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ABSTRACT

Infrared scintillation measurements were obtained along a 7.2 km path over San Diego Bay, concurrently with mean meteorological and turbulence measurements obtained from a buoy located along the path. Bulk estimates and turbulence measurements of $C_n^2$ were computed from the buoy data and compared with the optical scintillation-derived $C_n^2$ values. Similar to the results of previous experiments, the bulk $C_n^2$ estimates agreed well with both the scintillation and turbulence measurements in unstable conditions, increasingly underestimated $C_n^2$ as conditions approached neutral, and agreed less well with scintillation and turbulence $C_n^2$ values in stable conditions. The mean differences between bulk $C_n^2$ estimates and both the turbulence and scintillation measurements when conditions were not near-neutral exhibited an air-sea temperature difference and wind speed dependence, possibly indicating that the forms of the empirical stability functions used by the bulk model are incorrect. The turbulent $C_n^2$ measurements from the buoy showed excellent agreement with the scintillation values in unstable conditions, but had surprisingly large differences in weakly stable conditions. This disagreement may be related to the fact that humidity fluctuations begin to increasingly influence refractive index fluctuations when the air-sea temperature difference is small and are not properly taken into account by the sonic temperature measurements. As the absolute air-sea temperature difference approaches zero the bulk $C_n^2$ estimates decrease much more rapidly and to much smaller values than either the scintillation or turbulence measurements. Fortunately, in such near-neutral conditions scintillation is usually small enough to have little effect on many optical system applications.

Keywords: scintillation, optical turbulence, refractive-index structure parameter ($C_n^2$), bulk models

1. INTRODUCTION

Electro-optical (EO) signals propagating through the atmosphere exhibit intensity fluctuations caused by turbulence in the intervening atmosphere. This phenomenon is known as scintillation. Scintillation is directly related to the refractive index structure parameter, $C_n^2$, therefore knowledge of $C_n^2$ is essential to evaluate and predict the effects of scintillation on EO system performance. Since direct measurements of $C_n^2$ over the ocean are difficult and expensive to obtain, it is useful to be able to estimate $C_n^2$ from routinely measured environmental parameters. Bulk models have been developed to estimate near surface $C_n^2$ values from mean meteorological and sea temperature measurements, which can be made relatively easily from ships, buoys and ocean towers. Important uses of bulk $C_n^2$ models include the ability to predict $C_n^2$ values from numerical weather prediction model outputs, to construct $C_n^2$ climatologies from historical marine meteorological data bases, and to use real-time, in situ meteorological measurements to produce $C_n^2$ estimates to assist operational personnel in optimally employing their EO systems in the current environment.

The goal of this study is to determine how accurately path-averaged optically-derived $C_n^2$ values can be estimated near the ocean surface from routine single-point meteorological measurements using bulk models under a variety of environmental conditions. This study is based on data obtained during a propagation field experiments conducted in San Diego Bay in May 2005. Bulk $C_n^2$ estimates computed from mean environmental measurements obtained on a buoy are compared with concurrent single-point turbulence $C_n^2$ values and optical scintillation-derived $C_n^2$ measurements along an over-water propagation path to determine how closely the different methods agree under various conditions and to illustrate areas of the bulk models that need improvement.

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2. THEORETICAL BACKGROUND

The turbulent fluctuation component of the refractive index of air, \( n \), can be expressed to a first order approximation as a function of turbulent air temperature and specific humidity fluctuations, as follows\(^1\):

\[
n' = A(\lambda, P, T, q)T' + B(\lambda, P, T, q)q'
\]

where

\[
A = \frac{\partial n}{\partial T} = -10^{-6} \frac{P}{T^2} \left[ m_1(\lambda) + \left[ m_2(\lambda) - m_1(\lambda) \right] \frac{q}{\varepsilon \gamma} \right],
\]

\[
B = \frac{\partial n}{\partial q} = 10^{-6} \left[ m_2(\lambda) - m_1(\lambda) \right] \frac{P}{T \varepsilon \gamma^2}.
\]

and \( \lambda \) is the optical wavelength, \( P \) is atmospheric pressure, \( T \) is the air temperature, \( q \) is specific humidity, \( \varepsilon = 0.622 \), and \( \gamma = (1 + 0.61q) \). \( m_1 \) and \( m_2 \) are empirical functions of wavelength. For the wavelength we will be examining in this study, 1.62 \( \mu \)m, \( m_1 = 77.66 \) and \( m_2 = 65.09 \).

Within the inertial-subrange of the atmospheric turbulence spectrum, the refractive index structure parameter, \( C_n^2 \), is defined as:

\[
C_n^2 = \frac{(n'(0) - n'(r))^2}{r^{3/5}},
\]

where \( n'(0) \) and \( n'(r) \) are the turbulent fluctuation values of \( n \) at two points separated by a distance \( r \) along the mean wind direction and the overbar denotes an ensemble average. In practice \( r \) is generally taken to be on the order of roughly 10 cm, therefore \( C_n^2 \) as defined by Eq. (4) is a statistical description of small-scale refractive index fluctuations. \( C_n^2 \) can also be expressed in terms of the structure parameters for temperature, \( C_T^2 \), specific humidity, \( C_q^2 \) and the temperature-specific humidity cross-structure parameter, \( C_{Tq} \), all defined similar to Eq. 4, as follows\(^1\):

\[
C_n^2 = A^2 C_T^2 + 2ABC_{Tq} + B^2 C_q^2.
\]

The first term on the right-hand side of Eq. (5) represents refractive index fluctuations caused by temperature fluctuations and is always positive, the second term represents the correlation of temperature and humidity fluctuations and can be positive or negative, while the third term represents humidity fluctuations and is always positive.

\( C_n^2 \) values can also be determined by optical systems from the normalized variance of the measured intensity fluctuations in a signal that has propagated through the turbulent atmosphere, \( \sigma_{\ell}^2 \), using the generalized relation:

\[
C_n^2 = 2\sigma_{\ell}^2 \left( \frac{2\pi}{\lambda} \right)^{7/6} L^{11/6} F
\]

where \( \lambda \) is the optical wavelength, \( L \) is the propagation path length and \( F \) is a dimensionless function which incorporates the effects of the turbulence strength and aperture averaging for finite size incoherent source and receiver apertures.

Equations (4-5) and (6) represent two very different means of determining \( C_n^2 \). Equations (4-5) are for single-point atmospheric turbulence measurements which are highly dependent upon the specific height above the surface and horizontal point in space where the measurements are taken, whereas Eq. (6) is a path-averaged measurement which includes the effects of horizontal variations in atmospheric turbulence along the path and also variations in turbulence levels at different heights above the surface as the optical rays are refracted through the atmosphere. We would expect the two methods to agree best when atmospheric conditions approach horizontally homogeneity.
3. THE BULK $C_n^2$ MODEL

Near the surface, Monin-Obukhov similarity theory (MOST) can be used to relate the structure parameters $C_T^2$, $C_q^2$, and $C_{Tq}^2$ in Eq. (5) to the mean properties of the atmospheric surface layer. According to MOST, conditions are assumed to be horizontally homogeneous and stationary; the turbulent fluxes of momentum, sensible heat and latent heat are assumed to be constant with height in the surface layer; and all dynamical properties within the surface layer, when scaled by the proper parameters, are assumed to be a dimensionless function of $\xi$, defined as:

$$\xi = \frac{z}{L_{MO}} = \frac{z \theta_k (\theta + 0.61 T \bar{u}_*^2)}{\theta_u \bar{u}_*^2},$$

(7)

where $z$ is the height above the surface, $L_{MO}$ is the Monin-Obukhov length scale, $k$ is the von Karman constant (= 0.4) and $\theta$, $T$, and $u_*$ are the scaling parameters for temperature, humidity, and wind speed, respectively. The ratio $\xi$ is often referred to simply as the ‘stability’, and is negative in unstable conditions, zero in neutral conditions, and positive in stable conditions. The surface layer scaling parameters can be expressed as:

$$x_s = (\Delta \sigma) k \left[ \ln(z/z_{\text{ou}}) - \psi_T(\xi) \right]^{-1},$$

(8)

where $x_s$ represents wind speed ($u$), temperature ($T$) or specific humidity ($q$) and the symbol $\Delta$ denotes the mean air-sea difference. The $\psi$ functions are the integrated dimensionless profile functions. We have made the common assumption that $\psi_T = \psi_q$. The parameters $z_{\text{ou}}$, $z_{\text{ot}}$, and $z_{\text{sq}}$ are known as the ‘roughness lengths,’ and are determined by the bulk surfacelayer model formulated by Fairall et al.2.

The structure parameters for temperature ($C_T^2$) and specific humidity ($C_q^2$) and the temperature-specific humidity cross-structure parameter ($C_{Tq}^2$) can be expressed in terms of the surface layer scaling parameters as follows:

$$C_T^2 = \frac{T^2 z^{-2/3} f_T(\xi)}{2},$$

(9a)

$$C_{Tq}^2 = r_{Tq} T q z^{-2/3} f_{Tq}(\xi),$$

(9b)

$$C_q^2 = \frac{q^2 z^{-2/3} f_q(\xi)}{2},$$

(9c)

where $r_{Tq}$ is the temperature-specific humidity correlation coefficient with a value of about 0.8, and $f_T$, $f_{Tq}$, and $f_q$ are dimensionless functions of $\xi$ that have been determined empirically, as follows1:

$$f_T(\xi) = f_q(\xi) = \begin{cases} 5.9 (1 - 8\xi)^{-2/3}, & \xi \leq 0 \\ 5.9 (1 + 2.4 \xi^{2/3}), & \xi \geq 0 \end{cases}.$$  

(10)

We can express $C_n^2$ in terms of mean meteorological properties by combining Eqs. (5, 7-10), resulting in:

$$C_n^2 = \frac{f(\xi) k^2 \left[ A T^2 + 2 ABr_{Tq} T \Delta q + B^2 \Delta q^2 \right]}{z^{-2/3} \left[ \ln(z/z_{\text{ou}}) - \psi_T(\xi) \right]^2}.$$  

(11)

and

$$\xi = \frac{z \theta_k (\Delta \theta + 0.61 T \Delta q) \left[ \ln(z/z_{\text{ou}}) - \psi_T(\xi) \right]^2}{\theta_u (\Delta U)^2 \left[ \ln(z/z_{\text{ou}}) - \psi_T(\xi) \right]^2}.$$  

(12)

Once the required model inputs ($\Delta T$, $\Delta q$, $\Delta U$) are known, $C_n^2$ can be estimated by solving Eqs. (11-12) by an iterative process. Full details on the Naval Postgraduate School’s bulk $C_n^2$ model are provided by Frederickson et al (2000)3.
The dependence of the bulk $C_n^2$ estimates on the air – sea temperature difference ($\Delta T$) is shown as a function of wind speed and relative humidity in Figs. 1a and 1b, respectively. The $C_n^2$ estimates generally increase as $|\Delta T|$ increases. The $C_n^2$ estimates increase with wind speed for negative $\Delta T$ values, and generally decrease with wind speed when $\Delta T$ is positive. Wind speed variations have the largest effect on $C_n^2$ for large $|\Delta T|$ values and are slightly larger when $\Delta T < 0$. The bulk $C_n^2$ estimates decrease with relative humidity for negative $\Delta T$ values and generally increase with relative humidity when $\Delta T$ is positive. The minimum $C_n^2$ values increase and occur at larger $\Delta T$ values as relative humidity decreases. The effects of relative humidity variations on the bulk $C_n^2$ estimates are largest for small $|\Delta T|$ values.

### 4. THE EXPERIMENT

During an ongoing U. S. Navy-sponsored experiment, low-level infrared scintillation measurements are being obtained by the SPAWAR Systems Center, San Diego (SSC-SD) along a propagation path over San Diego Bay, while concurrent meteorological and ocean surface measurements are being collected by the Naval Postgraduate School’s (NPS) buoy, located along the propagation path (Fig. 2). Measurements of wind speed, wind direction, air temperature, relative humidity, atmospheric pressure and sea temperature are obtained every second on the buoy. These 1 Hz data were then averaged over 15 minute intervals centered about the scintillation measurement times and bulk $C_n^2$ estimates were computed from these averaged values. Since $C_n^2$ is height dependent, the bulk $C_n^2$ estimates were adjusted for tidal sea level variations using tide data obtained from the National Ocean Service acoustic tide gauge located in San Diego Harbor.
High frequency (10 Hz) sonic temperature measurements are obtained on the NPS buoy from a Solent sonic anemometer mounted 5.25 m above the waterline. The sonic temperature structure parameter, $C_n^2$, was computed from power spectral densities of the sonic temperature, $S_f(f)$, using the expression:

$$
C_n^2 = 4 \left( \frac{2\pi f}{U} \right)^{2/3} S_f(f) f^{3/5},
$$

where $U$ is the mean wind speed and $f$ is the frequency. Direct turbulent estimates of $C_n^2$ were obtained from the relationship $C_n^2 = A^2 C_f^2$, which assumes that humidity fluctuation effects on both $C_n^2$ and $C_f^2$ are negligible compared to temperature fluctuations.

Infrared (IR) scintillation measurements are being obtained by SSC-SD every 15 minutes, when the amplitude of an infrared signal is recorded at a 300 Hz rate for a 109 second period in two wavelengths; near IR (1.06 µm) and short-wave IR (1.62 µm). The broad-beam IR transmitting source consists of 18 halogen lamps mounted inside a circle 25 cm in diameter and modulated by a 690 Hz chopper wheel. The receiver system consists of a telescope with a 20 cm diameter primary mirror, a beam splitter that separates the incoming beam to two 3 mm diameter photodiode detectors, one for each wavelength. A reference signal from the chopper blade is transmitted by radio to a synchronous detector at the receiver. The signal from the detectors is separated from the chopped carrier waveform by means of a lock-in amplifier system. $C_n^2$ values were calculated from the normalized variance of the measured signal amplitude, using a specialized form of Eq. (6), which takes into account the effects of aperture averaging. For more details on the SSC-SD equipment and procedures, see Zeisse et al. (2000). The transmitting source is located at the Naval Amphibious Base, Coronado, California, at a height of ~6.5 m above mean sea level (see Fig. 3). The receiver is located 7.2 km from the transmitter at the Submarine Base, Point Loma, California, at a height of ~11.5 m above mean sea level. The transmission path is over-water for its entire length except for very short distances at each end point.

5. RESULTS

The data examined in this study were obtained during an intensive observation period from 1 to 31 May 2005. Only data for 1.62 µm will be shown, since the results at 1.06 and 1.62 µm were nearly identical. First, we will examine the behavior of scintillation $C_n^2$ values as a function of the environmental conditions observed at the buoy. From Fig. 3 we can see that, as expected, the scintillation $C_n^2$ values clearly increase as the absolute value of the air-sea temperature difference (ASTD) increases. In unstable conditions (negative ASTD) the $C_n^2$ values increase systematically with wind speed, while in stable conditions (positive ASTD) a clear dependence upon wind speed is not apparent. These results qualitatively follow the behavior of the bulk model, as shown in Fig. 1a. An unexpected feature of this plot is the ‘bump’ seen in the bin-averaged scintillation log($C_n^2$) values for low wind speed cases when $0 < |\Delta T| < 1$. The cause of this ‘bump’ is not known, but indications of a similar feature have been observed in previous scintillation measurements made across San Diego Bay (see Fig. 12 of Frederickson et al. [2000]). From Fig. 3b we can see that there is no clear dependence of $C_n^2$ upon the relative humidity. This result is not very surprising, given that the bulk model predicts only a very weak relative humidity dependence, except for very small values of $|\Delta T|$. Such a weak dependence could very easily be lost amid the inherent noise and errors in the different measurements and the varied secondary effects that many different environmental parameters might have upon $C_n^2$. The above results demonstrate that the variations in scintillation-derived $C_n^2$ values at a given height level above the ocean surface depend primarily upon ASTD and, at least in unstable conditions, also to a significant degree upon wind speed.

Next, we examine the behavior of the sonic temperature structure parameter measurements from the NPS buoy. Fig. 4 demonstrates that there is a very similar qualitative dependence of the turbulent $C_f^2$ values upon both ASTD and wind speed as that shown above for the scintillation $C_n^2$ values. The turbulent $C_f^2$ values even exhibit a small ‘bump’ for low wind speed cases when $0 < |\Delta T| < 1$, similar to the scintillation $C_n^2$ data. Fig. 5 shows the mean differences between the bulk estimates and turbulent measurements of $C_f^2$ as a function of ASTD and wind speed. The agreement between the two methods is very good for unstable conditions with $\Delta T < -1 ^\circ C$. The difference between the bulk and turbulence measurements is generally constant with ASTD in these unstable conditions, but does have a weak wind speed dependence, indicating that the bulk model does not reflect wind speed effects upon $C_f^2$ entirely correctly. In the near-
neutral regime, with \(-1 < \Delta T < 0.5 \, ^{\circ}{\text{C}}\), the bulk model increasingly underestimates \(C_T^2\) as \(|\Delta T|\) approaches zero. When the mean vertical temperature gradient is nonexistent, the bulk model predicts there are no temperature fluctuations, and thus \(C_T^2\) vanishes. In stable conditions, with \(0.5 < \Delta T < 4 \, ^{\circ}{\text{C}}\) and wind speeds greater than 2.5 m/s, the bulk model overestimates the turbulence measurements by a nearly constant factor of about 2.5. This disagreement is not too excessive, when one considers that \(C_T^2\) values can easily vary by several orders of magnitude. The bulk model performs more poorly in stable conditions with very low wind speeds. This result is not surprising, given that MOST begins to break down in very stable low wind conditions and the uncertainties in low-wind turbulence measurements.

The mean differences between the turbulent and scintillation \(C_n^2\) values are presented in Fig. 6. The agreement between the turbulent and scintillation \(C_n^2\) values is uniformly excellent in unstable conditions. At the point where ASTD changes from negative to positive, however, the turbulence values suddenly begin to increasingly underestimate the scintillation measurements, reaching a maximum difference at \(\Delta T \approx 1\) before beginning to agree better with the scintillation values again as conditions move to even more stable stratification. A possible explanation for this behavior is that the sonic temperature-derived values determined by the relation \(222 \frac{s}{T} C_{AC} = C_n^2\) are an approximation of \(C_n^2\) that includes an incorrect humidity dependence, which would probably become more important with small values of \(|\Delta T|\).

We would expect that the bulk model would more accurately predict single-point turbulence measurements of \(C_T^2\) than path-averaged scintillation measurements of \(C_n^2\). This is because the path-averaged scintillation values include the integrated effects of horizontally varying atmospheric conditions along the propagation path, which at any point might depart significantly from conditions at the buoy, and also the varying heights above the ocean surface which a refracted beam takes while propagating along the path. In Fig. 7 we compare the bulk \(C_n^2\) estimates computed from the buoy data with the path-averaged optical scintillation measurements. Again, the bulk model generally performs well in unstable conditions where \(\Delta T < -1 \, ^{\circ}{\text{C}}\), although the differences between bulk and scintillation \(C_n^2\) values have a small ASTD and wind speed dependence. As expected, the bulk model increasingly underestimates \(C_n^2\) as the absolute value of ASTD approaches zero. In stable conditions, with \(\Delta T > 1 \, ^{\circ}{\text{C}}\), the bin-averaged bulk-scintillation log(\(C_n^2\)) differences do not have a constant bias, but rather exhibit a strong, almost linear, ASTD dependence. This result could indicate that the general forms of the empirical stability functions used in the bulk model are not correct.
Figure 4. Turbulence measurements of $\log(C_n^2)$ from the NPS buoy plotted versus the air – sea temperature difference. Values shown have been averaged into ASTD bins for different wind speed intervals, as indicated by the different symbols.

Figure 5. Difference of bulk estimates and turbulence measurements of $\log(C_n^2)$ from the NPS buoy plotted versus the air – sea temperature difference. Values shown have been averaged into ASTD bins for different wind speed intervals, as indicated by the different symbols.

Figure 6. Difference of turbulence measurements of $\log(C_n^2)$ from the NPS buoy and scintillation $\log(C_n^2)$ measurements, plotted versus the air – sea temperature difference. Values shown have been averaged into ASTD bins for different wind speed intervals, as indicated by the different symbols.

Figure 7. Difference of bulk model estimates of $\log(C_n^2)$ from the NPS buoy and scintillation $\log(C_n^2)$ measurements, plotted versus the air – sea temperature difference. Values shown have been averaged into ASTD bins for different wind speed intervals, as indicated by the different symbols.
6. CONCLUSIONS

This study has confirmed the results of many previous experiments, that in unstable conditions (negative air-sea temperature differences) both path-averaged scintillation $C_n^2$ and single-point turbulent $C_T^2$ measurements can be estimated over the ocean with good accuracy from routinely obtained meteorological measurements using bulk methods. This result is due primarily to the fact that Monin-Obukhov similarity theory in general has been found to describe the dynamic characteristics of fully-turbulent unstable surface layers quite well and much better than weakly-turbulent stable surface layers.

The comparisons shown above demonstrate that in near-neutral conditions the bulk estimates increasingly underestimate both the scintillation $C_n^2$ and turbulence $C_T^2$ measurements as the absolute mean value of ASTD approaches zero. The probable reason why directly measured structure parameters are not observed to approach zero for near-zero mean ASTD values is that the instantaneous scalar gradient between the measurement height and the surface may actually fluctuate between positive and negative values, leading to a near-zero mean air-sea difference measurement and thus a near-zero bulk structure parameter estimate. However, in the presence of turbulent vertical wind fluctuations the non-zero instantaneous scalar gradients can lead to significant scalar fluctuations and thus much larger structure parameter values than predicted by the bulk model. These results demonstrate that bulk methods for estimating scalar structure parameters based upon mean air-sea differences are not appropriate when the mean air-sea difference approaches zero. It must also be recognized that the signal-to-noise ratio inherent to turbulence and scintillation sensors places a lower limit on the measured values of the structure parameters. The only fortunate aspect of the poor bulk model performance in near-neutral conditions is that the very low $C_n^2$ values observed in such conditions generally indicate such weak scintillation conditions as to have very little practical effect on optical systems in many applications.

In general, MOS theory has been only marginally successful in describing stable surface layers. When conditions become more stable the stratification increasingly suppresses any vertical mixing, and with low winds little mechanical mixing can be generated in any case. Such a situation can allow the atmosphere to become effectively decoupled from the surface, thereby violating the MOS assumptions upon which the bulk models are based. These problems are demonstrated by the above comparisons, which show that the bulk models have much poorer agreement with scintillation and turbulence-derived structure parameters in stable conditions than in unstable. The ASTD and wind speed dependence of the bulk-scintillation $C_n^2$ and bulk-turbulence $C_T^2$ differences indicate that the forms of the dimensionless stability functions used in the bulk model may be incorrect. An interesting aspect of this study is that even the turbulence-derived $C_n^2$ estimates did not agree well with the scintillation measurements in stable conditions. It is possible that this is due to the incorrect humidity dependence upon the refractive index when using sonic temperature measurements to determine $C_n^2$.

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