Transient temperature distributions produced in a two-layer finite structure by a dithering or rotating laser beam

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ABSTRACT
In this paper we provide numerical solutions for a transient temperature distribution produced in a two-layer finite structure that is induced by a dithering or rotating laser beam. Our results show that the maximum temperature rise can be reduced significantly by combining coating and motion of the object.

Keywords: rotating or dithering Gaussian laser beam; non-homogeneous heat equation; two-layer structure

1. INTRODUCTION
Direct energy laser weapons deliver high-intensity beams to a target and can quickly destroy or burn it. In order to protect objects, such as Navy ships and missiles, one possible counter measure is to add a coating skin to the surface of the target or let the target rotate or shake. The main purpose of this work is to investigate the effects of coating and motion of the target on the maximum temperature rise on the target induced by a laser beam.

Many works can be found in the literature which are devoted to the theoretical and numerical modeling of temperature profiles induced by laser radiation in solids (Araya and Gutierrez, 2006; Bertolotti and Sibilia., 1981; Burgener and Reedy, 1982; Calder and Sue, 1982; Cline and Anthony, 1977; Lax, 1977 and 1978; Moody and Hendel, 1982; Sanders, 1984; Sistaninia et al., 2009) and temperature distributions produced in a two-layer structure by a scanning laser beam (Burgener and Reedy, 1982). However, there is little work on the combined factor of coating and target movement on the temperature profile induced by laser beams. Recently we have given a detailed study of the temperature rise induced by a rotating or dithering laser beam on a semi-infinite domain (Zhou, 2011) and a solid with finite geometry (Zhou and Tan, 2011). Here we extend our work to study transient temperature rises on a two-layer finite structure.
We organize our paper as follows. In Sections 2-3 we present the numerical solutions of the temperature distributions in a two-dimensional and three-dimensional two-layer structure induced by a dithering or rotating laser beam, respectively. Finally, conclusions are drawn in Section 4.

2. 2-D MATHEMATICAL SIMULATIONS

We first construct a time-dependent model of a two-dimensional (2-D) film made of one material which is coated with a thin layer of another material. Laser beams are shone on the top surface of the film. Figure 1 depicts the model geometry and boundary conditions. We divide the finite structure into two regions: Region 1 is a film of thickness \(a\) in the \(y\) direction, which is on a substrate of dissimilar material (Region 2).

![Figure 1: A schematic diagram of a two-dimensional two-layer structure.](image)

Mathematically, the temperature distribution of the film in the two-layer structure can be modeled as follows. In Region 1, the governing equation is

\[
\frac{\partial u_i}{\partial t} = \alpha_{T,1} \left( \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} \right) + \frac{\alpha_{T,1}}{K_{T,1}} q(x, y, t),
\]

where \(u_i(x, y, t)\) denotes the temperature rise of the thin film at position \((x, y)\) and time \(t\), \(\alpha_{T,1}\) is the thermal diffusivity of the material in Region 1, \(K_{T,1}\) is the thermal conductivity,
and \(q(x,y,t)\) is the energy distribution of the moving laser beam. In the case of a dithering laser beam shown in Figure 1, \(q(x,y,t)\) can be expressed as

\[
q(x,y,t) = \frac{I_0}{r_0} \exp\left[-\frac{k}{r_0^2} (x-x_c(t))^2\right] \delta(y),
\]

(2)

\[
x_c(t) = x_0 + a \sin \frac{2\pi t}{T},
\]

where \(x_c(t)\) is the position of the dithering Gaussian beam, \(x_0\) is the initial position of the laser beam, \(I_0\) is the intensity of the laser beam, \(r_0\) is the effective radius of the laser beam, \(k\) is a constant used for the Gaussian model and \(\delta(y)\) is the Dirac delta function. In Region 2, the equation for the temperature rise \(u_2(x,y,t)\) where the material has the thermal diffusivity \(\alpha_{r,2}\) and thermal conductivity \(K_{r,2}\) is

\[
\frac{\partial u_2}{\partial t} = \alpha_{r,2} \left( \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} \right).
\]

(3)

At the interface between the two regions, we require that the energy is conserved for heat flow across the interface between Regions 1 and 2:

\[
K_{r,1} \frac{\partial u_1}{\partial y} = K_{r,2} \frac{\partial u_2}{\partial y} \quad \text{at the interface between Regions 1 and 2}
\]

(4)

The initial condition for \(u_1(x,y,t), u_2(x,y,t)\) is zero which assumes that the film has the same temperature as the ambient initially. The boundary conditions at the air/material interface impose that the film is insulated at the edges. The boundary conditions reflect the assumption that no energy escapes into the ambient at the air/material interface. This is a good approximation for most materials under consideration because heat flow by conduction through the material is usually much bigger than heat loss by radiation or convection at the air/material interface.

The materials we use here are copper, graphite and aluminum, whose thermal properties are listed in Table 1. We use the commercial software COMSOL to solve the equations (1)-(4) with the given initial and boundary conditions.
Table 1: Thermal properties for Copper, Graphite and Aluminum

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Copper</th>
<th>Graphite</th>
<th>Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat Capacity (C&lt;sub&gt;p&lt;/sub&gt;) (unit: J/(kg*K))</td>
<td>385</td>
<td>846</td>
<td>900</td>
</tr>
<tr>
<td>Density ((\rho)) (unit: Kg/(m^3))</td>
<td>8700</td>
<td>1100</td>
<td>2700</td>
</tr>
<tr>
<td>Thermal Conductivity (K&lt;sub&gt;T&lt;/sub&gt;) (unit: W/(m*K))</td>
<td>400</td>
<td>500</td>
<td>160</td>
</tr>
<tr>
<td>Thermal Diffusivity ((\alpha)) (unit: m^2/s)</td>
<td>1.1942e-4</td>
<td>5.3729e-4</td>
<td>6.5844e-5</td>
</tr>
<tr>
<td>Melting Point (unit: K)</td>
<td>1356</td>
<td>3500</td>
<td>933</td>
</tr>
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</table>

In Figure 2 we compare the temperature rise of an aluminum film induced by a Gaussian beam with the temperature rise when the film is coated with a thin layer of copper on the top edge. Figure 1(a) plots the temperature rise for a pure aluminum film at t = 1s and the maximum temperature rise is 2903K. Figure 1(b)-(d) depict the corresponding temperature rise of the film when it is coated with 1%, 5% and 10% of copper layer on the top edge. The maximum temperature rise falls to 1813K, 1143K and 867K, respectively.

In Figure 3 we plot the maximum temperature rise in the two-layer structure in Figure 2 as a function of the thickness of the coating layer. As shown here, the maximum temperature rise decreases as the thickness of the coating layer increases. The asymptotic behavior is predicted in Figure 3(b) using the least square fitting method.

Figure 4 is similar to Figure 2, but the coating material is graphite whose thermal conductivity is bigger than that of copper. As expected, a comparison of Figure 2 and Figure 4 shows that when the thermal conductivity of the coating materials is bigger, the maximum temperature rise can be reduced more.

Figure 5 is also similar to Figure 2, but the Gaussian beam is now dithering. The properties of the dithering Gaussian beam are given in Table 2. Comparing Figure 2(a) and Figure 5(a), we can see that once the beam moves, the maximum temperature rise is reduced more. The combined effect of coating and dithering is to reduce the maximum temperature rise further, as shown in Figure 5 (b)-(d).
Figure 2: The temperature rise of an aluminum film with dimension 2cm x 2cm at t=1s induced by a Gaussian beam. The aluminum structure is coated with (a) 0% (b) 1% (0.2mm) (c) 5% (1mm) and (d) 10% (2mm) layer of copper skin.
Figure 3: (a) The maximum temperature rise of the two-layer structure in Figure 2 as a function of the thickness of the coating layer. (b) The asymptotic behavior of the maximum temperature rise as a function of the coating thickness. The solid curve is the least-square fitted function $T_{\text{max}} = \frac{-49.5852}{d^2} + \frac{423.7815}{d} + 781.4141$ where $d$ is the thickness of the coating.
Figure 4: The temperature rise of an aluminum film with dimension 2cm x 2cm at t=1s induced by a Gaussian beam. The aluminum structure is coated with (a) 1% (0.2mm) (b) 2% (0.4mm) (c) 3% (0.6mm) and (d) 4% (0.8mm) layer of graphite skin.
Figure 5: The temperature rise of an aluminum film with dimension 0.02m x 0.02m at t=1s induced by a dithering Gaussian laser beam. The aluminum structure is coated with (a) 0% (b) 1% (0.2mm) (c) 5% (1mm) and (d) 10% (2mm) layer of copper skin.

Table 2: Parameters of the dithering laser beam used in Figure 5

<table>
<thead>
<tr>
<th>Laser Input</th>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude of Gaussian beam</td>
<td>$I_0$</td>
<td>$2 \times 10^9$</td>
<td>$W/m^2$</td>
</tr>
<tr>
<td>Effective radius of Gaussian beam</td>
<td>$r_0$</td>
<td>$10^{-4}$</td>
<td>m</td>
</tr>
<tr>
<td>Dithering frequency</td>
<td>$1/T$</td>
<td>1</td>
<td>Hz</td>
</tr>
<tr>
<td>Center of dithering</td>
<td>$x_0$</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Dithering radius</td>
<td>$a$</td>
<td>0.002</td>
<td>m</td>
</tr>
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</table>
3. 3-D MATHEMATICAL MODELING

Now we simulate the temperature rise induced by a Gaussian beam in a three-dimensional body. In Figure 6 we give the temperature rise in an aluminum box at $t=1s$. The maximum temperature (1609K) is reached at the top surface which is hit directly by the laser beam. The temperature rise decreases in the $z$-direction, as shown in Figure 6 (b)-(d).

![Figure 6: The temperature distribution of an aluminum box induced by a Gaussian beam. (a) 3-D view; (b) A horizontal slice through $z = 0.995$; (c) A horizontal slice through $z = 0.99$; (d) A horizontal slice through $z = 0.95$.](image)

Now we add a thin layer of copper coating (1%) on the top surface of the aluminum box and compute the temperature rise at $t=1s$. Our results are depicted in Figure 7. The maximum temperature rise drops from 1609K in Figure 6(a) to 552K in Figure 7(a) due to the application of copper skin. It is remarkable that 1% coating leads to nearly 66% reduction in the maximum temperature rise. Figure 7(b)-(d) show how the temperature rise decreases in the $z$-direction.
Figure 7: The temperature distribution of an aluminum box with 1% copper skin on the top surface \((z = 1.0)\) induced by a Gaussian beam. (a) 3-D view; (b) A horizontal slice through \(z = 0.995\); (c) A horizontal slice through \(z = 0.99\); (d) A horizontal slice through \(z = 0.95\).

To study the effect of the thickness of coating layer, we compute the temperature rise due to 0.1%, 0.2%, 0.3% or 0.4% copper coating layer in Figure 8. When there is 0.1% coating, the maximum temperature rise is 1352K whereas the maximum temperature rise is reduced to 1165K, 1023K and 912K when the thickness of the coating layer is increased to 0.2%, 0.3% and 0.4%, respectively. Figure 9(a) gives the detailed relationship between the maximum temperature rise and the thickness of the coating layer whereas Figure 9(b) predicts the asymptotic behavior with the least square fitting.

In Figure 10 we apply a dithering Gaussian beam on the top surface of the aluminum box which has a 1% copper coating on the top surface. Figure 10(a) indicates that the maximum temperature rise is reduced significantly now to 113K whereas Figure 10(b)-(d) give the temperature rise distributions on several lower layers.
Figure 8: The temperature distribution of an aluminum box with (1) 0.1% (2) 0.2% (3) 0.3% (4) 0.4% copper skin on the top surface ($z=1.0$) induced by a Gaussian beam.

In the simulations here we model the distribution of a dithering or rotating Gaussian beam as $q(x,y,z,t) = f(x,y,t) \delta(z-1)$.\hspace{1cm} (5)

Here $f(x,y,t)$ is given by

$$f(x,y,t) = \frac{I_0}{2\pi d^2} \exp \left[ -\frac{(x-x_c(t))^2 + (y-y_c(t))^2}{2d^2} \right],$$

$$x_c(t) = x_0 + a \cos \frac{2\pi t}{T}, \quad x_0 = \frac{L_x}{2},$$

$$y_c(t) = y_0 + b \sin \frac{2\pi t}{T}, \quad y_0 = \frac{L_y}{2}.$$ \hspace{1cm} (6)

In equation (6) $a=b=0$ corresponds to a Gaussian beam, $a \neq 0, b=0$ describes a dithering Gaussian beam, and $a=b \neq 0$ gives a rotating Gaussian beam. The delta function in $z$ expresses the assumption that all the energy is absorbed at the surface $z=1$ which is hit directly by the beam. The boundary conditions are insulating and the initial condition is zero.
Figure 9: (a) The maximum temperature rise of the two-layer structure in Figure 8 as a function of the thickness of the coating layer. (b) The asymptotic behavior of the maximum temperature rise as a function of the coating thickness. The solid curve is the least-square fitted function

$$T_{\text{max}} = -5079.6 \frac{1}{d^2} + 4187.4 \frac{1}{d} + 189.3$$

where $d$ is the thickness of the coating.

Figure 11(a) shows the temperature rise distribution induced by a rotating Gaussian beam. The maximum temperature rise gets a large reduction and it is about 28K. The temperature rise distributions in several lower layers are plotted in Figure 11(b)-(d). Figure 11 indicates that the temperature rise induced by a rotating Gaussian beam is smaller than that induced by a dithering Gaussian beam.

The parameters of the dithering and rotating Gaussian beam used for Figures 10-11 are given in Table 3.
Figure 10: The temperature distribution of an aluminum box with 1% copper skin on the top surface \((z = 1.0)\) induced by a dithering Gaussian beam. (a) 3-D view; (b) A horizontal slice through \(z = 0.995\); (c) A horizontal slice through \(z = 0.99\); (d) A horizontal slice through \(z = 0.95\).
Figure 11: The temperature distribution of an aluminum box with 1% copper skin on the top surface \((z=1.0)\) induced by a dithering Gaussian beam. (a) 3-D view; (b) A horizontal slice through \(z = 0.995\); (c) A horizontal slice through \(z = 0.99\); (d) A horizontal slice through \(z = 0.95\).

Table 3: Parameters of the 3-D dithering/rotating laser beam in Figures 10-11

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<tr>
<td>(I_0)</td>
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<td>(W / m^2)</td>
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<tr>
<td>(d)</td>
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<td>m</td>
</tr>
<tr>
<td>(L_x)</td>
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<td>m</td>
</tr>
<tr>
<td>(L_y)</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>(x_0)</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>(y_0)</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>(a)</td>
<td>0.25</td>
<td>m</td>
</tr>
<tr>
<td>(b)</td>
<td>0 (dithering) (0.25 ) (rotating)</td>
<td>m</td>
</tr>
<tr>
<td>period</td>
<td>1</td>
<td>s</td>
</tr>
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4. CONCLUSIONS  
We have calculated the temperature rise induced by a rotating or dithering Gaussian laser beam for a two-layer finite body. Our numerical results show that the maximum temperature
rise can be reduced significantly by either coating or moving the beam (which is equivalently
to moving the target).

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