demonstrate the benefits of adaptive radar transmit signal design on friendly communication systems. This work should lead to new system designs that will assist in the arduous task of managing the RF spectrum on the modern battlefield. This work may also assist future radar designers in understanding the limitations of the adaptive radar waveform design explored here.
II. TRADITIONAL RADAR DETECTION

A. THE RADAR EQUATION

This chapter is a review of traditional radar presented in [1]. We begin this chapter by looking at the response to a point target of traditional radar. It is assumed that the radar transmits a pulse of constant frequency and with power $P_T$ and that the target is some range $R$ from the radar. If the antenna is isotropic, meaning the power is spread with spherical symmetry, the signal arriving at the target has a power density of

$$PD = \frac{P_T}{4\pi R^2}.$$  \hspace{1cm} (2.1)

Since the antenna in most radar systems is not isotropic but directional, we describe the antenna as having some gain $G$ over the isotropic power in the direction of the antenna’s main lobe. This gives the transmitted signal a peak power density of

$$PD = \frac{P_T G}{4\pi R^2},$$  \hspace{1cm} (2.2)

as it arrives at a target within the main lobe of the transmit antenna.

Every point target has a radar cross-section which is described by the variable $\sigma$. This radar cross-section is a function of the target’s relative size, the reflectivity of its materials, its orientation to the antenna and the wavelength $\lambda$ of the signal. The radar cross section is different for every target and can even change with time and movement of the target. The important thing to note is that $\sigma$ describes the size of an isotropic reflector that returns the same power as the target. Since this power is described as being reflected isotropically, the power density arriving back at the transmitter can be described as

$$PD = \frac{P_T G \sigma}{(4\pi R^2)^2},$$ \hspace{1cm} (2.3)

Through antenna theory it is known that the effective area of a parabolic reflector antenna is given by

$$A = \frac{G \lambda^2}{4\pi}$$ \hspace{1cm} (2.4)
where the aperture efficiency is $\approx 1[2]$. Multiplying (2.3) by (2.4), we get the radar return power $P_R$ of

$$P_R = \frac{P_T G^2 A^2 \sigma}{(4\pi)^3 R^4}.$$  \hspace{1cm} (2.5)

The result in (2.5) is an important equation as it relates the power of the radar return to the transmit power, gain of the antenna, wavelength of the pulse, radar cross section of the target and the target’s range from the antenna. [1] Of course in practice the transmit signal encounters various atmospheric phenomena that cause further attenuation of the radar return as well as multipath and distortion effects. These effects are outside the scope of this work and are ignored in this thesis.

**B. THERMAL NOISE**

It is important to note that this radar return is always received accompanied by noise. This noise has many sources. The first source, and one that is always present, is thermal noise. Thermal noise in the receiver is primarily due to two causes. The first is the noise created by the random motion of electrons in the resistive components in the receiver. The second is the antenna noise which consists of sky noise created by warm objects such as stars and the surface of the earth that emit radiation proportional to their temperatures.[3]

Due the random nature of the electron motion causing this noise, the power spectral density (PSD) for frequencies $f < 1000$ GHz is very well approximated as a constant given by

$$S(f) = \frac{KT}{2},$$ \hspace{1cm} (2.6)

where $K$ is Boltzmann’s constant and $T$ is the noise temperature of the environment in Kelvin[3]. The total antenna noise temperature is summed by multiplying the thermal noise of each object in the environment and the percent of the field-of-view of the antenna that each object occupies. If the antenna scene consisted of 90 percent sky with a noise temperature $10K$ and 10 percent ground with a noise temperature $290K$, the equivalent antenna noise temperature becomes

$$T_A = 0.9 \times 10 + 0.1 \times 290 = 38K.$$ \hspace{1cm} (2.7)
This equivalent antenna noise temperature then leads to a total antenna noise of

\[ S_A(f) = \frac{KT_A}{2}, \]  
(2.8)

which describes the noise level at the input to the receiver.

This noise is propagated through the receiver. At each component in the receiver the previous noise temperature is added to the noise temperature of that component multiplied by the loss and divided by the gain of each component encountered until we arrive at an equivalent noise temperature \( T_E[3] \). In this manner we get a total thermal noise PSD of

\[ S_E(f) = \frac{KT_E}{2}. \]  
(2.9)

At the input to the detector (an example being the envelope detector described in the next section and depicted in Figure 1), the total noise power is

\[ N = \int_{-B}^{B} S(f) df = \int_{-B}^{B} \frac{KT_E}{2} df = KT_E B, \]  
(2.10)

where \( B \) is the signal bandwidth.

Other sources of noise include the many RF transmitters and electronic devices that either knowingly or unknowingly introduce RF transmissions into the environment. For the purposes of this investigation, only the friendly communications signal in our frequency band of interest is considered. It should be noted, however, that the framework and results of this study can be applied to these RF interferers.

C. PULSE DETECTION

The receiver tries to detect the radar return in the presence of noise via the detection circuit. A basic detection circuit is shown in Figure 1. This circuit consists of a bandpass filter (BPF), an envelope detector to extract the signal from the noise and, finally, a threshold detector. An envelope detector is chosen to simplify the detector such that a circuit to receive the pulse coherently is not needed. Here the output of the envelope detector is compared against some predetermined threshold voltage \( V_T \). Whenever the threshold voltage is exceeded, a target is declared.
In (2.5) it can be seen that the power of the radar return drops as the target’s distance from the antenna increases. This along with other factors causes a reduction in the SNR, and some target returns do not meet the threshold for detection, especially at longer ranges. These targets go undetected leading to a probability of detection $P_D$ that is always less than one.

Another phenomenon that occurs is when a noise realization surpasses the threshold voltage, causing detection where no target is present. This is called a false alarm. Anytime the possibility for a false alarm exists, i.e., there is noise, interference or clutter present, the probability of false alarm $P_{FA}$ is greater than zero.

Lowering the detection voltage increases the probability of detection but increases the $P_{FA}$. Another method to increase the $P_D$ is to increase the SNR. Consequently, there is a three-way relationship between the $P_D$, $P_{FA}$, and SNR.

To develop this relationship, we start by looking at additive white Gaussian noise (AWGN). When passed through a narrow BPF, the output can be described by

$$n_0(t) = X(t) \cos(\omega_c t) + Y(t) \sin(\omega_c t)$$  

(2.11)

where $\omega_c$ is the center frequency of the BPF and $X(t)$ and $Y(t)$ are two Gaussian independent random variables with zero mean and variance of $\sigma_n^2$ [1].

The target signal, which is a convolution of the transmit signal with an impulse, is a sine wave with center frequency $\omega_c$ and is described by

$$s_0(t) = a \cos(\omega_c t) + b \sin(\omega_c t),$$  

(2.12)

where $\sqrt{a^2 + b^2}$ is the magnitude of the return signal at the receiver. These two signals are added together at the input to the receiver and become
\[ e_0(t) = n_0(t) + s_0(t) = [a + X(t)] \cos(\omega_c t) + [b + Y(t)] \sin(\omega_c t). \] (2.13)

This combined signal \( e_0(t) \) when passed through a linear envelope detector yields
\[ r(t) = [X_1^2(t) + Y_1^2(t)]^{1/2} \] (2.14)

where
\[ X_1(t) = a + X(t) \]
\[ Y_1(t) = b + Y(t). \] (2.15)

It is now easily seen that \( X_1 \) and \( Y_1 \) are now independent Gaussian random variables with means of \( a \) and \( b \), respectively, and variance \( \sigma_n^2 \). These two Gaussian random variables can now be described by their respective probability density functions (PDFs)
\[ p_1(X_1) = \frac{1}{\beta(2\pi)^{1/2}} \exp\left(-\frac{(X_1 - a)^2}{2\beta^2}\right) \] (2.16)
\[ p_2(Y_1) = \frac{1}{\beta(2\pi)^{1/2}} \exp\left(-\frac{(Y_1 - b)^2}{2\beta^2}\right) \] (2.17)

where
\[ \beta = \left[n_0^2(t)\right]^{1/2}. \] (2.18)

Here we see that the standard deviation of \( X_1 \) and \( Y_1 \) are equal to the RMS value of the noise \( n_0(t) \) [1].

For example we start by looking at the PDFs of the signal amplitude of bandpass filtered AWGN and a signal plus noise with SNR = 8.0. In Figure 2 the detection threshold voltage \( V_t \) that corresponds to a \( P_{FA} \) of 0.01 is presented. The \( P_D \) is the area under the target return PDF that is above the threshold voltage, depicted in green. The \( P_{FA} \) is the area under the noise PDF that is above the threshold voltage, depicted in red. It is seen that moving the threshold voltage to the left increases the \( P_D \) but also increases the \( P_{FA} \). The alternate method mentioned above of increasing the SNR moves the target return PDF to the right. This increases the \( P_D \) while having no effect on the \( P_{FA} \).
Figure 2. PDF of AWGN noise and a target return with a SNR of 8.

Figure 3. PDF of noise and return signals with SNR ranging from 1 to 12.
In Figure 3 the target present PDFs are shown with SNR varying from 1 to 12. The $V_t$ selected for this diagram gives a $P_{FA}$ of 0.01. It can be seen that with a SNR = 4.0 the $P_D$ is approximately only 50 percent; however, when the SNR reaches 12, the $P_D$ is 98 percent. This diagram leads to the question of how much signal power is required for acceptable detection of a target at a given $V_t$ or $P_{FA}$.

![Required SNR vs. detection probability for single pulse detection.](image)

Figure 4. Required SNR vs. detection probability for single pulse detection.

From the extensive Monte Carlo simulations performed we are able to plot $P_D$ versus SNR for various $P_{FA}$s. In Figure 4 a required SNR can be determined for a given $P_{FA}$ and $P_D$ requirement. It should be noted that these comparisons are for single pulse detection and do not take into consideration the effects of integration of multiple returns from the same target [1].
D. INTERFERENCE

After presenting various $P_D$ versus SNR curves for single pulse detection in thermal noise environment, it is important to realize that most radar systems do not operate in a thermal-noise only environment. In fact, as discussed in the introduction there are multiple systems on the battlefield emitting RF energy, and they add to the thermal noise of the radar receiver. Some of these systems are jammers (both friendly and enemy) designed to deny the use of certain portions of the spectrum. For this research we assume that the radar system has knowledge of the spectra of interference and is able to use the least noisy portion of the spectrum. Given the growing number of RF denial systems and systems designed for friendly communications, it is proposed that the radar system produce a waveform such that a friendly communication signal is less affected spectrally.

In this work we assume that a friendly communications system employing a QPSK modulation technique occupies some portion of the available spectrum. This signal is defined in complex baseband notation as

$$q(t) = V_s \cos \varphi(t) + iV_s \sin \varphi(t)$$

(2.19)

where $\varphi(t)$ represents the phase of the QPSK signal and $V_s^2$ represents the average power of the signal. The noise is be defined as

$$n(t) = \sqrt{\frac{P_n}{2}} X(t) + i \sqrt{\frac{P_n}{2}} Y(t)$$

(2.20)

where $X(t)$ and $Y(t)$ are normally distributed random variables with a zero mean and a standard deviation of one, and $P_n$ is the power of the noise. The radar signal is defined as

$$r(t) = \sqrt{P_r} e^{-i\frac{\pi}{4}t}$$

(2.21)

where $r(t)$ is the radar return and $P_r$ is the power of the radar return. The radar receiver has to detect the radar return $r(t)$ in the presence of $q(t)$ and $n(t)$. To help visualize this problem, an illustration of the signal space (i.e., phase constellation) is provided.
In Figure 5 the magnitude of the radar threshold voltage $V_t$ is depicted in the complex plain as a green circle around the origin of the signal space graphs. The cloudy spread of the signal constellations are noise realization vectors due to thermal noise with a SNR of 10 dB. In Figure 5A the QPSK signal and noise are mostly outside of the detection circle, and these returns are detected as targets. This is the signal space representation when no targets are present which “jams” the radar by causing it to continuously detect targets that are not present. In Figure 5B the result of a radar return with the same power as the QPSK signal is shown. The only difference between the radar return and the QPSK return is that the phase of the radar return is centered on $\pi/4$ since this is the phase of the transmit signal and is also assumed to be the phase of the radar return. In practice the initial phase of the radar return may be random. In Figure 5C the signal space when these two signals are added together is shown. Here it is seen that the QPSK signal has caused nearly $1/4$ of the returns to go undetected, namely, whenever
the QPSK phase is $5\pi/4$. When this occurs the two signals are in opposite directions and add to zero, leaving the constellation centered at the origin.

These effects can also be seen by looking at the PDFs of the magnitude of the two signals. Here a Monte Carlo simulation is performed as opposed to the theoretical curves in Figure 2.

![Figure 6](image)

Figure 6. (A): PDF of magnitude of AWGN. (B): PDF of magnitude of QPSK signal in AWGN. (C): PDF of magnitude of target return in AWGN. (D): PDF of magnitude of target return plus QPSK in AWGN.

In Figure 6A the PDF of the magnitude of AWGN is demonstrated, which confirms the noise only representation in Figure 2. Since the PDFs show only the magnitude of the signal and not the phase, the QPSK signal in Figure 6B has an identical PDF to that of the radar return in AWGN shown in Figure 6C. This is because the radar signal and interference are both given a SNR of 10 dB in this simulation. What is most interesting is
the addition of the QPSK and target return shown in Figure 6D. Here we see the effects of the differing phases of the QPSK signal since they result in different magnitudes when added to the target return of constant phase. The first relative maximum in this PDF is due to the portion of the QPSK signal that is opposite in phase to the radar signal. The second relative maximum is created by the two QPSK signals that are perpendicular in phase to the radar signal, and finally, the third relative maximum is caused by the portion of the QPSK signal that is in the same phase as the radar signal.

From these diagrams it can be seen that there is no good place to choose a $V_t$ that yields sufficient $P_D$ while maintaining an acceptable $P_{FA}$. If $V_t$ is at the original level, approximately 2.15, it can be seen in Figure 6B that the $P_{FA}$ suffers dramatically since no target is present but 90 percent of the return is to the right of this level. In order to maintain the $P_{FA}$ of 0.01, $V_t$ must be moved out to approximately 5.7. It can also be seen from the PDF in Figure 6D that the $P_D$ suffers dramatically as well. With the $V_t$ set for 0.01 $P_{FA}$, the $P_D$ is approximately 0.5. In this case we may say that the radar is successfully jammed.

It should be noted that the traditional radar field has dealt with this problem in the past. Most of the work has been applied to receiver processing where the radar attempts to recognize interference and then subtract it from the received signal. Some methods such as polarization filters, chirp signals, and even spectrum avoidance have made an effort at adjusting the transmit signal to mitigate interference. The problem with these types of systems and their method of noise mitigation is that they do not in any way help a communication signal that ends up in the band of the radar.

E. EFFECTS OF THE RADAR ON THE COMMUNICATION SYSTEM

The effects of traditional radar on a communication system are even more pronounced. Experience shows that when a nearby radar dish points in the direction of a radio antenna, the communications link is usually lost. This experience is commonly noted by helicopter pilots as they fly to a ship. When the ship’s radar turns in their direction, the squelch breaks and the radio data become momentarily unreadable. Then as
soon as it turns away, the radio goes back to working order. This phenomenon is easily explained by the broadband pulse sent out by the radar that saturates the radio receiver.

We begin the analysis of the communications receiver’s performance with the well-known result that the standard QPSK receiver has a SER of

\[ P_s = 2Q \left( \frac{E_s}{N_0} \right), \]  

(2.22)

where \( E_s \) is the energy of a symbol and \( N_0 \) is the noise density at the receiver [3]. The \( Q(X) \) function used here is the Gaussian integral function that defines the area under a Gaussian distribution function from \( X \) to infinity. This SER equation is the starting point for the communication engineers who develop legacy QPSK communications systems. In this situation it is assumed that the bandwidth of the radar \( W_R \) is much larger than the bandwidth of the bandwidth of the QPSK receiver \( W_q \). It may appear that in the band of the communications receiver the spectrum of the radar signal is flat because of this relationship. Since it is really not flat, take a maximum of the radar spectral density and call it \( N_i \). For the purpose of finding an approximate expression for the SER or \( P_s \) in the presence of the radar signal, assume for a moment that the radar spectral density is indeed flat with a value of \( N_i \). If the radar signal is random (Guassian), then the approximate SER for the QPSK receiver is

\[ P_s = 2Q \left( \frac{E_s}{N_0 + N_i} \right), \]  

(2.23)

which is the worst case approximation given that the PSD of the radar signal is not actually flat. From (2.23) it can be seen that as the power of the interference increases, in this case the radar signal, the argument of the Q function decreases and the resulting \( P_s \) increases. For numerical analysis consider the case where \( E_s \) is twice \( N_0 \) and equal to \( N_i \). The SER prior to the radar signal being present is

\[ P_s = 2Q \left( \frac{E_s}{N_0} \right) = 2Q \left( \frac{2N_0}{N_0} \right) = 2Q(\sqrt{2}) = 0.157. \]  

(2.24)

With the radar signal present, the SER becomes
\[ P_s = 2Q \left( \sqrt{\frac{E_s}{N_0 + N_i}} \right) = 2Q \left( \sqrt{\frac{2N_0}{3N_0}} \right) = 2Q \left( \frac{2}{3} \right) = 0.414. \quad (2.25) \]

In this analysis the SER went from 16 to 41 percent.

In practice typical communications and radar signals both have much higher power than 3 dB above the noise level to allow for better \( P_D \) and \( P_s \). A good rule of thumb for a communications system employing basic modulations such as QPSK is a SNR \((E_s/N_0)\) of 13 dB. This results in an approximate SER of \(1.0 \times 10^{-5}\). If more stringent SER is required, then higher SNR maybe required or error correction coding and other methods can be used to achieve reliable communications. Radar signal power is usually higher and is often stronger than the communications signal being received; in other words, \(N_i\) can be much larger than \(N_0\). Thus, (2.23) becomes

\[ P_s \approx 2Q \left( \sqrt{\frac{E_s}{N_i}} \right). \quad (2.26) \]

If the radar signal is in fact much stronger than the communications signal, the argument of the \(Q\)-function becomes much less than one resulting in a \(P_s\) that approaches 0.5, effectively jamming reliable communications.

The radar usually does not transmit continuously. In fact the ratio of the radar’s transmitted pulse width to pulse repetition time, a quantity known as the duty cycle \(d_c\), is typically between 0.001 and 0.03 for most pulsed radar systems [1]. The approximate symbol error rate for the communication system becomes

\[ P_s \approx (1 - d_c)2Q \left( \sqrt{\frac{E_s}{N_0}} \right) + d_c2Q \left( \sqrt{\frac{E_s}{N_i}} \right). \quad (2.27) \]

Since the SER in the first part of (2.27) is the design requirement of the communication system and the SER in the second part results in unreliable communications, the resulting probability of symbol error is driven more by the duty cycle of the radar than by the signal levels. The probability of symbol error effectively becomes

\[ P_s \approx (1 - d_c)(1 \times 10^{-5}) + d_c(0.5), \quad (2.28) \]
and since the typical duty cycle is greater than $1 \times 10^{-3}$, the resulting SER is now two orders of magnitude too high. The design goal for the adaptive radar discussed in the next chapter is to decrease the SER during the transmit portion of the duty cycle to a level that still allows for effective communications.
III. ADAPTIVE RADAR

A. WAVEFORM DESIGN

To begin the analysis of the waveform design we first note that there is an initial period when the radar receiver is turned on prior to the transmission of the radar pulse. At this point the received signal is represented by

\[ x(t) = q(t) + n(t) \]  \hspace{1cm} (3.1)

where \( q(t) \) is the time domain representation of the friendly communications signal and \( n(t) \) represents AWGN noise of the receiver. In the frequency domain this becomes

\[ X(f) = Q(f) + N(f). \]  \hspace{1cm} (3.2)

In order to find an effective transmit signal, a SNR-based method is used. This method is referred to as the “waterfilling” technique and has its roots in information theory [4]. This technique examines the interference spectrum and places the energy of the transmitted signal where the total interference is spectrally low. To execute this technique the radar transmitter starts with the energy or power constraint, which is the amount of energy that the radar can place in one transmit pulse. The effective power is given by

\[ \varepsilon = \int_{-w/2}^{w/2} |S(f)|^2 df \]  \hspace{1cm} (3.3)

where \( S(f) \) is the time normalized Fourier transform of the transmit waveform \( s(t) \). Of course, the equivalent energy constraint is nothing but power multiplied by the time support. Consequently, the transmitter then waterfills the energy spectrum as dictated by

\[ \varepsilon_s(f) = T|S(F)|^2 = \max \left( \frac{\sqrt{P_x(f)/\lambda} - P_x(f)}{P_h(f)}, 0 \right) \]  \hspace{1cm} (3.4)

where \( P_h(f) \) represents the PSD of the clutter response which is sometimes present (e.g., ground-looking radar) and \( P_x(f) \) represents the PSD of the interference plus noise. The transmitter must then solve for \( \lambda \) that satisfies the energy constraint of the radar transmitter and divide by \( T \) to arrive at the optimal transmit PSD \( |S(f)|^2 \) [4].
It should be noted that (3.4) is only valid within certain conditions. First from the numerator of (3.4) it can be seen that

$$\sqrt{P_x(f)/\lambda} > P_x(f)$$

(3.5)

for the output to remain above zero; otherwise, (3.4) results in zero. By squaring both sides and dividing by $P_x(f)$ (3.5) becomes

$$\frac{1}{\lambda} > P_x(f),$$

(3.6)

which is a result noted in [4]. Since $P_x(f)$ is the PSD of interference plus noise, its minimum occurs at or above the noise only value of $N_0$ where we assume $P_n(f) = N_0$. Therefore, the minimum value of $1/\lambda$ in order for the equation to have an output is $N_0$.

The upper bound for $1/\lambda$ is much more difficult to see. For the result to be valid and to waterfill the spectrum due to $P_x(f)$, the output of the equation must be monotonically decreasing with increasing values for $P_x(f)$ until the output reaches zero, at which point the maximum function of (3.4) outputs zero for further increases in $P_x(f)$.

For ease of analysis let $A = 1/\lambda$, $B = P_h(f)$ and $X = P_x(f)$. The non-zero part of (3.4) becomes

$$\frac{\sqrt{AX} - X}{B}$$

(3.7)

where it is now easier to determine the slope as a function of $X$. To this end the first derivative of the function with respect to $X$ is given by

$$\frac{\sqrt{A}}{2B\sqrt{X}} - \frac{1}{B}$$

(3.8)

Given a fixed value for $A$ and $B$, it can be immediately determined that as $X$ approaches zero from the right the slope of this function is positive and approaching infinity as dictated by the square root of $X$ in the denominator of the derivative. Since (3.4) needs to be applied in portions where its output values are monotonically decreasing, the variables $1/\lambda$ and $P_h(f)$ must be such that the first derivative remains negative throughout the range of $P_x(f)$. To insure this we set the first derivative less than zero, yielding
\[ \frac{\sqrt{A}}{2B\sqrt{X}} < \frac{1}{B} \]  

which can be simplified to the result

\[ \frac{A}{4} < X. \]  

(3.10)

Substituting the original variables back into (3.10) indicates that (3.4) decreases with increasing \( P_x(f) \) as long as

\[ 4 \min P_x(f) > \frac{1}{\lambda}. \]  

(3.11)

Therefore, the bounds of the waterfilling variable are determined to be

\[ \min P_x(f) < \frac{1}{\lambda} < 4 \min P_x(f). \]  

(3.12)

Figure 7. Demonstration of waterfill technique.
In Figure 7 it can be seen how the function is waterfilled to create $\varepsilon_s(f)$ along with the maximum and minimum values of $1/\lambda$. From this illustration it is easy to see that nothing is waterfilled in $\varepsilon_s(f)$ if $1/\lambda$ is chosen to be below $\min P_x(f)$.

In Figure 8 we see an actual example of what happens when the waterfilling variable $1/\lambda$ becomes greater than $4 \min P_x(f)$.

Figure 8. An example of a “waterfilled” output when $1/\lambda$ limits are not followed. (A): The QPSK signal with $1/2$ the bandwidth of the radar. (B): The resulting radar spectrum using the waterfilling technique.

For this example the values of $P_x(f)$, $P_h(f)$ and $\varepsilon$ are chosen such that the waterfilling variable $1/\lambda$ is 14.85 to complete the waterfilling procedure. For convenience we set noise PSD to one making the min value of $P_x(f)$ equal to one, which is much less than $14.85/4$. Consequently, for all values of $P_x(f) < 3.71$ the output is flipped, placing more energy in the transmit spectrum at frequencies where there is
already more energy in the original interference signal. Once $P_x(f)$ reaches and exceeds 3.71 this effect is reversed and less energy is placed at frequencies with increasing values of $P_x(f)$ as seen in Figure 8. If $1/\lambda$ is chosen such that $1/\lambda > 4 \max P_x(f)$, then the entire spectrum is filled incorrectly, and the radar places most of its power in the middle of the communications spectrum causing negative results for both systems.

Consider a friendly communications system using a QPSK modulation whose spectrum is shown in Figure 9A where AWGN noise PSD is added. From (3.4), the optimum transmit power spectrum can be calculated. The resulting power spectrum of the transmit pulse is shown in Figure 9B. It is clear that the peaks of this waveform occur at the nulls of the QPSK signal, indicating that the waterfilling variable is correctly chosen. It can also be seen that there could be many waveforms that fit the spectrum since the optimal spectrum is phase tolerant. For the simulations in this work, it is sufficient to choose one realization of the many possible.

Figure 9. (A): PSD of QPSK with 1/5 the bandwidth of radar signal with AWGN noise PSD added. (B): PSD of resulting radar transmit signal.
B. RADAR RECEIVER

Given a realization of the transmit waveform, the two detection hypotheses are given by

\[ H_0: x(t) = s(t) \ast h(t) + q(t) + n(t) \]  \hspace{1cm} (3.13)

\[ H_1: x(t) = As(t) + s(t) \ast h(t) + q(t) + n(t) \]  \hspace{1cm} (3.14)

where \( x(t) \) is the received signal, \( s(t) \) is the transmitted signal, \( As(t) \) is the deterministic radar response of a point target, \( h(t) \) is the clutter response, the convolution \( s(t) \ast h(t) \) is the clutter echo, \( q(t) \) is the QPSK random communication signal, and \( n(t) \) is AWGN noise. For a point target \( A \) is defined to be an amplitude such that the radar response becomes simply a scaled copy of the transmit signal \( s(t) \). The symbol \( H_0 \) represents the return when no target is present, and \( H_1 \) represents the return when a point target is present. In this application, emphasis is given to the effect of QPSK interference rather than signal-dependent clutter. Thus, it is assumed that \( P_x(f) \gg P_h(f) \) in the frequency spectrum, and it is assumed \( h(t) \) is sufficiently small in (3.13) and (3.14). Moreover, a convenient transition to discrete-time signal model is made. Of course, proper time sampling as dictated by the Nyquist sampling theorem must be maintained. We assume unit target amplitude for convenience and simulation purposes, i.e., \( A = 1 \). Thus, the detection hypothesis are

\[ H_0: \tilde{x} = \tilde{q} + \tilde{n} \]  \hspace{1cm} (3.15)

\[ H_1: \tilde{x} = \tilde{s} + \tilde{q} + \tilde{n}. \]  \hspace{1cm} (3.16)

If the total interference is assumed to be Gaussian, then a generalized match filter detector is presented in [5]. The optimum detector for this problem incorporates the fact that the QPSK interference is random (i.e., four phases) and, thus, the proper distribution. The proper distribution may also be a function of the relative received timing between the radar pulse and random interference symbol. In other words, the optimum detector is difficult to derive due to the addition of non-Gaussian interference to the additive white Gaussian noise of the receiver. Here, a suboptimal detector is proposed. Note that even though the friendly QPSK interference is not Gaussian, it is nonetheless a random signal whose autocorrelation function can easily be calculated. The total interference is
temporarily assumed to be Gaussian such that the generalized matched filter detector for correlated Gaussian noise can be used.

The detection performance can be calculated for the purposes of generating interim performance curves (via theoretical calculations) since the correlation matrix of the QPSK signal $C_q$ is known. The correlation matrix $C_q$ is given by the matrix with the primary diagonal equal to $\sigma_q^2$ and each successive diagonal decreasing by $\sigma_q^2 / N$, where $N$ is the number of radar samples in a single QPSK symbol. Thus, the interference correlation matrix is given by

$$C_q = \begin{bmatrix}
\sigma_q^2 & \sigma_q^2 - \frac{1}{N} \sigma_q^2 & \sigma_q^2 - \frac{N-1}{N} \sigma_q^2 & 0 \\
\sigma_q^2 - \frac{1}{N} \sigma_q^2 & \sigma_q^2 & \sigma_q^2 - \frac{N-2}{N} \sigma_q^2 & \sigma_q^2 - \frac{N-1}{N} \sigma_q^2 \\
\sigma_q^2 - \frac{N-1}{N} \sigma_q^2 & \sigma_q^2 - \frac{N-2}{N} \sigma_q^2 & \sigma_q^2 & \sigma_q^2 - \frac{1}{N} \sigma_q^2 \\
0 & \sigma_q^2 - \frac{N-1}{N} \sigma_q^2 & \sigma_q^2 - \frac{N-2}{N} \sigma_q^2 & \sigma_q^2 \\
\end{bmatrix}. \quad (3.17)$$

Now let $C$ be the correlation matrix of interference plus noise. Since the correlation matrix of AWGN noise is $C_n = \sigma_q^2 I$, then the correlation matrix is given by

$$C = C_n + C_q. \quad (3.18)$$

To produce more accurate performance curves, Monte Carlo simulations are performed and compared to the theoretical results. We start by looking at the PDFs of the two hypotheses assuming the total interference to be a correlated Gaussian process which are given by

$$p(\bar{x}|H_0) = \frac{1}{\pi^N \text{det}(C)} \exp[-\bar{x}^H C^{-1} \bar{x}] \quad (3.19)$$

$$p(\bar{x}|H_1) = \frac{1}{\pi^N \text{det}(C)} \exp[-(\bar{x} - \bar{s})^H C^{-1} (\bar{x} - \bar{s})] \quad (3.20)$$

where $H$ is the conjugate transpose. We decide $H_1$, i.e., a target is present, if

$$\frac{p(\bar{x}|H_1)}{p(\bar{x}|H_0)} > \gamma \quad (3.21)$$

which is easily reduced to [5]
It can be shown that the approximate theoretical probability of detection \( P_D \) and probability of false alarm \( P_{FA} \) are given by

\[
P_D = Q\left(\frac{\gamma' - \hat{s}^H C^{-1} \hat{s}}{\sqrt{\hat{s}^H C^{-1} \hat{s}/2}}\right)
\]

(3.23)

\[
P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\hat{s}^H C^{-1} \hat{s}/2}}\right).
\]

(3.24)

Solving (3.24) for \( \gamma' \), we get

\[
\gamma' = \frac{\sqrt{\hat{s}^H C^{-1} \hat{s}}}{2} Q^{-1}(P_{FA}).
\]

(3.25)

and substituting (3.25) back into (3.23), we get

\[
P_D = Q \left( Q^{-1}(P_{FA}) - \sqrt{d^2} \right)
\]

(3.26)

where the deflection coefficient is given by [5]

\[
d^2 = 2\hat{s}^H C^{-1} \hat{s}.
\]

(3.27)

In Figure 10 the theoretical detection curves for the radar based on (3.26) are shown. Due to the fact that we assumed the total interference to be Gaussian, (3.26) is only a theoretical approximation. To account for the fact that the total interference is the sum of non-Gaussian QPSK random symbols and Gaussian receiver noise, extensive Monte Carlo experiments were performed to produce more accurate detection curves. The detection curves are close, albeit the approximation curves are clearly pessimistic results compared to the actual Monte Carlo results. In this simulation the SIR is the ratio of the radar signal power to the interference power of the QPSK signal. It should now be clear that the performance differences are attributed to the fact that correlated Gaussian interference was assumed in calculating \( \gamma \) (the detection threshold) when in fact our QPSK interference is a non-Gaussian random information signal. In performing Monte Carlo simulations, \( \gamma \) is adjusted manually until the desired \( P_{FA} \) is attained.
Figure 10. Probability of detection versus SIR for each selected probability of false alarm. Dotted lines indicate performance due to traditional pulsed radar response, dashed lines are the theoretical predictions from (3.26) and the solid lines are the demonstrated performance of the optimum spectrum waveform.

Finally, the detection performance of the optimum transmit waveform can be compared to the detection performance of a single-pulse radar with QPSK interference present. The goal of the optimum transmit waveform is to mitigate the effect of the interference (QPSK in this case). In Figure 10 its actual performance is shown with solid lines and is better than the theoretical results shown as dashed lines. There is also a marked improvement in detection performance over that of a single-pulse radar shown as dotted lines. For example with a selected $P_{FA}$ of 0.01 and a symbol-to-interference ratio (SIR) of 3 dB, we get an improvement in the probability of detection from 0.12 with the pulsed waveform to 0.40 when the optimum spectrum waveform is used, which shows a three to one improvement in detection.

Of course this improved detectability comes at some expense. As illustrated in Figure 9, the radar is not using the entire bandwidth at its disposal. This loss of bandwidth results in lower radar resolution sensitivity [1]. Of course for maximum resolution the radar has to use the entire bandwidth. If the bandwidth of the radar is large
enough, then the loss of bandwidth still allows for acceptable resolution sensitivity. The compromise mentioned above is better than increasing the $P_{FA}$ to improve $P_D$ as with the pulse radar case.

C. EXISTING COMMUNICATION SYSTEM PERFORMANCE

Another advantage of the optimum transmit waveform design is that it effectively mitigates the disruptive effect to the friendly communication system. In the case of a system employing QPSK modulation, our goal is to minimize the effect on the SER or probability of correct symbol detection $P_s$. The idea is not to make any system changes (software or hardware) to the legacy communication system; i.e., the communications system is unaware of the radar's beneficial waveform design. This means the communications receiver uses the matched filter detector for QPSK without any additional signal processing. With the use of optimum transmit radar waveform, the result is the mitigation of effect on $P_s$ performance.

In Figure 9 we see that the radar puts very little energy in the central part of the band for the communications receiver, allowing the receiver to perform much better than when the traditional pulse waveform is used. In fact when the radar is able to avoid the band of the communication receiver (almost entirely in this case), the performance of the communications receiver is not appreciably affected at all.

In Figure 11 the correct symbol probability $P_s$ of a legacy QPSK system is shown. Here a SNR of 3 dB for the QPSK signal is selected for performance discussion. The red line indicates the performance of the system when no radar signal is present. A pulse radar signal is then introduced, and the probability of detecting the correct symbol with radar to QPSK power ratios ranging from -15 dB to +20 dB is simulated. As depicted by the blue line, the pulse has a disruptive effect on the communication system at about the -5 dB radar pulse to QPSK signal power ratio. Next, the performance of the un-altered QPSK detector is shown when the spectral shaped transmit waveform from the adaptive radar is used. From the green plot it can be seen that this waveform does not begin to interfere with the communications detector until the 10 dB radar pulse to QPSK signal
power ratio is reached. This is a marked improvement of the communication detector's ability to correctly demodulate the communication signal. In other words, this improvement comes with no adjustments to the legacy communication system but is simply a result of our optimum shaped radar pulse.

![Graph showing SER performance curves](image)

**Figure 11.** Demonstrated performance of friendly communications system employing QPSK in presence of radar interference.

In Figure 12 the SER performance curves of a QPSK receiver when the radar signal is present are shown. Here we can compare the SER performances when the shaped waveform signal is present and when traditional pulse radar transmit signal is present. To measure the effectiveness of the shaped transmit waveform in minimizing its effect on the communications systems, we also plot the ideal SER for the scenario where there is only white noise, i.e., no radar present as the baseline performance. Clearly, if the SER corresponding to the shaped waveform is close to the former then the shaped waveform is highly effective. In Figure 12 the radar power to noise ratio is 9 dB. Baseline performance without a radar signal present is depicted in green for comparison.
As can be seen from the Figure 12, the communication receiver works very well as if no radar transmit signal is present with even a 9 dB radar signal-to-noise ratio provided the radar is using the optimum shaped transmit pulse. In contrast, the communications receiver’s performance is severely limited if a traditional pulse transmit waveform is used as depicted in red.

![Figure 12](image)

Figure 12. Performance curves for QPSK receiver with a 9 dB radar-to-noise interference present.
IV. MULTIPLE RADAR SCENARIOS

For further analysis of the adaptive radar performance in a communications channel, we now look at its performance and the performance of the communications receivers in various conditions.

A. WIDEBAND RADAR

For the purposes of this research wideband radar is defined as a radar that has a transmit spectrum at its disposal that is at least ten times the null to null bandwidth of the communications signal in its frequency range.

1. Single Communication Signal Present

The first radar signal considered is a wideband signal with 20 times the available bandwidth of a single communications interferer in its spectrum. The spectrum of the QPSK interferer is shown in Figure 13A with a QPSK-to-noise ratio of 10 dB. In Figure 13B the radar is able to waterfill the spectrum of the resulting transmit signal using a 40 dB SNR without putting energy in 95 percent of the spectrum used by the communications receiver. As expected from earlier results, this situation allows the two systems to perform almost as if the other signal is not present as seen in Figure 14 and Figure 15.

In Figure 14 the performance of the radar when the QPSK interference is 10 dB above thermal noise level is depicted. With a $P_{FA}$ of $10^{-3}$, the performance of the radar with a 3 dB SIR improves from a $P_D$ of 0.09 to 0.78 by using the adaptive radar waveform. This is a marked improvement over the pulse radar case.

In Figure 15 the performance curves of the QPSK receiver are depicted for this scenario. The performance curve for the legacy system with the shaped radar signal present very closely follows the theoretical predictions of (2.22) where no interference is present. This result indicates that the communication system works as if no radar is present even during the radar transmit time.
The clutter response $P_h(f)$ also plays an important role in deciding the shape of the spectrum. In Figure 16 we see what happens when $P_h(f)$ is decreased, causing (3.4) to fill the spectrum quicker and, consequently, be more sensitive to the values of $P_x(f)$. Since the communications signal is mostly unaffected by what happens in these side lobes and the radar power is usually many orders of magnitude stronger, it is beneficial for simulation to set the $P_h(f)$ so that this ringing does not occur too far past the main lobe of the communications signal. In reality, we do not have control over $P_h(f)$ and such ringing may occur.

Figure 13. (A): PSD of QPSK with bandwidth of $1/20^{th}$ of the available radar bandwidth and AWGN. (B): PSD of resulting radar transmit signal.
Figure 14. Probability of detection versus SIR for each selected probability of false alarm for wideband radar with one interferer. Dotted lines indicate performance due to traditional pulsed radar response, dashed lines are the theoretical predictions from (3.26) and the solid lines are the demonstrated performance of the optimum spectrum waveform.

Figure 15. Performance curves for QPSK receiver with 9 dB interference-to-noise ratio for wideband radar with one interferer.
Figure 16. (A): The same QPSK signal used in Figure 13. (B): The resulting waterfill with a lower value of $P_h(f)$.

2. Multiple Communication Signals Present

Now we look to a much more likely scenario in that there are multiple communication signals present in the spectrum of the wideband radar pulse as shown in Figure 17. In this simulation the radar’s available bandwidth is still 20 times that of the communications systems; however, now we assume that there are nine communications signals present in that bandwidth. The waterfilling $P_h(f)$ is set to a level that still allows the radar to effectively use the remaining spectrum without causing undue interference for the communications systems.
In Figure 17 it can be seen that the radar places its energy in the gaps between the communications signals and, therefore, once again the two systems effectively do not interfere with one another.

Since the radar’s matched filter effectively suppresses energy in the bands of the communication signals, the communications signals have little effect on the radar’s ability to detect targets. It is evident in Figure 18 that the performance curves for the pulse radar (shown here with dotted lines) suffer dramatically as the threshold voltage is adjusted to account for the increased interference. An effect that is also evident in this scenario from Figure 17 is that the loss of bandwidth which is the single biggest cost of the adaptive radar waveform.
Figure 18. Probability of detection versus SIR for each selected probability of false alarm for wideband radar with nine interferers. Dotted lines indicate performance due to traditional pulsed radar response, dashed lines are the theoretical predictions from (3.26) and the solid lines are the demonstrated performance of the optimum spectrum waveform.

It should be noted in Figure 17 that the radar waveform starts to place some of its energy in the main lobes of the nine communication signals. For simulation purposes only one of the nine communications signals is checked for SER in Figure 19, and the performance curve presented is for that one system. This analysis is sufficient since we can see in Figure 17 that the interference is relatively the same for each of the communications channels. The fact that more radar waveform energy is now in the main lobe of each communication system makes performance of a communications system suffer slightly at lower signal-to-noise ratios. Once SNR reaches a level of greater than 10 dB, the performance is once again on par with only having AWGN present as shown by the green line.
Figure 19. Performance curves for QPSK receiver with 9 dB interference to noise ratio for wideband radar with nine interferers.

B. NARROWBAND RADAR

1. Multiple Communication Signals Equally Spaced

In Figure 20 we see that the multiple communications signals are equally spaced and their null-to-null bandwidths almost collectively fill the spectrum of the radar. When the waterfilling variable $1/\lambda$ is relatively small, the radar is still able to place the energy away from the center frequency of the communications signals. As the radar reaches higher power, it requires a higher value of $1/\lambda$ to complete the waterfilling procedure and, consequently, fills more and more of the spectrum of the communications system with energy.
Figure 20. (A): PSD of multiple QPSK signals with bandwidth of $1/5$ of the radar signal. (B): PSD of the resulting radar transmit signal.

Figure 21. Probability of detection versus SIR for each selected probability of false alarm for narrowband radar with five interferers. Dotted lines indicate performance due to traditional pulsed radar response, dashed lines are the theoretical predictions from (3.26) and the solid lines are the demonstrated performance of the optimum spectrum waveform.
In Figure 21 it can be seen that the shaped radar waveform’s performance suffers when compared to the wideband radar in Figure 18 due to some the communication signal energies being present in the radar waveform spectrum. Conversely, in Figure 22 the performance of one of the communications system starts to suffer compared to the baseline (or noise-only interference) performance as more and more of the radar’s energy finds its way into the bandwidth of the communications signal.

![Figure 22. Performance curves for QPSK receiver with 9dB interference to noise ratio for narrowband radar with five interferers.](image)

2. Radar Bandwidth Equal To The Communications Bandwidth

In this scenario, we consider the case when the radar bandwidth and the communications bandwidth are equal. This case is not very likely since radars are typically designed to have a large bandwidth to allow for better range resolution; however, these results are treated as worst case scenario.
In Figure 23 it can be seen that all of the radar’s energy is now in the spectrum of the communications signal. With an appropriately scaled value of $1/\lambda$ we are still able to keep the energy out of the center of the main lobe, but as the clutter response of the radar grows relative to the energy of the communications signal, the radar begins to fill energy into the center of the communications systems frequency band. This has a negative effect on the communications systems ability to operate in this band as well as the radar. Usually, the negative effects are much greater for the communications receiver because of power differences in the two signals and the fact that the radar’s match filter naturally weights those portions least effected by the QPSK interference.
In Figure 24 the shaped waveform radar’s performance is still about the same as that of the multi-signal case in Figure 21. When the radar energy or power constraint is small relative to the interfering communications signal, it has a distinct spectral shape that focuses the radar’s energy away from the center of the communications signal; as the radar signal power grows, its shape is less important to the radar as its power simply saturates the communication signal.

The communications receiver has a good performance in Figure 25 when the shaped waveform is present as depicted with the blue dashed lines. Here the communications signal is not appreciably interfered with as the radar has placed its energy away from the center frequency of the communications signal. As the clutter response, (which is held constant throughout these simulations), begins to grow the radar begins to place energy in the central portion of the communications signal and, therefore, cause much more interference to the communications system. Here the QPSK matched
filter is able to suppress the energies of the radar signal sufficiently to still identify the correct symbols with minimal effect compared to the baseline performance.

Figure 25. Performance curves for QPSK receiver with 9 dB interference to noise ratio for narrowband radar with one interferer.
V. CONCLUSION

From the scenarios in Chapter IV much can be learned about how the adaptive radar can coexist with a communications system in the same band. If the adaptive radar’s bandwidth is much larger than the bandwidths of the communications signals present, then it is amenable to the existence of the communications signals in the spectrum without appreciable interference between the systems. The more bandwidth that is available to the radar when creating its adaptive waveform, the better the performance of both the radar and the communications system is preserved. The fact that the radar does not interfere with the communications system allows the radar designers to spread the signal over more of the spectrum to almost eliminate the negative effect on its detection probability, recognizing that lost bandwidth affects range resolution. In Table 1 the results of the four scenarios are summarized for easy reference.

Table 1. Conditional SER of the QPSK receiver for the given scenarios.

<table>
<thead>
<tr>
<th>SNR</th>
<th>No interference</th>
<th>Pulse radar</th>
<th>Wideband with one interferer present</th>
<th>Wideband with multiple interferers</th>
<th>Narrowband with multiple interferers</th>
<th>Equal bandwidth of the two systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 dB</td>
<td>0.046</td>
<td>0.75</td>
<td>0.057</td>
<td>0.11</td>
<td>0.093</td>
<td>0.050</td>
</tr>
<tr>
<td>9 dB</td>
<td>0.0051</td>
<td>0.75</td>
<td>0.0071</td>
<td>0.014</td>
<td>0.0088</td>
<td>0.0064</td>
</tr>
<tr>
<td>12 dB</td>
<td>$7 \times 10^{-5}$</td>
<td>0.74</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.9 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

It is important to realize that the SERs shown in Table 1 are the conditional $SER_c$ during the actual transmit portion of the radar signal. As mentioned in Chapter II, the radar is expected to transmit with a duty cycle on the order of 0.01. With this information
we can now calculate the total $SER_T$ for the communications receiver using 0.01 as the duty cycle of the proposed radar. This is performed using

$$SER_T = (1 - d_c)(SER_N) + d_c(SER_C)$$  \hspace{1cm} (3.29)

where $SER_N$ is the SER in only thermal noise. The total SER for the communications receiver is presented in Table 2.

<table>
<thead>
<tr>
<th>SNR</th>
<th>No interference</th>
<th>Pulse radar</th>
<th>Wideband with one interferer present</th>
<th>Wideband with multiple interferers</th>
<th>Narrowband with multiple interferers</th>
<th>Equal bandwidth of the two systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 dB</td>
<td>0.046</td>
<td>0.053</td>
<td>0.046</td>
<td>0.047</td>
<td>0.046</td>
<td>0.046</td>
</tr>
<tr>
<td>9 dB</td>
<td>0.0051</td>
<td>0.013</td>
<td>0.0051</td>
<td>0.0052</td>
<td>0.0051</td>
<td>0.0051</td>
</tr>
<tr>
<td>12 dB</td>
<td>$7 \times 10^{-5}$</td>
<td>0.0075</td>
<td>$7.1 \times 10^{-5}$</td>
<td>$7.1 \times 10^{-5}$</td>
<td>$7.1 \times 10^{-5}$</td>
<td>$7.1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

These results suggest that the radar can coexist with the communications system and not have appreciable impact on that system at some bandwidth cost to the radar.

A. SUMMARY

This thesis began with a review of the traditional pulsed radar system. The radar equation was derived, and the effects of noise were reviewed. A method of pulse detection was discussed, and a QPSK signal was introduced into the channel. The received signal was analyzed and possible methods of mitigation were mentioned. The primary drawback to all of the methods is that they do not assist in the arduous task of spectrum allocation; rather, they allow the radar to operate while diminishing the ability of other systems to use that portion of the spectrum.
For this reason the adaptive radar waveform was theorized as a possible answer to the problem. The adaptive radar waveform design was discussed, and a new bound on the waterfilling variable $1/\lambda$ was derived. The lower bound of $1/\lambda$ was confirmed as the lowest value of the received signal including the interference. This bound is intuitive when looking at the diagram of the waterfilling technique and had been presented in other works. The upper bound of $1/\lambda$, which had to be investigated since we concentrated on signal–independent interference, was derived to be four times this level in order to complete the waterfilling correctly.

A sub-optimum matched filter receiver was presented, and a decision variable based on the $P_{FA}$ was presented. Using this decision variable and the match filter, we presented a set of performance curves for the radar when QPSK was present. After giving consideration to how the radar would perform with the communications signal present, the performance of the communications receiver was examined. A set of performance curves for a QPSK receiver with the radar signal present were derived via Monte Carlo simulations and presented.

We considered four different radar-communications scenarios based on their available bandwidth ratio. These four scenarios approximated four radar and communications type scenarios that might be encountered. First, wideband radar was considered with a single communications signal present. It was shown that both systems perform at an improved level, and their coexistence has only the effect of reducing the bandwidth available to the radar system. Second, wideband radar was considered with multiple communications signals. In this scenario the radar and communications performance were slightly worse compared to that of the single communications signal since the radar was still able to place most of its energy outside the bandwidth of the communications signals. The third scenario was a narrow bandwidth radar with several communications signals present. In this scenario the radar had no choice but to place some energy in the bandwidth of the communications system and, therefore, began to interfere with communications. The waterfilling procedure for this scenario showed that the radar was able to avoid the center of the main lobe of the communications systems, which reduced the interference effects compared to the traditional pulse radar. The final
example considered the scenario where the radar bandwidth and the communications bandwidth were the same. Here we saw that all of the radar’s energy was in the same band as the communications receiver; however, since it was concentrated away from the central portion of the communications frequency band, it still had very little negative effect on the communications receiver.

Finally, a table of results were presented in Table 1 that related the SER of the communications receiver at 6, 9 and 12 dB SNR when the radar was on the transmit portion of its duty cycle. Those results were then analyzed accounting for the radar’s duty cycle where we assumed a radar duty cycle of 0.01 In Table 2 the resulting $SER_T$ was presented showing the overall performance of the communications receiver. The results of Table 2 illustrate the superior effects of the adaptive waveform over pulsed radar. Also in Table 1 and Table 2, we can see that the radar should have a large enough bandwidth such that there is minimal effect on $P_D$ and range resolution while allowing an SER that is acceptable for the communications system.

This study helps to demonstrate the benefits of adaptive radar transmit signal design on friendly communication systems. This work should lead to new system designs that assist in the arduous task of managing the RF spectrum on the modern battlefield. This work may also assist future radar designers in understanding the limitations of the adaptive radar waveform design explored here.

Future work in this area should include a study of how the adaptive radar waveform affects other forms of communications signals. A physical implementation of this technique should also be implemented to confirm the results of the simulations found in this thesis. This would confirm the adaptive radar waveform design as a viable asset in a complex communications environment.
LIST OF REFERENCES


INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California