Optimal routing of ice reconnaissance aircraft

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THESIS

OPTIMAL ROUTING OF ICE RECONNAISSANCE AIRCRAFT

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The United States Coast Guard (USCG) conducts the International Ice Patrol (IIP) in the North Atlantic. The primary mission of the IIP is to identify the Limits of All Known Ice (the southeastern, southern and southwestern limits of the iceberg region in the vicinity of the Grand Banks of Newfoundland) and to disseminate this information to mariners. The IIP routinely flies reconnaissance missions during the ice season to help locate the Limits of All Known Ice. This thesis develops an algorithm that, given a set of priorities, determines the optimal routes to fly during these reconnaissance missions. The algorithm relies on partitioning the operation area into squares where the length of each square's side is the HP's radar or visual identification range. Each square has a reward assigned using IIP priorities which include location of the node, its proximity to the Limits of All Known Ice, whether or not known icebergs are near it, and the time since it was last visited. The algorithm picks the route that conforms to IIP operating procedures with total greatest reward for nodes searched. The algorithm enumerates all routes obeying IIP operational procedures within a few seconds guaranteeing an optimal solution. When compared to actual flights flown by the IIP, routes produced by the algorithm better satisfy USCG defined priorities.
ABSTRACT

The United States Coast Guard (USCG) conducts the International Ice Patrol (IIP) in the North Atlantic. The primary mission of the IIP is to identify the Limits of All Known Ice (the southeastern, southern and southwestern limits of the iceberg region in the vicinity of the Grand Banks of Newfoundland) and to disseminate this information to mariners. The IIP routinely flies reconnaissance missions during the ice season to help locate the Limits of All Known Ice. This thesis develops an algorithm that, given a set of priorities, determines the optimal routes to fly during these reconnaissance missions. The algorithm relies on partitioning the operation area into squares where the length of each square’s side is the IIP’s radar or visual identification range. Each square has a reward assigned using IIP priorities which include location of the node, it’s proximity to the Limits of All Known Ice, whether or not known icebergs are near it, and the time since it was last visited. The algorithm picks the route that conforms to IIP operating procedures with total greatest reward for nodes searched. The algorithm enumerates all routes obeying IIP operational procedures within a few seconds guaranteeing an optimal solution. When compared to actual flights flown by the IIP, routes produced by the algorithm better satisfy USCG defined priorities.
THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While effort has been made, within the time available, to ensure the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.
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EXECUTIVE SUMMARY

The International Ice Patrol's (IIP) primary mission is to identify the Limit of All Known Ice (the southeastern, southern, and southwestern limits of the iceberg region in the vicinity of the Grand Banks of Newfoundland) and to disseminate this information to mariners. The IIP routinely flies reconnaissance missions during the ice season to help locate the boundaries of the ice region. This thesis develops an algorithm that, given a set of priorities, determines the optimal routes to fly during these reconnaissance missions. The algorithm relies on partitioning the operation area into squares where the length of each square's side is the IIP's radar or visual identification range. Each square has a reward assigned using IIP priorities which include location of the node, its proximity to the Limits of All Known Ice, whether or not known icebergs are near it, and the time since the node was last visited. The algorithm picks the route that conforms to IIP operating procedures with total greatest reward for nodes searched. The algorithm is capable of enumerating all possible routes obeying IIP operational procedures within a few seconds guaranteeing an optimal solution.

Using data from the 1995 ice season, this thesis presents a comparison between the actual flights flown by the IIP and routes produced by the algorithm. It also presents the
advantages one obtains when routes use other than cardinal headings (North, South, East, West) and use different spacing between flight legs.

In all instances, the algorithm produces better flight routes, with respect to general IIP defined priorities, than those flown by the IIP. Also, allowing for non-cardinal heading flights increases the quality of the algorithm routes. A numerical comparison shows the magnitude of the advantage one obtains when increasing the distance between flight legs during a search.

The algorithm is an excellent tool for pre-flight planning. It displays to the IIP where to fly to obtain the optimal return on a flight. With this information the IIP can either fly the recommended routes or tailor their routes using the algorithm's information to get as close to the optimal return as possible.
I. LOOKING FOR ICEBERGS

The International Ice Patrol's (IIP) primary mission is to identify the Limits of All Known Ice (the southeastern, southern, and southwestern limits of the iceberg region in the vicinity of the Grand Banks of Newfoundland) and to disseminate this information to mariners. The IIP routinely flies reconnaissance missions during the ice season to help locate the boundaries of the Limits of All Known Ice (LAKI). This thesis develops an algorithm that, given a set of priorities, determines the optimal routes to fly during these reconnaissance missions.

A. INTERNATIONAL ICE PATROL

Following the sinking of RMS Titanic in 1912, the IIP was formed to track icebergs and provide warnings to vessels in the trans-Atlantic shipping lanes over the Grand Banks of Newfoundland. The United States Coast Guard (USCG) manages and operates the IIP.

The IIP area of responsibility in the North Atlantic (Figure 1) is approximately 544,320 square miles. Calved from glaciers on the west coast of Greenland, icebergs, many the size of a city block, are carried along by ocean currents. The primary force that carries the icebergs south into the Grand Banks region is the Labrador current (Figure 2). It is
Figure 1. The IIP area of responsibility in the North Atlantic. From Ref. (U.S. Coast Guard, 1991).

Figure 2. The Labrador Current in the IIP operating area in the North Atlantic. From Ref. (U.S. Coast Guard, 1991).
near the Grand Bank that the Labrador Current and the Gulf Stream meet producing dense fog due to the difference (up to 20 degrees Celsius) in water temperatures. Of most concern to the IIP are the icebergs that pass south of the 48th parallel. These uncontrollable moving icebergs, along with the fog, pose the greatest threat to shipping lanes, oil platforms and fishing vessels in the area. The IIP defines the severity of the ice season on the number of icebergs that drift south of this parallel. An average ice season can have between 300-600 icebergs passing south of 48°N, some drifting as far south as 42°N as seen in Figure 3.

Figure 3. Distribution of icebergs in the IIP operating area on 30 July 1991. Icebergs are seen located as far south as below 42° latitude. After Ref. (U.S. Coast Guard, 1991).
B. INTERNATIONAL ICE PATROL MISSION

The primary mission of the IIP is to publish the LAKI along the southeastern, southern, and southwestern edges of the ice region as shown in Figure 4. The USCG broadcasts twice daily to all interested mariners an ice bulletin and facsimile chart that contain the current LAKI.

![Distribution of icebergs in the IIP area of responsibility on 30 July 1991 along with the LAKI. After Ref. (U.S. Coast Guard, 1991).](image)

The IIP uses two computer models to predict the future position of icebergs. A drift model simulates the movement of icebergs through the IIP operation area and a deterioration model predicts the amount of iceberg melt. These models use several sources of information on current iceberg sightings as
input data. The most vital and accurate information entered into the models comes from IIP sightings. Every other week during the ice season, the USCG sends an Ice Reconnaissance Detachment (ICERECDET), consisting of one aircraft and crew, to ST. John’s Newfoundland for about 9 days. During this time, it flies roughly 4-5 sorties to patrol around the LAKI. The aircraft is a HC-130H equipped with a pair of AN/APS-135 Side Looking Airborne Radars (SLARs) and one AN/APS-137 Forward Looking Airborne Radar (FLAR). The IIP Standard Operating Procedures (SOP’s) (U.S. Coast Guard, 1992) require 200% SLAR coverage in order to delete icebergs and radar targets within 60 nautical miles (nm) of the LAKI. In other words, to eliminate any iceberg assumed to exist within 60 nm of the LAKI, the area requires two scans, with no detection by the radar, before the iceberg’s assumed location can be deleted. Also, to simplify the interpretation of results obtained from the SLAR, flights are usually flown on cardinal headings (North,South,East,West). The spacing between search legs depends on the flight conditions. For visual operations, search legs are spaced 20 nm apart and for the more common FLAR or SLAR operations the spacing is 25 nm apart (U.S. Coast Guard, 1994). Thus, flight legs during a search must be of equal length and parallel (Figure 5).

The goal of the ICERECDET is complete coverage of the LAKI and as much of the interior of the region as possible. However, due to the large area encompassed by the LAKI,
especially during the middle of the ice season, the perimeter is too big to permit complete coverage during an ICERECDET. Thus, the IIP must determine what sections of the LAKI have the highest priority and visit as much of these areas as possible.

C. CURRENT FLIGHT PLANNING

Current detachment flight planning involves manually plotting the flight tracks using the current LAKI from the models as a guide and a computer spread sheet to determine distance and flight time. The ICERECDET crew determines the areas to be searched using their past experience and knowledge
of environmental conditions. Factors taken into consideration by the crew include the location of an area, an area's distance from the LAKI, whether or not icebergs are near it, and time since the area was last visited. Icebergs in the Labrador current move south faster than those icebergs located on top the Grand Bank and hence have a higher priority to track. Also of importance is the location of the icebergs within the operating area. The IIP divides the operating area into south, southeast, southwest, and east regions (Figure 6) with the south and southeast regions having the highest priority. In the past this flight planning has provided routes with gaps in coverage which result in icebergs going undetected and reported outside the published LAKI.

D. OBJECTIVE OF CURRENT RESEARCH

The objective of this thesis is to develop an algorithm to optimize the ICERECDET sorties to obtain as much coverage of high priority areas as possible during a detachment.

E. THESIS OUTLINE

Chapter II discusses related studies on the tracking of icebergs and also presents the problem of tracking icebergs as an orienteering problem. Chapter III presents a description of the optimal flight route algorithm (OFRA) to solve the problem. Chapter IV presents the results of the OFRA applied to actual data from the 1995 ice season and Chapter V follows
Figure 6. Shown are the boundary lines that divide the IIP operating area into sections. Each section has a different priority. After Ref. (U.S. Coast Guard, 1991).

with conclusions. Appendix A contains the data used in determining a node’s reward. Appendix B has the supporting functions needed by the algorithm and Appendix C includes a formulation of the problem in Naval Postgraduate School (NPS) format.
II. RELATED STUDIES

A. ICEBERG TRACKING

A review of the literature reveals numerous studies related to icebergs and prediction of iceberg movements. Post (1956) shows the relative strengths of the Gulf Stream and the Labrador Current control the drift of icebergs south of latitude 48°N in the North Atlantic. Dempster (1974) concludes the effect of wind on iceberg movement is negligible unless the speed of the wind is greater than 25 to 35 knots and blowing from a constant direction over several hours. He observes ocean currents are the main effect on iceberg drift and with seven-eighths of the volume of an iceberg below water and variations in currents with depth, precise drift prediction won’t be achieved until currents can be measured more accurately. Cheema and Ahuja (1978) develop a kinematic model to analyze the drift of icebergs that includes the influences of ocean currents, wind-generated currents and other variables. They conclude, among other things, that precise location of a tracked iceberg is very important. They also recommend that location of icebergs be reported to within seconds of a degree instead of minutes of a degree as done by the IIP. They argue that a difference of 1 minute in latitude can result in an error of 1 nautical mile in the observed distance.

The IIP uses a model by Mountain (1980) to predict the
drift of an iceberg. Inputs to the model include initial location and size of the iceberg, forecast wind data, a mean current field and cross sectional area and mass data for the icebergs. Model inaccuracies are believed to stem from inaccuracies in model inputs (currents and wind data) and not in the formulation of the physics of the model itself. Murphy and Anderson (1985) use four case studies to first, test the accuracy of Mountain's drift model and second, to see how the accuracy changes when on-scene measured wind and current data are used. Because of the small data set, no firm conclusions can be drawn but the results support collecting up-to-date data as close as possible to the tracked iceberg.

Washburn (1995) develops a model that simulates the number and distribution of unidentified icebergs within the IIP area of responsibility. The model simulates the addition, movement (using the actual IIP drift model), deterioration and identification of icebergs in the operating area. A plot of the density of the unidentified icebergs gives the IIP estimated location information on the unidentified icebergs.

Any model to track the drift of an iceberg is only as accurate as the input information. Especially important in predicting the future location of an iceberg is precise accuracy in the iceberg’s initial location. Because of speed, range and accurate navigational systems, sighting by aircraft is the preferred method used by the IIP in locating icebergs. The literature, however, failed to produce any method
developed for aircraft to optimally search a given area for icebergs.

B. ORIENTEERING PROBLEM

The task of determining which sections of the North Atlantic the IIP should visit and search for icebergs can be approached as an orienteering problem. The orienteering problem (Tsiligirides, 1984) involves a set of nodes, each with an associated reward. The goal is to find a route which visits a subset of these nodes while maximizing the total reward, the route’s total distance from beginning node (b) to ending node (e) being within an allotted limit. The distance between any two nodes is known.

Tsiligirides (1984) solves the orienteering problem using a heuristic which generates a large number of possible routes and then selects the best one. Tsiligirides applies his heuristic to three problems consisting of 32, 21, and 33 nodes respectively and receives what he considers acceptable results. Golden, Levy, and Vohra (1987) show the orienteering problem as being NP-hard, and develop another heuristic to solve it. They use Tsiligirides’ three problems to show how their center-of-gravity heuristic performs better than Tsiligirides’ heuristic. The heuristic is written in FORTRAN 77 and runs on a UNIVAC 1190. Run times are not reported in Tsiligirides’ paper but Golden, Levy, and Vohra estimate the run times between the two heuristics to be similar. The
longest run time by Golden, Levy, and Vohra occurs in the 33 node problem and is just under 10 seconds.

Golden, Wang, and Liu (1988) present a heuristic that not only includes randomness and center-of-gravity, but also two new features referred to as subgravity and learning. The authors use the same problems mentioned in Tsiligirides’ paper to compare their new heuristic with Tsiligirides’ heuristic and the center-of-gravity heuristic. The new heuristic, written in FORTRAN and run on a UNIVAC 1100/92, obtains better solutions in much faster time than the other two heuristics.

Golden, Wang, and Liu report their run times in total CPU time to run all the instances of each problem where an instance is a specific maximum route length. For problem 1, which includes 32 nodes and 18 instances, the total time is 17.95 seconds. For problem 2, which includes 21 nodes and 11 instances, the total run time is 4.98 seconds and for the third problem with 33 nodes and 20 instances, the total time is 25.98 seconds.

Even though the above heuristics perform reasonably well, their results are for fairly small problems. This thesis approaches the problem of optimizing routes of flight as an orienteering problem involving over 2600 nodes. Fortunately the IIP SOP’s presented in Chapter I, such as the 200% SLAR coverage, result in a number of additional constraints to the generic orienteering problem. These additional constraints reduce the number of possible route combinations to examine,
thus enabling the enumeration of all possible routes within a reasonable amount of time and the guarantee of an optimal solution.
III. OPTIMAL ROUTING MODEL

A. PROBLEM DESCRIPTION AND FORMULATION

Limited by the number of sorties per detachment to ST. John’s and coupled with the limited range of the HC-130H and an area of responsibility of approximately 544,320 square miles, the IIP must decide which sections of the area to search and which to leave alone until a future date. To model this, this thesis develops a network by partitioning the IIP operating area into nodes with each node representing a fixed number of square miles. Figure 7 depicts a section of the operating area divided up into nodes. Using priorities set by the IIP, each node has an associated reward. The OFRA uses these node rewards to determine the optimal routes of flight.

A sortie (route of flight) divides into three phases. This thesis refers to these phases as "transit-in", "search", and "transit-out". The transit-in phase consists of that portion of the route of flight from ST. John’s directly to the starting point (node 2 in Figure 7) of the search phase. The search phase is when the search for icebergs begins. It is during this phase that the route of flight must conform to a structure that satisfies all IIP SOP’s (Figure 5). To maintain this structure and to ensure complete enumeration of all possible feasible routes in the network, this thesis uses the following method.
Figure 7. This figure shows part of the IIP operating area divided up into nodes. The numbering of some of the nodes is used to explain how the OFRA determines a feasible route.

A feasible route has a total allowable flight distance of 1700 nm and is produced by first selecting a node which can be reached in the transit-in phase (node 2 in Figure 7). From this start node, the route first proceeds in the east direction, (defined as the first search in the X direction where search in the X direction is movement along lines of latitude.) First search in the X direction in Figure 7 is movement from node 2 to node 3. The length of flight in the X direction, (referred to as a flight leg) is limited to
between 3 and 15 nodes (75 - 375 nm). (This limit can be easily changed but all computational experience reported in this thesis uses these limits since they bound previous flown routes by the IIP).

A feasible route must proceed north, (defined as search in the Y direction) after completing its initial flight leg. Search in the Y direction is movement along lines of longitude and in Figure 7 is movement from node 7 to node 15. The route can only proceed one node in this direction to preserve the required distance between flight legs. The route then proceeds back in the X direction (opposite the previous X direction, in this case west for the second leg, which is from node 15 to node 10 in Figure 7) and then again one node in the Y direction (node 10 to node 18 in Figure 7) and so on. This continues until reaching the last node in the search route (node 42) that enables the aircraft to still return to ST. John’s within 1700 nm. The transit-out phase is the route of flight that starts at the last node of the search phase and proceeds directly back to ST. John’s.

Assuming all routes are within 1700 nm for the network in Figure 7, the number of feasible routes from start node 2 would be 4 (routes with flight leg lengths of 3, 4, 5, and 6); the number of feasible routes from start node 3 would be 3 (routes with flight leg lengths of 3, 4, and 5); and the number of feasible routes from start node 6 would be 0 since the route is unable to proceed to the east (right) the minimum
of 3 nodes before continuing in the north (up) direction.

The above description of a feasible route is only one of several ways to determine a route that complies with IIP SOP's. Instead of proceeding to the "right and up" as described above, one could have easily chosen to proceed, for example, to the "left and down", or some other combination. In any case, approximately the same area can be covered using any number of similar processes and therefore this thesis restricts feasible routes to the form described above.

B. AN OPTIMAL ALGORITHM

The optimal flight route algorithm (OFRA) determines the optimal route by enumerating all feasible routes, and picking the one with the most favorable total reward. A route's total reward is found by adding the rewards from nodes forming the search phase of the route. (Appendix A describes the process of calculating a node's reward).

The OFRA only needs to determine two primary parameters to describe a feasible route; a feasible start node and a feasible length for the flight legs during the search phase. For each feasible start node (a feasible start node being a node that can be reached on a round trip flight from ST. John's without exceeding 1700 nm), the OFRA proceeds to the right and up until no other nodes can be visited without violating the 1700 nm total flight distance (it is assumed that flight from ST. John's to and from the search route is
The OFRA enumerates all feasible flight leg lengths from each start node.

C. SUPPORTING FUNCTIONS

The iceberg program contains the procedures that process the required data used by the OFRA. Included in the program are separate procedures (see Appendix B descriptions) which determine the LAKI, build the OFRA networks, and enable the OFRA to produce routes using other than cardinal headings. The OFRA itself is a separate procedure in the iceberg program.
IV. RESULTS

This chapter presents the comparison between the actual flight routes flown on ICERECDET from ST. John's Newfoundland and the flight routes produced by the OFRA. It also presents several different scenarios including cases where flying the LAKI is the only priority (producing non-cardinal headings), and cases using different spacings (20 nm and 25 nm) between flight legs. The data used, which consists of the active iceberg listings and the IIP actual routes of flight, is from 3 consecutive ICERECDET during the 1995 ice season. The algorithm is written in standard Pascal and compiled using the Silicon Valley Software (SVS) 32 bit compiler (SVS, 1991). The computer used is a 486 DX66 MHZ with 16MB RAM.

A. ACTUAL IIP FLIGHTS VS. OFRA FLIGHTS

The results of the OFRA are dependent on the specific reward assigned to the nodes in the network. Using the hierarchy of node attributes described in Appendix A and guidelines supplied by the IIP, results use the node rewards shown in Table 6 of Appendix A. These rewards combine to provide the total reward from flights actually flown by the IIP and the flight routes from OFRA. Although the IIP prefers to fly cardinal headings, to investigate potential added benefit, the OFRA examines routes flown at 15 degree increments (0 to 90 degrees). In all cases, the OFRA produces
flight routes (one route for each actual flight flown by the IIP) with higher rewards than those of the IIP (Table 1). The routes produced by the OFRA are near the southern LAKI, within the Labrador Current, and flown in an east-west (0 degrees) direction. Except for minor differences, the routes produced by the OFRA cover the same area in all 11 instances. Figure 8 shows one of the routes. All routes are similar since the data is from three consecutive ICERECDETs where the LAKI does not change substantially. The only change to the rewards, using the weights of Table 6 in Appendix A, is the change in

<table>
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<tr>
<th>ROUTE</th>
<th>IIP REWARD</th>
<th>OFRA REWARD</th>
<th>OFRA HEADING (DEGREES)</th>
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</thead>
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<tr>
<td>ICERECDET 1a</td>
<td>5.28</td>
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<td>ICERECDET 1b</td>
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<td>ICERECDET 1c</td>
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</tr>
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<tr>
<td>ICERECDET 3d</td>
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<td>16.87</td>
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</table>

Table 1. This table shows the reward obtained from the actual flights flown during ICERECDETs 1, 2, and 3, by the IIP and routes produced by the OFRA. The OFRA routes outperform the IIP routes and use a cardinal heading of 0 in all cases. Computations for the above table used the rewards in Table 6 of Appendix A.
the iceberg locations (the time since last visit for each node is set to 14 days). Therefore the optimal area to search, when looking at flying only one route independent of all other routes, remains reasonably constant. The comparison of individual routes is not necessarily valid since previously flown routes by the IIP are not taken into consideration. The section below investigates this dependence on previously flown routes.

A comparison is made between the total rewards from all flights of each ICERECDET and the total from the same number of flights produced by the OFRA. For example, from the three flights flown during ICERECDET 1, the total reward from these three flights is compared to the total reward produced from the OFRA which determines the three consecutive optimal flight routes. (To produce the three consecutive optimal flight routes, OFRA first determines the best single flight route; assuming this route is flown it then determines the best single flight route; assuming the past two calculated routes are flown, it then determines the best single flight route. Planning three routes concurrently may produce routes with a higher total reward but OFRA calculates only one route at a time to match IIP route planning under highly variable conditions). As shown in Table 2, the OFRA again produces routes that outperform the actual flights flown. In one instance, the OFRA produces a route to be flown at 45 degrees along the LAKI, thus taking advantage of non-cardinal
<table>
<thead>
<tr>
<th>ICERECDET</th>
<th>NUMBER OF FLIGHTS</th>
<th>IIP TOTAL REWARD</th>
<th>OFRA TOTAL REWARD</th>
<th>OFRA ROUTE HEADINGS (DEGREES)</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>65.99</td>
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<td>4</td>
<td>52.34</td>
<td>62.74</td>
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</tr>
</tbody>
</table>

Table 2. This table shows the number of flights flown by the IIP during the ICERECDETs. Also depicted are the total rewards for the routes flown by the IIP and routes from the OFRA for each ICERECDET. The headings for each route from the OFRA are shown. The routes from the OFRA have greater reward in all cases than those flown by the IIP.

headings. Figures 9 to 11 show the search routes flown by the IIP while Figures 12 to 14 show the routes produced by the OFRA.

B. FLYING LAKI

Using the rewards in Table 6 of Appendix A, the OFRA produces routes that not only include a section of the LAKI but also include areas such as the Labrador current. There may be times, such as in the middle of the ice season when the perimeter of the LAKI is considerably large, that patrolling the LAKI is the only priority (border patrol). It then must be decided which areas of the LAKI should be searched. To reflect the higher priority of searching the LAKI, the rewards in Table 7 of Appendix A are assigned. Because the focus is now solely on the LAKI, the OFRA can take full advantage of
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the non-cardinal headings in determining the routes. In other words, routes that parallel the LAKI are the most desirable (Figures 15 and 16). Using these rewards, a comparison is made between routes that can have non-cardinal headings with routes that are strictly flown on cardinal headings. Also a comparison is made once again between the total of the rewards from all flights of each ICERECDET and the total from the same number of flights produced by the OFRA. The results in Table 3 and Table 4 support the obvious advantage of routes that parallel the LAKI. Figures 17 to 19 depict the OFRA routes for the three ICERECDETs using the rewards of Table 7 of Appendix A.

C. 20 nm VS. 25 nm DISTANCE BETWEEN FLIGHT LEGS

As previously mentioned, the IIP flies the search phase of a flight using a constant distance between flight legs. Normally, because of adverse weather conditions, the distance between flight legs is 25 nm. When icebergs need to be located visually, flight leg spacing is 20 nm. It is obvious that as the distance between flight legs is increased, the amount of area that can be covered during a flight also increases. Using the rewards obtained from visiting nodes, a numerical comparison is made of the area that can be covered when using 20 nm and 25 nm flight leg spacing (Table 5). The number of nodes that can be visited flying 20 nm between flight legs is approximately 64 percent of the number that can
### Table 3
This table shows the reward obtained from the actual flights flown during ICERECDETs 1, 2, and 3 by the IIP and routes from the OFRA. The OFRA routes outperformed the IIP routes using non-cardinal headings most of the time (the reward obtained from OFRA using cardinal heading 0 degrees is also given in parenthesis). Computations for the above table used the rewards in Table 7 of Appendix A.

<table>
<thead>
<tr>
<th>ROUTE</th>
<th>IIP REWARD</th>
<th>OFRA REWARDS</th>
<th>OFRA HEADING (DEGREES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICERECDET 1a</td>
<td>5.55</td>
<td>21.04 (18.77)</td>
<td>45 (0)</td>
</tr>
<tr>
<td>ICERECDET 1b</td>
<td>13.99</td>
<td>21.60 (20.42)</td>
<td>45 (0)</td>
</tr>
<tr>
<td>ICERECDET 1c</td>
<td>14.40</td>
<td>22.40 (21.43)</td>
<td>45 (0)</td>
</tr>
<tr>
<td>ICERECDET 2a</td>
<td>20.11</td>
<td>20.30 (19.51)</td>
<td>60 (0)</td>
</tr>
<tr>
<td>ICERECDET 2b</td>
<td>12.78</td>
<td>23.00 (21.86)</td>
<td>60 (0)</td>
</tr>
<tr>
<td>ICERECDET 2c</td>
<td>12.74</td>
<td>21.73 (20.22)</td>
<td>60 (0)</td>
</tr>
<tr>
<td>ICERECDET 2d</td>
<td>7.18</td>
<td>21.46 (19.32)</td>
<td>75 (0)</td>
</tr>
<tr>
<td>ICERECDET 3a</td>
<td>16.79</td>
<td>21.29 (20.04)</td>
<td>15 (0)</td>
</tr>
<tr>
<td>ICERECDET 3b</td>
<td>13.03</td>
<td>22.07 (20.65)</td>
<td>60 (0)</td>
</tr>
<tr>
<td>ICERECDET 3c</td>
<td>8.24</td>
<td>19.84 (19.84)</td>
<td>0 (0)</td>
</tr>
<tr>
<td>ICERECDET 3d</td>
<td>16.46</td>
<td>19.95 (19.95)</td>
<td>0 (0)</td>
</tr>
</tbody>
</table>

### Table 4
This table shows the number of flights flown by the IIP on each ICERECDET. Also depicted are the total rewards for the routes flown by the IIP and routes from the OFRA for each ICERECDET. The headings for the OFRA routes are given. The OFRA routes are better in all cases than those flown by the IIP. Computations for the above table used rewards in Table 7 of Appendix A.

<table>
<thead>
<tr>
<th>ICERECDET</th>
<th>NUMBER OF FLIGHTS</th>
<th>IIP TOTAL REWARD</th>
<th>OFRA TOTAL REWARD</th>
<th>OFRA ROUTE HEADINGS (DEGREES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>33.94</td>
<td>58.88</td>
<td>45,75,0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>52.81</td>
<td>78.54</td>
<td>60,0,90,30</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>54.52</td>
<td>72.00</td>
<td>0,45,75,0</td>
</tr>
</tbody>
</table>
Table 5. This table shows the rewards obtained for each route produced by the OFRA for the ICERECDETs. One set of routes used 20 nm spacing between flight legs while the other set used 25 nm. The results show the advantage of flight routes with greater distance between legs.

be reached using 25 nm leg spacing. Hence, the obvious advantage of increasing the distance between flight legs.

D. EXECUTION TIME

The formulation in Appendix C was implemented in GAMS (Brooke, Kendrick, and Meeraus, 1992) and run using the OSL solver (OSL, 1991). Computational time for a network containing 28 nodes with a route limit of 15 nodes, was approximately 1.71 hours. The run time appears to increase exponentially as the network or allowable route length
increases.

The constraints in Appendix C, which the reader can modify to examine other types of routes, greatly reduce the potential number of feasible routes. This led to the coding of a similar formulation in Pascal which produced comparable results in considerably faster run times through complete enumeration. The majority of the time needed to produce an optimal route is not in the OFRA but in the data processing that leads up to the OFRA. Extracting iceberg locations from the active iceberg listing, developing the network, assigning icebergs to nodes in the network, determining the LAKI, and calculating the weight for the nodes, takes approximately 2 minutes and 15 seconds. The time required for the OFRA to determine the optimal route on a network containing over 2600 nodes is less than five seconds. Compare this with the times reported in previous work presented in chapter III and the advantage of the added constraints to the generic orienteering problem becomes obvious. Another factor influencing the total execution time is the number of non-cardinal headings the OFRA examines. For this study, 15 degree increments are used but 1 degree increments are conceivable. Each increment adds approximately 13 seconds to the run time to update the network after it is rotated prior to the execution of the OFRA and calculate the optimal route.
V. CONCLUSIONS

Provided with a break down of the IIP operating area in terms of area priorities and an active iceberg listing, the OFRA produces the optimal route of flight. This makes the OFRA an excellent tool for pre-flight planning. These routes, of course, are for the ideal situation and do not take into consideration such factors as current weather conditions which can disrupt even the best planned flights. Even so, the OFRA provides the IIP with a starting point in the flight planning process. The flexibility of the OFRA, in its ability to allow changes in the priorities that it uses, can help even when the ideal situation does not exist. For example, if the weather report is unfavorable in the south, the IIP can change the priority of the southern area from top priority to lowest priority and run the OFRA to determine the optimal route of flight for the updated priority list.

The OFRA also shows the IIP the benefits that can be obtained when non-cardinal headings are flown. This is especially true when coverage of the LAKI is paramount and the LAKI is not orientated along cardinal headings.
APPENDIX A. NODE REWARD CALCULATIONS

This appendix presents the hierarchy of node attributes used to determine a node’s reward. Also included are Tables 6 and 7 which contain the reward values that this thesis uses in all computations of route rewards. The following is an example of calculating a node’s reward; A node, which has not been visited in 8 days, located in the southern part of the operating area (inside the Labrador Current), less than 30 nm from the LAKI would have a score of 0.51 (0.07 + 0.14 + 0.06 + 0.24).

A. HIERARCHY OF NODE ATTRIBUTES

Using inputs from the IIP, this thesis uses the following hierarchy of node attributes (nodes in the south area having the highest priority followed by nodes in the southeast, southwest and east):

1. Nodes (with icebergs) <= 30 nm from LAKI.
2. Nodes (without icebergs) <= 30 nm from LAKI.
3. Nodes (with icebergs) > 30 nm & <= 60 nm from LAKI.
4. Nodes (without icebergs) > 30 nm & <= 60 nm from LAKI.
5. Nodes (with icebergs) > 60 nm & <= 90 nm from LAKI.
6. Nodes (without icebergs) > 60 nm & <= 90 nm from LAKI.
7. Nodes (with icebergs) > 90 nm from LAKI.
8. Nodes (without icebergs) > 90 nm from LAKI.

Nodes in the Labrador Current have a higher priority than
nodes outside the current and nodes on top of the Grand Bank. Finally, nodes that have been visited most recently, have the lowest priority.
### Table 6. Rewards calculated using inputs from the IIP.

<table>
<thead>
<tr>
<th>Node</th>
<th>Area Location</th>
<th>Node Location</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 30 nm from LAKI with bergs</td>
<td>S (0.14) + (0.24) = 0.38</td>
<td>SE (0.07) + (0.24) = 0.31</td>
<td>SW (0.03) + (0.24) = 0.27</td>
</tr>
<tr>
<td>31 to 60 nm from LAKI with bergs</td>
<td>S (0.14) + (0.12) = 0.26</td>
<td>SE (0.07) + (0.12) = 0.19</td>
<td>SW (0.03) + (0.12) = 0.15</td>
</tr>
<tr>
<td>61 to 90 nm from LAKI with bergs</td>
<td>S (0.14) + (0.06) = 0.20</td>
<td>SE (0.07) + (0.06) = 0.13</td>
<td>SW (0.03) + (0.06) = 0.09</td>
</tr>
<tr>
<td>&gt; 90 nm from LAKI with bergs</td>
<td>S (0.14) + (0.03) = 0.17</td>
<td>SE (0.07) + (0.03) = 0.10</td>
<td>SW (0.03) + (0.03) = 0.06</td>
</tr>
<tr>
<td>Ocean region</td>
<td>In Lab: nodes score + 0.06</td>
<td>Out Lab: nodes score + 0.03</td>
<td>Banks: nodes score + 0.01</td>
</tr>
</tbody>
</table>

Time since node visited
- 0..6 days: nodes score + 0.00
- 7..13 days: nodes score + 0.07
- >= 14 days: nodes score + 0.13

Ocean region

<table>
<thead>
<tr>
<th>Ocean region</th>
<th>In Lab:</th>
<th>Out Lab:</th>
<th>Banks:</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Lab:</td>
<td>nodes score + 0.06</td>
<td>Out Lab:</td>
<td>nodes score + 0.03</td>
</tr>
<tr>
<td>Node</td>
<td>Area Location</td>
<td>Node Location</td>
<td>Total</td>
</tr>
<tr>
<td>--------------------------</td>
<td>---------------</td>
<td>---------------</td>
<td>---------</td>
</tr>
<tr>
<td>&lt; 30 nm from LAKI with bergs</td>
<td>S (0.14) + (0.45) = 0.59</td>
<td>SE (0.07) + (0.45) = 0.52</td>
<td>SW (0.03) + (0.45) = 0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.45) = 0.46</td>
<td></td>
</tr>
<tr>
<td>&lt; 30 nm from LAKI with no bergs</td>
<td>S (0.14) + (0.34) = 0.48</td>
<td>SE (0.07) + (0.34) = 0.41</td>
<td>SW (0.03) + (0.34) = 0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.34) = 0.35</td>
<td></td>
</tr>
<tr>
<td>31 to 60 nm from LAKI with bergs</td>
<td>S (0.14) + (0.00) = 0.14</td>
<td>SE (0.07) + (0.00) = 0.07</td>
<td>SW (0.03) + (0.00) = 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.00) = 0.01</td>
<td></td>
</tr>
<tr>
<td>31 to 60 nm from LAKI with no bergs</td>
<td>S (0.14) + (0.00) = 0.14</td>
<td>SE (0.07) + (0.00) = 0.07</td>
<td>SW (0.03) + (0.00) = 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.00) = 0.01</td>
<td></td>
</tr>
<tr>
<td>61 to 90 nm from LAKI with bergs</td>
<td>S (0.14) + (0.00) = 0.14</td>
<td>SE (0.07) + (0.00) = 0.07</td>
<td>SW (0.03) + (0.00) = 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.00) = 0.01</td>
<td></td>
</tr>
<tr>
<td>61 to 90 nm from LAKI with no bergs</td>
<td>S (0.14) + (0.00) = 0.14</td>
<td>SE (0.07) + (0.00) = 0.07</td>
<td>SW (0.03) + (0.00) = 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.00) = 0.01</td>
<td></td>
</tr>
<tr>
<td>&gt; 90 nm from LAKI with bergs</td>
<td>S (0.14) + (0.00) = 0.14</td>
<td>SE (0.07) + (0.00) = 0.07</td>
<td>SW (0.03) + (0.00) = 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.00) = 0.01</td>
<td></td>
</tr>
<tr>
<td>&gt; 90 nm from LAKI with no bergs</td>
<td>S (0.14) + (0.00) = 0.14</td>
<td>SE (0.07) + (0.00) = 0.07</td>
<td>SW (0.03) + (0.00) = 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E (0.01) + (0.00) = 0.01</td>
<td></td>
</tr>
<tr>
<td>Ocean region</td>
<td>In Lab: nodes score + 0.06</td>
<td>Out Lab: nodes score + 0.03</td>
<td>Banks: nodes score + 0.01</td>
</tr>
<tr>
<td>Time since node visited</td>
<td>0..6 days: nodes score + 0.00</td>
<td>7..13 days: nodes score + 0.07</td>
<td>&gt;= 14 days: nodes score + 0.13</td>
</tr>
</tbody>
</table>

Table 7. Rewards calculated when flying LAKI is the only priority.
This appendix provides descriptions of the functions located in the iceberg program. These functions are essential in processing the data required by the OFRA.

A. DETERMINING THE LAKI

The iceberg program reads into a file the latitude and longitude of each iceberg from the IIP’s Active Berg Listing. (The Active Berg Listing is a list of all icebergs currently present in the IIP’s drift and deterioration models). From this file of icebergs, the iceberg program then determines the LAKI which defines a convex region that encompasses all the icebergs (U.S. Coast Guard, 1992).

The icebergs furthest north, east, south, and southwest, referred to here as the extreme icebergs, connect and form the initial LAKI. The next step is to determine if any icebergs fall outside this initial LAKI. The LAKI procedure accomplishes this one section at a time, starting with the northeastern part of the area, and proceeding clockwise around to the southwestern area. The northeastern section (using an example in which the extreme northern iceberg’s longitude is greater than the extreme eastern iceberg’s longitude) will be explained in detail. The first step in locating icebergs outside of the initial LAKI in the northeast section is to determine all icebergs that are within the "box" formed using
the extreme northern iceberg as the upper left hand corner and
the extreme eastern iceberg as the lower right hand corner.
Lines are then drawn from each of the icebergs in the box to
the extreme eastern iceberg. The slopes of these lines are
compared to the base-slope (which is the slope of the line
connecting the two extreme icebergs). If the slope is greater
than the base-slope then the iceberg corresponding to that
slope is located outside the LAKI (since we are assuming the
extreme northern iceberg's longitude is greater than the
extreme eastern's longitude). This produces a set $A$, of
icebergs outside the initial LAKI. Next, the slopes produced
from drawing a line from the extreme northern iceberg to each
iceberg in set $A$ are examined. The iceberg producing the
smallest slope is then added to the LAKI with the new boundary
of the LAKI now being drawn from the original extreme northern
iceberg, through this new iceberg and down to the extreme
eastern iceberg. The next step is to determine if any of the
original icebergs outside the initial LAKI (set $A$) are still
outside the new boundary produced by the new northern iceberg
and the original extreme eastern iceberg. If such icebergs
exist, then the above process starts over and continues until
all icebergs are within the LAKI. The above procedure then
continues with the other sections of the area. Once the above
process of determining the LAKI is complete, nodes (containing
latitudes and longitudes) are then placed on the boundaries to
be used as references when calculating distances to the LAKI.
B. SMALL NETWORK

With the establishment of the LAKI, the iceberg program then divides the IIP operating area up into nodes, with each node representing a fixed number of square miles. This set of nodes is called the "small" network (Figure 20). A number of procedures in the iceberg program provide each node in the small network with the following information:

- The latitude and longitude which is centered within the area defined by the node.
- The number of icebergs within the area the node represents.
- The node's location within the operating area (E, SE, S, or SW) as defined in Figure 6.
- The ocean region where the node is located (inside the Labrador current, outside the current or on top of the Grand Bank).
- Distance from the LAKI.
- Time since it was last visited.
- Node's reward ($R_{ij,t}$).

To determine each node's latitude and longitude a loop starts in the upper left hand corner of the operating area (52°N, 57°W), and proceeds left to right down to the lower right hand corner of the area (38°N, 39°W) incrementing the latitude and longitude the required number of degrees in order to maintain the desired distance between nodes (i.e. 25 nm or 20 nm). To determine the number of icebergs located near a particular node one simply takes each iceberg from the
iceberg file (created earlier) and determines which node it is closest to by calculating the distance using the latitudes and longitudes from the nodes and iceberg. The IIP provides coordinates that divide the operation area into sections (Figure 6) and that define the Labrador current and Grand Bank. Using these coordinates and each node’s own position, the iceberg program determines in what section of the area and ocean region the nodes lie. To calculate the distance of each node from the LAKI, the iceberg program uses the latitudes and longitudes of the nodes in the small network and the nodes that were placed on the LAKI after it was created. Finally, the time since the node was last visited is initialized at 14, representing the average number of days between visits to
any one particular area.

C. BIG NETWORK

The big network is 56 rows by 47 columns (when using 25 nm between nodes) and "lies" centered on top of the small network. The size of the big network enables it to be rotated at any angle about its center without exposing the small network. In other words, the smallest side of the big network is greater than the largest diagonal of the small network. Each node in the big network contains the following information:

- The latitude and longitude which is centered within the area defined by the node.
- Neighborhood nodes (nodes that are located on all sides of the node of interest).
- Distance from the node to ST. John’s

A looping procedure determines the latitude and longitude of each node in the big network. The loop starts at 56.9166°N, 62.0952°W, and proceeds left to right downward until it is large enough to cover the small network at any rotated angle. The nodes adjacent to each node (a node’s neighbors) are determined during the creation of the network, recording the nodes that occur on all four sides of each node. The distance from each node to ST. John’s (47.37°N, 52.45°W) is calculated using euclidean distance.

To evaluate routes flown at other than cardinal headings
(between 0 and 90 degrees), the big network is rotated that many degrees above the small network. Once rotated, each node in the small network assigns it's score to the closest node in the big network. To rotate the big network, a procedure in the iceberg program takes each individual node in the big network and determines it's position relative to the center of the operating area (45°N, 48°W). Using trigonometric functions, each node moves "around" the center of the area by the number of degrees desired in the rotation. It is this rotation of the big network that enables the examination of other than cardinal headings. Once rotated, the OFRA is then implemented to find the feasible route which produces the highest score.
The problem of determining the optimal route of flight is presented below in NPS format after the introduction of appropriate notation.

Indices:

- $b,e$ beginning and ending nodes;
- $i,j$ node;
- $s$ search mode \{TIN = Transit in, FSX = First search in X direction, FSY = First search in Y direction, SX = Search in X direction, SY = Search in Y direction, FTOUT = First transit out, TOUT = Transit out\};
- $x_l$ Latitude; and
- $t,t'$ time ($t = 0,1,2,3,...T$; where $T$ equates to the maximum allowable flight distance).

Data:

- $R_{i,j,s,t}$ reward for going from node $i$ to node $j$ during search mode $s$ at time $t$;
- $\text{Lines}_{x_l}$ set of all nodes on Latitude $x_l$;
- $\text{OUT}_i$ set of all nodes that can be reached from node $i$ in one step (step refers to a transition in one time period);
- $\text{OUTX}_i$ set of all nodes which can be reached from node $i$ in one step along X direction;
OUTY$_i$, set of all nodes which can be reached from node i in one step along Y direction;
IN$_i$, set of all nodes that can reach node i in one step;
INX$_i$, set of all nodes that can reach node i in one step along the X direction;
INY$_i$, set of all nodes that can reach node i in one step along the Y direction;
LengthX, maximum allowable length of flight leg in direction X;
ABOVE$_1j$, node one step directly above node j on same longitude as node j;
BELOW$_1j$, node one step directly below node j on same longitude as node j; and
BELOW$_j$, set of all nodes below node j on same longitude as node j.

Decision Variables:

$X_{i,j,s,t}$, 1 if go from node i in search mode s to node j at time t, and 0 otherwise;

Formulation:

\[
\text{MAXIMIZE } \sum_{i} \sum_{j} \sum_{s} \sum_{t} R_{i,j,s,t} X_{i,j,s,t}
\]

Subject to:
\[ \sum_{j \in \text{OUT}_b} X_{b,j,TIN,1} = 1 \] (6)

\[ \sum_{i \in \text{IN}_j} X_{i,j,TIN,t} = \sum_{i \in \text{OUT}_j} X_{j,i,TIN,t+1} + \sum_{i \in \text{OUT}_j} X_{j,i,FSX,t+1} \forall j \neq b, e, 1 \leq t < T \] (7)

\[ \sum_{i \in \text{IN}_j} X_{i,j,FSX,t} + \sum_{i \in \text{IN}_j} X_{i,j,FSY,t} + \sum_{i \in \text{IN}_j} X_{i,j,SX,t} + \sum_{i \in \text{IN}_j} X_{i,j,SY,t} = \] (8)

\[ \sum_{i \in \text{OUT}_j} X_{j,i,SX,t+1} + \sum_{i \in \text{OUT}_j} X_{j,i,SY,t+1} + \sum_{i \in \text{OUT}_j} X_{j,i,FTOUT,t+1} + \sum_{i \in \text{OUT}_j} X_{j,i,FSY,t+1} \forall j \neq b, e, 1 \leq t < T \] (9)

\[ \sum_{i \in \text{IN}_j} X_{i,j,FTOUT,t} + \sum_{i \in \text{IN}_j} X_{i,j,TOUT,t} = \sum_{i \in \text{OUT}_j} X_{j,i,TOUT,t+1} \forall j \neq b, e, 3 \leq t \leq T \] (10)

\[ \sum_{j \in \text{IN}_e} \sum_{t=3}^{T} X_{j,e,FTOUT,t} + \sum_{j \in \text{IN}_e} \sum_{t=3}^{T} X_{j,e,TOUT,t} = 1 \] (11)

\[ \sum_{i \in \text{IN}_j} \sum_{s} \sum_{t} X_{i,j,s,t} \leq 1 \quad \forall j \] (12)

\[ \sum_{i \in \text{IN}_j} X_{i,j,FSY,t} + \sum_{i \in \text{IN}_j} X_{i,j,SY,t} + \sum_{i \in \text{OUT}_j} \sum_{t} X_{j,i,SY,t} \leq 1 \quad \forall j \] (12)
\[ \sum_{(i,j) \in \text{lines}_{x}} \sum_{t} (X_{i,j,sx,t} + X_{i,j,FSX,t}) \leq \text{Length}_{X} \quad \forall x \]  \hspace{1cm} (13) \]

\[ \sum_{i} \sum_{j} \sum_{s} X_{i,j,s,t} \leq 1 \quad \forall t \hspace{1cm} (14) \]

\[ \sum_{i \in \text{IN}_{j}} \sum_{t} X_{i,j,FSX,t} + \sum_{i \in \text{IN}_{j}} \sum_{t} X_{i,j,sx,t} \leq \sum_{i \in \text{INA}_{j}} \sum_{t} X_{j,i,sx,t} \]
\[ + \sum_{i \in \text{IB}_{j}} \sum_{t} X_{j,i,sx,t} + \sum_{i \in \text{INA}_{j}} \sum_{t} X_{j,i,FSX,t} + \sum_{i \in \text{IB}_{j}} \sum_{t} X_{j,i,FSX,t} \quad \forall j \hspace{1cm} (15) \]

\[ X_{i,j,SY,t} \leq \sum_{i \in \text{BELOW}_{j}} \sum_{t} X_{i,j,FSY,t} + \sum_{i \in \text{BELOW}_{j}} \sum_{t} X_{i,j,FSY,t} \quad \forall i,j,t \hspace{1cm} (16) \]

\[ \sum_{i} \sum_{j} \sum_{t} X_{i,j,FSX,t} = 1 \hspace{1cm} (17) \]

The constraints translate as follows:

(6) The route must leave b (ST. John's) during the transit-in phase.

(7) The route, while in the transit-in phase, can continue to transit-in or proceed to the first search in the X direction (FSX).

(8) The route, once in the search phase, can continue to
search or proceed to the first transit-out (FTOUT) step.

(9) The route, once in the transit-out phase, must continue to transit-out.

(10) The route must return to e (ST. John’s) either from the first transit-out step or subsequent transit-out steps.

(11) Visit node at most once per sortie.

(12) Limit search in Y direction to 1 node to maintain equal distances between search legs in X direction.

(13) Maintain length of flight legs within require limits (must be at least one because of the requirement of a first search in the Y direction which forces a first search in the X direction).

(14) Visit at most one node per time period.

(15) Requires each leg of the search in the X direction to have a corresponding leg either directly above or below it. Thus preventing search patterns like the one depicted in Figure 21.

(16) Requires each search in the Y direction to have located below it on the same longitude either the first search in the X direction (FSX) or the first search in the Y direction (FSY). Thus preventing search patterns like the one depicted in Figure 22.

(17) Must have an initial movement in the Y direction.
Figure 21. Search pattern prevented by constraint 15.

Figure 22. Search pattern prevented by constraint 16.
LIST OF REFERENCES


U.S. Coast Guard, 1992, Standing Orders for IIP Operations Center Duty Personnel, CIIPINST M3120B, Commander,
International Ice Patrol, Groton, CT.


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