P-vector method for determining Arctic Ocean circulation from the Joint US-Russian hydrographic data

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Motivation

• Improving the weakness of diagnostic initialization

• Numerical model is usually integrated from $(T, S)$ field (climatology or …)
  
  \[ u = v = w = 0 \]

Diagnostic Initialization
Basic Equations for OGCM

\[ \frac{\partial V}{\partial t} = -V \cdot \nabla V - w \frac{\partial V}{\partial z} - k_x f V - \frac{1}{\rho} \nabla p + \frac{\partial}{\partial z} \left( K_M \frac{\partial V}{\partial z} \right) + H_V \]

\[ \frac{\partial T}{\partial t} = -V \cdot \nabla T - w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) + H_T, \]

\[ \frac{\partial S}{\partial t} = -V \cdot \nabla S - w \frac{\partial S}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) + H_S, \]
Diagnostic Initialization

\[ \frac{\partial T}{\partial t} = -\mathbf{V} \cdot \nabla T - w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) + H_T + F_T, \]

- Keep \((T, S)\) constant

\[ \frac{\partial S}{\partial t} = -\mathbf{V} \cdot \nabla S - w \frac{\partial S}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) + H_S + F_S, \]

- Generate \((u, v, w)\) fields

\[ \frac{\partial T}{\partial t} = 0, \quad \frac{\partial S}{\partial t} = 0 \]
Extremely Strong Thermohaline Source/Sink Terms (Non-Physical)

\[
F_T \equiv \mathbf{V} \cdot \nabla T + w \frac{\partial T}{\partial z} - \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) - H_T,
\]

\[
F_S \equiv \mathbf{V} \cdot \nabla S + w \frac{\partial S}{\partial z} - \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) - H_S.
\]

Maximum Value = 503
Minimum Value = -751
unit: W/m²

σ = 0
Geostrophic Initialization

- Absolute geostrophic velocity computed from hydrographic data

\[(T, S) \rightarrow \text{Density} \rightarrow \text{Velocity}\]
First Thought - Thermal Wind Relation

\[ u = u_0 + \frac{g}{f\rho_0} \int_{z_0}^{z} \frac{\partial \rho}{\partial y} \, dz' \]

\[ v = v_0 - \frac{g}{f\rho_0} \int_{z_0}^{z} \frac{\partial \rho}{\partial x} \, dz' \]

How to determine \((u_0, v_0)\)?
Conservation of Mass and Potential Vorticity

\[ \nabla \cdot \nabla \rho = 0 \]

\[ \vec{V} \cdot \nabla q = 0, \quad q \equiv f \frac{\partial \rho}{\partial z} \]
Relationship Among Three Vectors

\[ \vec{V} \perp \nabla \rho \]

\[ \vec{V} \perp \nabla q \]

\[ \vec{V} \sim \nabla q \times \nabla \rho \]
P-Vector

\[ \vec{P} = \frac{\nabla q \times \nabla \rho}{|\nabla q \times \nabla \rho|} \]

\[ \vec{P} \parallel \vec{V} \]

\[ \vec{V} = r(\lambda, \phi, z)\vec{P} \]
Two-Step Inverse Method

(1) Density determines the P-vector.

(2) Thermal wind relation determines $\gamma$. 

P-Vector
Intersection of density and potential vorticity surfaces
Thermal Wind Relation
Determines $\gamma$

\[ r^{(k)} P^{(k)}_x - r^{(m)} P^{(m)}_x = \Delta u_{km} \]

\[ r^{(k)} P^{(k)}_y - r^{(m)} P^{(m)}_y = \Delta v_{km} \]

\[ \Delta u_{km} \equiv \frac{g}{f\rho_0} \int_{z_m}^{z_k} \frac{\partial \rho}{\partial y} \ dz' \]

\[ \Delta v_{km} \equiv -\frac{g}{f\rho_0} \int_{z_m}^{z_k} \frac{\partial \rho}{\partial x} \ dz' \]
Solution of $\gamma$

$$r^{(k)} = \frac{\begin{vmatrix} \Delta u_{km} & P_x^{(m)} \\ \Delta v_{km} & P_y^{(m)} \end{vmatrix}}{\sin(\alpha_{km})}$$

$$\begin{vmatrix} P_x^{(k)} & P_x^{(m)} \\ P_y^{(k)} & P_y^{(m)} \end{vmatrix} = \sin(\alpha_{km})$$

$\alpha_{km} \neq 0$
US-Russian Arctic Hydrographic Data

http://nsidc.org/
Temperature \((z = 0)\)
Temperature ( $z = -1000 \text{ m}$ )
Salinity (z = 0)
Salinity ($z = -1000$ m)
Mean Circulations ($z = -32$ m)

P-vector

7 years mean of POPS model results (Zhang, Maslowski, Semtner, 1999, JGR)
Mean Circulations ($z = -32$ m)

P-vector

7 years mean of POPS model results (Zhang, Maslowski, Semtner, 1999, JGR)
Conclusions

• (1) P-Vector method is effective to calculate the absolute velocity from hydrographic data.

• (2) The initial velocity filed can be calculated using the P-vector method (Geostrophic Initialization).