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In the light of the plethora of definitions for the hurricane boundary layer, we advocate a dynamical definition based on the distribution of agradient flow. We seek also to clarify the fundamental role of the boundary layer in the hurricane intensification process. In particular, we contrast the differences between unsteady boundary layers that are able to facilitate the spin-up of the vortex above and steady boundary layers that cannot. If slaved to the time-dependent vortex aloft, the latter can spin up the interior vortex only indirectly by changing its thermodynamic properties through vertical advection of these from below and adjustment to thermal wind balance. These differences are highlighted by an analytical demonstration that the application of a zero-vertical-gradient condition on velocity above a steady boundary layer does not provide a direct means of allowing the boundary layer to determine the flow in the interior vortex. This result assumes that frictional forces are negligible at this boundary. Finally, echoing a few previous insights, we question the applicability of conventional boundary-layer theory at radii of strong ascent into the eyewall, where the flow is akin to that of separation in aerodynamic boundary layers. Copyright © 2010 Royal Meteorological Society

Key Words: tropical cyclone; friction layer; agradient flow

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1. Introduction

The boundary layer of a mature hurricane has been long recognized to be an important feature of the storm as it strongly constrains the radial distribution of vertical motion at its top, as well as those of absolute angular momentum and moisture. Indeed, this idea was central to Emanuel's (1986) formulation of a steady-state hurricane model and to his formulation of a theory for the potential intensity of hurricanes (Bister and Emanuel, 1998, and references therein). There is mounting evidence also that the boundary layer plays a central role in the hurricane intensification process itself (e.g. Emanuel, 1997; Smith *et al.*, 2009, henceforth M3). In the light of current efforts to improve forecasts of hurricane intensity and rapid intensification in particular, we believe it is useful to re-examine the role of the boundary layer on the physics of hurricanes. Although parts of our discussion may be well known to some readers, we believe that the interpretations offer a broad context in which to understand the role of the boundary layer in hurricane forecast models.

A significant result of M3 was the identification of two mechanisms for the spin-up of the mean tangential circulation of a hurricane. The first involves convergence of absolute angular momentum *above* the boundary layer, where this quantity is approximately conserved. This mechanism acts to spin up the outer circulation at radii where the boundary-layer flow is subgradient and where there is subsidence into the boundary layer. The second mechanism involves the convergence of absolute angular momentum within the boundary layer, where it is not conserved, but where air parcels are displaced farther radially inwards than are those above the boundary layer. This mechanism is associated with the development of supergradient wind speeds in the boundary layer. Of course, the radial inflow in both mechanisms is ultimately linked to the overturning circulation forced by the local buoyancy of individual deep convective clouds in the inner-core region, but the boundary-layer inflow is considerably enhanced by the force imbalance brought about by surface friction (Montgomery *et al.*, 2009; Bui *et al.*, 2009). Thus the boundary layer is an essential ingredient of the spin-up of the inner-core region.

Smith and Vogl (2008) sought to develop an improved formulation of the slab model for a steady-state boundary layer with the initial aim of improving Emanuel's steady-state hurricane model. They found that when the boundary-layer depth is held constant as assumed by Emanuel, the slab model breaks down at a finite radius inside the radius of maximum tangential wind speed at the top of the boundary layer. This breakdown is associated with the development of supergradient winds, which lead to a rapid deceleration of the radial inflow. As a result, the inflow is brought rapidly to rest at a finite radius, leading to an unrealistically large vertical velocity near the radius of breakdown. Although this unrealistic behaviour can be mitigated by allowing the boundary-layer depth to decrease with decreasing radius*, the development of supergradient winds became problematic in developing a simple extension of Emanuel's model, because the angular momentum exiting the boundary layer cannot match the assumed winds above it, which were required to be in thermal wind balance.

The foregoing issues were articulated further in a paper by Smith *et al.* (2008), who showed that Emanuel's formulation is inconsistent in the hurricane inner-core region. In the paper, Smith *et al.* (2008) argued that a single-layer slab model was inadequate to represent the low-level flow in this region and that the surface-based inflow layer would need to be supplemented by an outflow layer on top of it to allow the flow to readjust to a near-gradient value† before ascending in the eyewall clouds (their Figure 6). Such a configuration finds strong support in the calculations of M3.

An immediate question arises: do similar difficulties arise in continuous models for the steady-state boundary layer such as that presented by Kepert and Wang (2001, henceforth KW01), or are such models able to allow the boundary layer to determine the radial distribution of radial and azimuthal momentum that exits the top of the boundary layer in the inner-core region? It is noteworthy that KW01 chose to apply a zero-vertical-gradient condition for both horizontal velocity components at the top of their model domain, presumably with the aim of allowing the region of upflow to be an 'outflow boundary'. However it is apparent from the flow fields shown in KW01's Figure 2 that the flow exits the top of their computational domain almost normally so that the radial flow there is almost zero. Assuming that the frictional forces at this altitude are small, which is consistent with the information given in the figure, it follows readily that the azimuthal flow is in close gradient wind balance with the prescribed pressure field.

One purpose of the present paper is to examine the related problem of defining the boundary layer in hurricanes as well as issues of parametrizing the boundary layer in this region in full time-dependent models.

A second purpose of the paper is to better understand steady-state boundary-layer calculations such as those of KW01 and to show *analytically* that, if the top of a boundary layer model is taken at or above the level at which the frictional force vanishes, the zero-vertical-gradient condition does not allow the model to determine the flow above it. Rather, the condition requires that the flow returns to gradient wind balance with zero radial velocity. Therefore, it would appear that continuous steady-state boundary-layer models in which frictional forces are small near their top are so constrained that they are unable to determine the radial distribution of radial and azimuthal momentum of air that ascends through their top. In such steady models, the outflow above the inflow layer can only bring bring the ascending air to the *prescribed* gradient wind above the boundary layer. These inferences help to understand KW01's Figure 2 as discussed above.

The paper is organized as follows. In section 2, we review briefly the relationship of the boundary-layer equations to the full equations for the motion of a turbulent, axisymmetric vortex. In section 3, we demonstrate that the zero-vertical-gradient conditions adopted by KW01 are almost the same as assuming that the radial wind is zero at the upper boundary of the domain and that the tangential wind component has its gradient value there, provided that the corresponding frictional forces are small. In section 4, we review some definitions of the boundary layer that have appeared in the literature and discuss their bearing on the issues discussed above. The conclusions are presented in section 5.

2. Conventional vortex boundary-layer theory

Traditionally, the boundary layer refers to the shallow region of flow adjacent to a rigid boundary (or fluid interface) where velocity gradients normal to the surface are relatively large on account of frictional effects. When the Reynolds number is large, a standard scale analysis for the boundary-layer flow indicates that the pressure gradient of the exterior flow is transmitted approximately unchanged through the boundary layer to the surface, a result that goes back to seminal work by Ludwig Prandtl first published in 1904‡. It is assumed also that the velocity blends smoothly into that of the free stream above the boundary layer.

The boundary layer of a hurricane is typically less than 1 km deep, so that the variation of density with height can be neglected to a good approximation. Assuming for the present that the turbulent momentum transfer may be represented in terms of a constant eddy diffusivity, K, the Navier–Stokes equations for an axisymmetric vortex may be

*The decrease was taken at a rate that is inversely proportional to the square root of the inertial stability parameter of the gradient wind, a rate that is suggested by a scale analysis of the full boundary-layer equations (Vogl and Smith, 2009).

†A scale analysis for a hurricane shows that the mean azimuthal flow is in approximate gradient wind balance in the free atmosphere at heights where the radial component of flow is small compared with the azimuthal component (Willoughby, 1979), a result supported by observations (Willoughby, 1990).

‡An excellent summary of conventional boundary-layer theory is given by Kundu (1990). He and Anderson (2005) give interesting historical reviews of Prandtl's pioneering work.

expressed in cylindrical polar coordinates, (r, λ, z) as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial r} + K\left(\nabla^2 u - \frac{u}{r^2}\right), \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} + fu = K\left(\nabla^2 v - \frac{v}{r^2}\right), \quad (2)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + K\nabla^2 w, \quad (3)$$

where (u, v, w) is the velocity vector, p is the perturbation pressure, and ρ is the density. The equations are supplemented by the continuity equation, which for a homogeneous fluid is

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0\,. \quad (4)$$

In the derivation of equations for the boundary layer, it is normally assumed that the tangential wind component at the top of the boundary layer, v_g, is at most a function of radius and time *and* that it is in gradient wind balance, i.e. it satisfies the equation:

$$\frac{v_g^2}{r} + fv_g = \frac{1}{\rho}\frac{\partial p}{\partial r}. \quad (5)$$

The fact that the radial pressure gradient throughout the boundary layer is to a close approximation equal to that at the top of the layer allows us to substitute for the pressure gradient in terms of v_g using (5). A scale analysis shows also that the friction terms are dominated by those involving vertical gradients (Vogl and Smith, 2009). Then, setting $v = v_g(r,t) + v'$, and allowing the eddy diffusivity, K, to be a function of height, Eqs (1) and (2) become:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v'^2}{r} - \xi_g v' = \frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z}\right), \quad (6)$$

$$\frac{\partial v'}{\partial t} + u\frac{\partial v'}{\partial r} + w\frac{\partial v'}{\partial z} + \frac{uv'}{r} + \zeta_g u = \frac{\partial}{\partial z}\left(K\frac{\partial v'}{\partial z}\right), \quad (7)$$

where

$$\xi_g = \frac{2v_g}{r} + f \quad \text{and} \quad \zeta_g = \frac{\partial v_g}{\partial r} + \frac{v_g}{r} + f, \quad (8)$$

are the absolute angular velocity and the vertical component of absolute vorticity of the gradient wind, respectively.

3. The upper boundary condition

We consider now a steady boundary layer with $\partial/\partial t \equiv 0$. Assuming that frictional forces can be ignored at the top of the boundary layer (the usual assumption of boundary-layer theory), Eqs (6) and (7) become

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v'^2}{r} - \xi_g v' = 0\,, \quad (9)$$

$$u\frac{\partial v'}{\partial r} + w\frac{\partial v'}{\partial z} + \frac{uv'}{r} + \zeta_g u = 0\,, \quad (10)$$

Following KW01, let us choose the upper boundary condition on u and v to be

$$\frac{\partial u}{\partial z} = 0 \quad \text{and} \quad \frac{\partial v}{\partial z} = 0 \quad \text{at} \quad z = h\,. \quad (11)$$

The second of these conditions is equivalent, of course, to $\partial v'/\partial z = 0$. Then Eqs (9) and (10) give

$$u\frac{\partial u}{\partial r} = \frac{v'^2}{r} + \xi_g v' \quad \text{at} \quad z = h\,, \quad (12)$$

$$u(\zeta' + \zeta_g) = 0 \quad \text{at} \quad z = h\,, \quad (13)$$

where $\zeta' = (1/r)\{\partial(rv')/\partial r\}$ is the vertical component of the relative vorticity of the agradient flow. Since the vertical component of the relative vorticity $\zeta' + \zeta_g$ at $z = h$ is typically not zero, Eq. (13) requires that $u = 0$, whereupon from Eq. (12), for a cyclonic vortex at the top of the boundary layer, $v' = 0$.

It follows from the analysis above that there is practically no difference between applying the zero-vertical-gradient condition (11) and applying the conventional boundary condition that the flow at the top of the boundary layer merges smoothly into the prescribed flow above, i.e. $u = 0$ and $v = v_g$ (or $v' = 0$) at $z = h$. Thus the zero-vertical-gradient condition does not provide a means of allowing a steady-state boundary-layer model to directly determine the flow in the vortex interior.

The foregoing result may be interpreted physically in terms of angular momentum. If frictional torques are negligible in an axisymmetric flow, the absolute angular momentum, $M = rv + fr^2/2$, is materially conserved. The zero vertical gradient condition on the tangential velocity component at the upper boundary implies that the vertical advection of absolute angular momentum, $w(\partial M/\partial z)$, is zero. Then, if the flow is steady ($\partial v/\partial t = 0$), the radial advection of absolute angular momentum, $u(\partial M/\partial r)$, must vanish also. If $\partial M/\partial r > 0$, i.e. if the flow is centrifugally stable, it follows that u must be zero. Note that $u(\partial M/\partial r) = 0$ is equivalent to $u(\zeta' + \zeta_g) = 0$ (cf. Eq. (13)), which states that the radial flux of absolute vorticity is zero.

The above result explains why the boundary-layer flow shown in Figure 2 of KW01 exits the calculation domain close to vertically. This figure shows that turbulent diffusion is small[§] at the top of the computational domain, so that the foregoing analytical derivation is applicable. The result suggests also why, in the linear model described by Eliassen and Lystadt (1977), Kuo (1982), Kepert (2001) and Vogl and Smith (2009), it is not possible to avoid satisfying the condition that $u = 0$ and $v = v_g$ at $z = h$ (or more precisely as $z \to \infty$).

Note that, in the solutions of the foregoing linear models, *there is no finite height at which the zero-vertical-gradient boundary condition on the horizontal velocity vector* (Eq. (11)) *is satisfied*. This was another reason that led us to examine the consequences of KW01's use of the condition.

4. The hurricane boundary layer revisited

There is some divergence of opinion in the literature on how to define the hurricane boundary layer. M3 adopts

[§]The caption to the figure states '... the solid heavy line marks the top of the layer in which vertical diffusion plays a marked role in the dynamics' and this line lies below the upper boundary.

a dynamical definition, using the term *boundary layer* to describe the shallow layer of *strong* inflow near the sea surface that is typically 500 m to 1 km deep and which arises *largely* because of the frictional disruption of gradient wind balance near the surface (Figure 6 of M3)[¶]. While this definition appears to be consistent with the descriptions provided in Kepert (2001) and KW01, these authors do not give an explicit definition for the boundary layer. The definition of M3 does not apply to the friction layer in axisymmetric balance models, which are founded on the assumption that the entire flow is in strict gradient wind balance (Smith and Montgomery, 2008). In these models, the boundary layer is the layer across which the vorticity influx is equal to the frictional torque resulting from surface friction. While the foregoing assumption might seem plausible, we are unaware of any rigorous justification of it and note that it is not supported by a scale analysis of the boundary-layer equations (Smith and Montgomery, 2008; Vogl and Smith, 2009). Recently, Bryan and Rotunno (2009) take the boundary layer to be the layer in which the turbulent force is important and strongly controlled by surface interaction. Based on axisymmetric numerical simulations, they showed that the height of the maximum tangential wind component is roughly equivalent to the top of the boundary layer so defined. This assumption was apparently needed 'to match the free atmosphere component to the planetary boundary-layer closure in Emanuel's potential intensity theory'. Other authors have adopted a thermodynamic definition of the boundary layer, characterized by the layer in which the virtual potential temperature is appreciably well mixed (e.g. Moss and Merceret, 1976), the lifting condensation level outside of deep convective regions (Moss and Rosenthal, 1975), or where the virtual potential temperature exceeds that of the air at the ocean surface by 0.5 K (Anthes and Chang, 1978).

The dynamical definition of M3 is uncontroversial in the outer regions of a hurricane, where there is subsidence into the boundary layer, but it has limitations in the inner-core region where boundary-layer air is being lofted into the eyewall clouds. In the latter region, conventional boundary-layer theory breaks down. For one thing, vertical perturbation pressure gradients may not be ignored there[‖]. The flow in this region is akin to that of separation in aerodynamic boundary layers. There is recent observational evidence showing that radial gradients of the vector momentum stress are important also in this region of the hurricane (Zhang *et al.*, 2010).

The idea of boundary-layer air erupting into the eyewall clouds is in the spirit of the discussion by Stull (1988) in relation to defining the boundary layer in the subtropical high pressure regions, where there is large-scale subsidence into the boundary layer and in the intertropical convergence zone (ITCZ) where the air is ascending into deep convective clouds (his Figure 1.6). Stull noted that the boundary layer was well defined in the former region, but not in the latter. Indeed, his Figure 1.6 is suggestive that the 'boundary layer' should be thought of as encompassing the entire troposphere in the ITCZ region. The problems were recognized long ago by Moss and Rosenthal (1975) and Shapiro (1983). For example, Shapiro wrote: '... as the radius of maximum tangential wind is approached, the boundary layer itself becomes ill defined, as air is pulled up into the active convection'. The M3 definition is limited also as it does not make reference to the layer of outflow that surmounts the inflow layer in the inner-core region of the hurricane. This outflow feeds smoothly into the eyewall (Marks *et al.*, 2008; Bell and Montgomery, 2008). Retrievals of turbulent kinetic energy from Doppler radar data in hurricanes suggest that boundary-layer turbulence is being lofted into the eyewall clouds in this region to supplement the turbulence generated locally within the eyewall itself (Lorsolo *et al.*, 2010). If this is the case, it is hard to imagine how 'the turbulent fluxes go to zero at $z = h$' (Bryan and Rotunno, 2009: their section 2b and footnote 5), h being the top of the boundary layer at which the tangential wind speed is presumed also to attain its maximum value.

The thermodynamic definitions may be useful in the outer regions of the vortex, but they are moot on the important dynamical processes described in M3. A recent observational study by Zhang *et al.* (2010) has highlighted the differences between the thermodynamic and dynamic definitions of the boundary layer. They showed that turbulent momentum fluxes decrease to zero, not at the mixed-layer top (about 300 m), but at a height of about 700 m, which is a little shallower than the depth of the inflow layer. While these observations are limited to a clear-air region between outer rainbands where mean wind speeds are of marginal hurricane strength, this finding highlights the limitations of a thermodynamic definition of the boundary layer.

Notwithstanding the difficulties of precisely defining the boundary layer in the ascent region of the hurricane inner core, it is undisputed that the boundary layer exerts an immense control on the swirling flow above it. In particular, it determines the radial distribution of absolute angular momentum, pseudo-equivalent potential temperature and turbulent kinetic energy. We have shown here analytically that if the lofting of turbulent kinetic energy is ignored, a continuous steady boundary-layer model is intrinsically unable to determine radial and tangential wind distributions that depart from gradient wind balance in the vortex above. The reason is that the outflow that surmounts the inflow layer adjusts the flow back to its gradient value. This limitation would not detract from their possible utility in constraining the free atmosphere in *steady* hurricane models of the type proposed by Emanuel (1986), providing the consequences of neglecting the vertical transport of turbulent kinetic energy are recognized. However, models like Emanuel's that assume gradient balance in the boundary layer are unable to capture the strong amplification of the tangential wind component in the inflow layer as demonstrated by Smith *et al.* (2008) and Bryan and Rotunno (2009).

The situation is different when the flow is time dependent, but because of the tight coupling between the flow in the boundary layer and that above, care must be exercised in constructing cause and effect arguments. With this note of caution, one might think of changes in the radial distribution of absolute angular momentum and pseudo-equivalent

[¶]While there is inflow throughout the lower troposphere in the calculations presented in M3, the largest radial wind speeds are confined within the lowest kilometre. The lower-tropospheric inflow results from a balanced response of the vortex to the radial gradient of the azimuthally averaged diabatic heating rate in the eyewall clouds. The strong inflow near the surface is not captured by a balance model (Bui *et al.*, 2009, Figures 5 and 6).

[‖]The scale analysis presented by Vogl and Smith (2009) does not apply in this region as the conventional boundary-layer assumption that the radial length-scale is much larger than the vertical length-scale is not valid.

potential temperature in the boundary layer as being lofted into the vortex above by the air ascending out of the boundary layer, much in the sense envisioned in Emanuel's (1997) formulation of a time-dependent model for the hurricane. In general, these changes will be accompanied by a change in the pressure field of the vortex itself, which, in turn, will change the degree of force imbalance in the boundary layer leading to a change in the inflow. A consequence of the result described in section 3 is that a steady-state boundary layer that is slaved to the flow above cannot directly alter the dynamics of that flow, but thermodynamic changes in the boundary layer can be communicated to the vortex above. These changes can have an indirect effect on the dynamics of the vortex through adjustments to thermal wind balance. If the boundary layer flow is allowed to vary with time as in M3, it can have a direct dynamical effect on the vortex aloft.

In hurricane forecast models, it is necessary to parametrize the effects of the turbulent transfer of heat, moisture and momentum across the boundary layer and their exchange across the air–sea interface. A range of schemes has been proposed for the atmosphere boundary layer in general (e.g. Stull, 1988), but not specifically for hurricanes in a high-wind-speed marine environment. While the schemes have various degrees of sophistication, most** seek to determine some local value of turbulent diffusivity, $K(r, z)$, to close Eqs (6) and (7). In some schemes, the determination is based on empirical formulae while in others it is based on a calculation of the turbulent kinetic energy, which may be carried as a prognostic quantity (e.g. in the Gayno–Seaman scheme; Shafran *et al.*, 2000). Normally, these schemes are applied in time-dependent models in which the boundary-layer flow has a direct influence on the dynamics and thermodynamics as discussed above, but they may be used also for diagnostic studies, an example being the steady-state model of KW01.

5. Conclusions

We have sought to clarify the fundamental role of the boundary layer in hurricane dynamics and intensification. We have examined the related problem of defining the boundary layer in hurricanes as well as issues of parametrizing the boundary layer in full time-dependent models. We advocate a dynamical definition of the boundary layer, that recognises the role of gradient wind imbalance resulting from surface friction. We note also the limitations of any definition at radii of strong ascent into the eyewall, where local vertical pressure gradients are expected to become important, invalidating a key assumption of boundary-layer theory that neglects such gradients. Other key assumptions of boundary-layer theory are violated also: (i) that the layer is a shallow transition zone through which turbulent momentum transport is important, and (ii) that this transport becomes negligible at the top of the layer.

In the light of these issues, we have revisited the problem of formulating a steady hurricane boundary-layer model. We have shown analytically that, if the top of the model is taken at, or above, the level at which the frictional force vanishes, the zero-vertical-gradient condition does not allow the model to determine the flow above it, but requires the flow to return to the prescribed gradient wind with zero radial velocity.

We have contrasted the differences between unsteady boundary layers that are able to directly spin up the vortex above and steady boundary layers that cannot. When slaved to the time-dependent interior vortex, steady boundary layers can spin up the vortex only indirectly by changing its thermodynamic properties through vertical advection from below and adjustment to thermal wind balance.

**One exception is the Blackadar boundary-layer scheme, which uses a non-local mixing algorithm in the dry convectively unstable regime. Nevertheless, this regime appears not to be invoked in the inner-core region of a tropical cyclone, say at radii < 200 km (Smith & Thomsen 2010).

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