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Military Cost-Benefit Analysis: A Multi-Attribute Three-Stage Procurement Model

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A Multi-Attribute Three-Stage Procurement Model

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Introduction

• Large & rising federal debt, shrinking discretionary budget
  – Budget uncertainty!
• Defense procurement typically requires vendors to submit bids which include
  – Price
  – Performance attributes
• Problem: Optimal vendor choice may change with changes in the budget!
Budget Constraint

• Based on an “Economic Evaluation of Alternatives” (EEoA)* approach:
  – The procurement agency buyer reveals desired attributes and the budget for the program
  – Vendor offers (bids) consist of product proposals to produce a set of performance attributes for a given budget authority
  – The procurement agency buyer selects a vendor according to the buyer’s (“secret”) weighting of the attributes (i.e. a multi-attribute value function)

Model Structure

Stage 1

Buyer specifies attribute set and budget information

Stage 2

Vendor 1 offers bid

Vendor i offers bid

Vendor n offers bid

Stage 3

Buyer selects winning bid according to its value function
Model

- $n$ vendors
- Set of attributes $A \ (1, \ldots, m)$
- Vendor $i$’s offer is $A_i = [a_{i1}, \ldots, a_{im}]$
- Buyer’s “secret” value function (MOE) is $V(A_i)$
- Budget level is $B$
- Buyer makes selection decision according to:

$$\max_i \quad V(A_i) = \sum_{j=1}^{m} w_j a_{ij}$$
Vendor’s Decision Problem

• Private information on production capabilities and costs:
  – Captured by cost functions \( c_{ij}(a_{ij}) \)
• Does not know \( V \), but forms beliefs about the buyer’s preferences
• “Best guess” \( \gamma_i = (\gamma_{i1}, \ldots, \gamma_{im}) \)
• Results in a hypothetical value function to maximize:
  \[
  Q(A_i) = \sum_{j=1}^{m} \gamma_{ij} a_{ij}
  \]
Vendor’s Decision Problem

• Vendor $i$’s problem can be expressed as:

$$\max_{a_{ij}} Q(A_i) = \sum_{j=1}^{m} \gamma_{ij} a_{ij}$$

s.t.  \( TC_i = \sum_{j=1}^{m} c_{ij} (a_{ij}) \leq B \)
Simplified Approach

• For the sake of clarity, the remainder of the analysis will assume:

  Two attributes

  Two vendors
Solution to Vendor’s Problem

• A vendor’s best offer (bid) will be a combination of attribute levels that uses the entire budget, and satisfies the condition:

\[
\frac{\gamma_i}{c_i(a_i)} = \frac{\gamma_j}{c_j(a_j)}
\]

• The buyer then chooses the vendor that maximizes its military effectiveness value, \( V \), for the planned budget, \( B \)
Budget Uncertainty

• Now, instead of $B$, consider a range of possible budgets: $B_1, \ldots, B_k$
• Each vendor submits an offer (bid) for each of the $k$ possible budgets
• This set of offers from a vendor constitutes an “expansion path”
Examples

• Let the vendors have cost functions of the form:

\[ c_{ij}(a_{ij}) = \alpha_{ij} e^{\beta_{ij}a_{ij}}, \text{ where } \alpha_{ij}, \beta_{ij} > 0 \]

• \( B_1 = 5, B_2 = 10, B_3 = 15, B_4 = 20, B_5 = 25, B_6 = 30 \)

• We will examine several cases where the vendors differ in their cost functions and/or beliefs about the weight the buyer places on the attributes
\[ \alpha_{11} = \alpha_{12} = 2.0, \beta_{11} = \beta_{12} = 0.6, \alpha_{21} = \alpha_{22} = 1.0, \beta_{21} = \beta_{22} = 1.0, \gamma_{11} = 0.7, \gamma_{21} = 0.7 \]
Expansion Paths - Differing Beliefs ($\gamma$)

$$\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 2.0, \beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} = 0.6, \gamma_{11} = 0.5, \gamma_{21} = 0.7$$
\[
\alpha_{11} = \alpha_{12} = 2.0, \beta_{11} = \beta_{12} = 0.6, \alpha_{21} = \alpha_{22} = 1.0, \beta_{21} = \beta_{22} = 1.0, \gamma_{11} = 0.5, \gamma_{21} = 0.7
\]
Switch to Budget-Value Space

• What is the value to the buyer (procurement agency; warfighter) provided by each vendor for a specific budget authority?
• What is the value to the buyer provided by each vendor over all possible budget levels?

• Assume the two vendors have the properties from the last graph, and that the buyer places a weight of 0.7 on attribute 1.
Traditional Price & Performance Bid

Value by Budget Level

- Vendor 1
- Vendor 2
Air Tanker Costs for Given Level of Effectiveness (Boeing vs. EADS?)

Value by Budget Level

Budget

Vendor 1
Vendor 2
Vendor Bids: Performance Offers over a Range of Budgets
Next Steps

• Model the budget uncertainty with a probability distribution, and determine the expected utility provided by each vendor

• Include uncertainty in vendor performance (quantity, quality, schedule) promises
  – May be framed as either cost uncertainty or performance uncertainty or both (depends on the particular contract structure)