Inexpensive global location and tracking systems using geostationary satellites.

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THESIS

INEXPENSIVE GLOBAL LOCATION AND TRACKING SYSTEMS USING GEOSTATIONARY SATELLITES

by

Danny L. DeFries

June 1989

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Danny L. DeFries

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Abstract
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Inexpensive Global Location and Tracking Systems Using Geostationary Satellites

by

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Inexpensive Global Location and Tracking Systems are currently being designed to provide the civilian market low-cost radio position determination. This paper discusses two possible designs. The first design employs 3 or 4 satellites, depending on whether altitude is known a priori, each transmitting continuous ranging signals. The user transceiver receives the ranging signals, measures the time differentials of the receipt of the signals and transfers this information to a control station via a satellite link. The control station computes the user position from this data and sends the position coordinates back to the user via another satellite link. In the second design, each user transceiver transmits a unique code to the control station via the 3 or 4 satellite links, again depending on whether the altitude is known a priori. The control station measures the time differentials of the receipt of the signals and determines the user position. This position information is then transmitted back to the user via a satellite link.
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I. INTRODUCTION

Today, the United States Department of Defense (DOD) is utilizing the NAVSTAR Global Positioning System (GPS) for accurate position determination. GPS employs 18 geosynchronous satellites within 6 different orbits with 3 satellites in each orbit. GPS provides highly accurate position determination to an unlimited number of users worldwide, although the position determination accuracy provided to the civilian community is intentionally degraded due to national defense reasons. The degraded mode still enables the civilian community to attain accuracies of 100 meters.

In addition to the high number of operational satellites that are required to support the 24-hour world-wide GPS system, the user equipment cost is also quite high due to its complexity. Even though the user equipment cost has decreased considerably since GPS was first introduced, the civilian community can not afford the user equipment. To lower the cost of the position determination system so that the civilian community can afford to install position determination equipment in one's vehicle, a simpler system has to be designed and the total number of satellites deployed has to be reduced.

This thesis presents two low-cost concepts of vehicle location and tracking using geostationary satellites. The first concept is borrowed from the concept of geostationary satellite navigation systems presented in [Ref. 1]. The second concept is a new concept that is presented in this thesis. For the systems to be utilized by the civilian community, the systems have to be lower in cost than GPS, provide reasonable position accuracy, and be reliable. To cover the United States, each system requires four satellites to determine longitude, latitude, and altitude, or three satellites if altitude is not measured. Like the geostationary satellite navigation system, the location and tracking systems will also deploy three or four geostationary satellites parked in orbits on the equator at a latitude of zero degrees and altitude of approximately 35,786 kilometers. The difference between the proposed location and tracking system and the geostationary navigation system is that the location and tracking system user does not determine his position but relies on the central control station to determine it. The satellites continuously and synchronously transmit a satellite-unique ranging code to the user on earth. The user receives the satellites signals at different times due to the path length differences between the user and each of the satellites. The transceiver measures the time differentials between a designated signal and the remaining signals and transmits this information to a
control station via a satellite communications link. The control station calculates the user position from the time differentials and transmits the user coordinates to the user via another satellite channel. By placing the computational burden on the central control station, the location and tracking transceiver can be made at a lower cost.

Position determination in a geostationary coordinate system is normally defined in a geocentric coordinate system as shown in Figure 1. The x-axis is the intersection of the equator plane (0° latitude) and the Greenwich meridian plane (0° longitude) and is oriented from the center of the earth, the z-axis is the polar axis oriented from south to north, and the y-axis completes the right-handed Cartesian coordinate system. We designate the unknown user position by \((x, y, z)\), the known positions of \(n = 3\) or 4 geostationary satellites by \((x_i, y_i, z_i)\) where \(i = 1, 2, ..., n\), and the known position of the central control station by \((x_0, y_0, z_0)\).

![Geocentric Coordinate System](image)

Figure 1. Geocentric Coordinate System: [Ref. 1]
The main function of the transceiver is to measure the differential distance

\[ a_i = d_i - d_i, \quad i = 1, 2, ..., n \]  

(1)

representing the differences in the distance to the user from one satellite, arbitrarily designated as satellite one, and from each of the other satellites. The variable \( a_i \) represents the difference in the distance of satellite one to the user and that of each of the other satellites to the user. The variable \( d_i \) represents the distance from each satellite to the user. The differential distance determination concept is discussed later. The squared distance from the user position to each geostationary satellite can be expressed by

\[ d_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2, \quad i = 1, 2, ..., n \]  

(2)

where \((x, y, z)\) represent the users coordinate position in the Cartesian coordinate system and \((x_i, y_i, z_i)\) represents each of the satellites’ positions.

For \(n = 4\), (1) and (2) combine to form a set of four independent equations and four unknowns, namely, \(x, y, z\), and \(d_i\). These four equations are

\[ (d_i - a)^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2, \quad i = 1, 2, ..., 4 \]  

(3)

which can be solved using the measured data \(a\), and the known satellite ephemeris \((x_i, y_i, z_i), i = 1, 2, 3, 4\). The value \(a\) is found through the use of a correlation register and ranging codes which will be discussed later. A detailed discussion of the concept can be found in [Ref. 1].

For the three satellite system, \(n = 3\), we use

\[ x^2 + y^2 + z^2 = (R + h)^2 \]  

(4)

where \(R = 6378\) km is the Earth’s mean equatorial radius, and \(h\) is the user altitude (assumed to be known), as the fourth equation in addition to the three equations

\[ (d_i - a)^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \quad i = 1, 2, 3. \]

The first system requires each satellite to broadcast a unique pseudo-noise (PN) code in the frequency band 2483.5 - 2500 MHz. The PN codes are selected from a set of Gold codes which allow minimum interference between signals from various satellites. The satellites employ extremely stable on-board atomic clocks which are mutually synchronized. A central control station which can view all satellites is responsible for the clock correction via a communications link in the frequency band 6526.5 - 6541.5 MHz. The user transceiver measures the time differentials between one satellite arbitrarily
designated as satellite one, and each of the other satellites. The accuracy of the measurements and their effect on the user position determination accuracy will be discussed later. The user transceiver then sends the measured time differentials to the central control station for computing the user coordinates via a code division multiple access (CDMA) uplink in the frequency band 1610 - 1626.5 MHz. The central control station calculates the user coordinates using the received time differentials and the predicted satellite ephemerides. It then relays the coordinates to the user via a time division multiplexed (TDM) downlink in the frequency band 2483.5 - 2500 MHz.

The second system requires the user transceiver to broadcast a PN code to the central control station via a code division multiple access uplink. The central control station calculates the user differential distances to the satellites with the available satellite ephemeris and then the user coordinates which are relayed to the user via the TDM downlink.
II. SYSTEM I

A. SYSTEM I DESCRIPTION

The frequency assignment and bandwidth allocated for each of the satellite links listed below are assigned by the Federal Communications Commission as listed in [Ref. 2].

- **Satellite-to-User Ranging Link**
  - Frequency: 2483.5 - 2493.73 MHz
  - Bandwidth: 10.23 MHz

- **User-to-Control Station**
  - *User-to-Satellite Uplink*
    - Frequency: 1610 - 1626.5 MHz
    - Bandwidth: 16.5 MHz
  - *Satellite-to-Control Station Downlink*
    - Frequency: 5150 - 5166.5 MHz
    - Bandwidth: 16.5 MHz

- **Control Station-to-User**
  - *Control Station-to-Satellite Uplink*
    - Frequency: 6524.5 - 6530.77 MHz
    - Bandwidth: 6.27 MHz
  - *Satellite-to-User Downlink*
    - Frequency: 2493.73 - 2500 MHz
    - Bandwidth: 6.27 MHz

- **Control Station-to-Satellite Command Link**
  - Frequency: 6526.5 - 6541.5 MHz
  - Bandwidth: 15 MHz

B. SATELLITE-TO-USER DOWNLINK

1. **Downlink Frequency and Bandwidth**

   The communications downlink from each of the satellites to the user occupies a bandwidth of 16.5 MHz from 2483.5 to 2500 MHz. Within this bandwidth, the satellites must transmit a pseudo-noise ranging code needed for user position determination and provide the downlink for the control station-to-user data link. In System I design, the satellites must employ two satellite-to-user channels, one for the sole purpose of transmitting the unique pseudo-noise code and one to perform as a relay for the data link from the control station to the user. The pseudo-noise code will be transmitted between the frequencies of 2483.5 and 2493.73 MHz in a bandwidth of 10.23 MHz. The control station-to-user data link will be transmitted between the frequencies of 2493.73 and 2500 MHz in a bandwidth of 6.27 MHz.
2. Pseudo-Noise (PN) Ranging Code

The PN ranging code transmitted by the satellites is received by the user and is used to determine time differentials. The accuracy of the measurements of the time differentials used to determine the user position is of prime importance to the overall position accuracy. The more precise the time measurement is, the more accurate the position determination will be. GPS uses code chip rates of 10.23 Mbit/s in its transmissions to the user. For our example, a given code length of \(2^{19} - 1 = 524,287\) chips and a chip rate of 10.23 Mbit/s results in a code period of \(524,287 / 10.23 \times 10^6 = 51.25\) ms. A lower code length of \(2^{16} - 1 = 65,535\) and a lower chip rate of 1.023 Mbit/s has a period of \(65,535 / 1.023 \times 10^6 = 64.06\) ms and a larger error in the differential distance measurement. The code tracking loop that would be used is able to maintain code alignment to approximately 0.01 chip. This is a conservative figure, for code tracking loops can be accurate to 0.001 chip. For a 0.01 alignment factor and a 10.23 Mbit/s chip rate, a ranging error of 0.29 meter would exist, where a 1.023 Mbit/s chip rate would have a ranging error of 2.9 meters [Ref. 1]. To better understand the fundamentals of PN ranging codes that determine position accuracy, Figure 2 shows the concept of differential distance measurement. To determine the differential distance, the user equipment must be able to perform a correlation operation among the satellites’ codes and internally stored copies of those codes. The receiver clock must be able to count the number of chips between the correlation peak of the first received code and the subsequent codes as shown in Figure 2. Since \(a\) may not be an integer multiple number of chips, the receiver may use the correlation value relative to the peak correlation voltage to determine a fraction of a chip in the measurement. As was stated earlier, a fraction of a chip of 0.001 can be measured.
Figure 2. Differential Distance Measurement: [Ref. 1]

In [Ref. 1], the PN ranging code and its effects on position determination accuracy is discussed in further detail.

C. USER-TO-CONTROL STATION LINK

1. Uplink Bandwidth

The frequency bandwidth allocated to the user-to-control station data link is 1610-1626.5 MHz. The user direct sequence spreads the data with a Gold code of period \(2^{10} - 1 = 1023\) chips which is a standard period for navigation systems. Thus for a bandwidth of 16.5 MHz from 1610-1626.5 MHz, the typical user data rate is approximately 16,129 bits/s when spread at 1023 chips per bit, if Quaternary Phase Shift Keying (QPSK) is used as a modulation scheme. That is, the chip rate is \((16,129)(1,023) = 16.5 \times 10^6\) chips. With QPSK, there are 2 bits per symbol and hence the symbol rate is \(8.5 \times 10^6\) symbols a second. The bandwidth required (double sideband for QPSK) is twice the symbol rate or equal to 16.5 MHz which is compatible with the available 16.5 MHz bandwidth. The users must share this available bandwidth with each other to transmit their IDs, time differentials, and other information to the control station.

The user-to-satellite uplink is the most critical link in the system. The user is not able to employ or utilize a high gain dish antenna like those used in the fixed stations. The maximum output RF power is also limited by the power available in a mobile vehicle. To calculate the Effective Isotropic Radiated Power (EIRP) of a vehicle,
Eq. (5) is used, where $P_t$ is the transmitter power out and $G_t$ is the effective gain of the antenna.

$$EIRP = P_tG_t$$

In our design, a transmitter power out of 80 watts which is equal to 19 dBW and an omni directional antenna with a gain of 4 dB will yield an EIRP of 23 dBW. These design values are selected with some judgment and are also being used in current navigation systems.

The Free Space Path Loss (FSPL) is calculated using Eq. (6).

$$FSPL(dB) = 10 \log \left( \frac{4\pi f d}{c} \right)^2$$

where $f$ is the carrier frequency, $d$ is the slant range, and $c$ is the speed of light. At a slant range of 35.786 km and a carrier frequency of 1618 MHz, a FSPL of approximately 190 dB is obtained.

A standard satellite $G/T$ of +3 dB K has been chosen for this design model and Boltzmann’s constant $k$ is given as -228.6 dBW/K-Hz. To calculate the uplink carrier-to-noise density ratio, Eq. (7) is used.

$$\frac{C}{N}(dB) = EIRP(dBW) - FSPL(dB) + \frac{G}{T}(dB/K) - k(dBW/K-Hz)$$

The EIRP of 23 dBW, FSPL of 190 dB, satellite $G/T$ of +3 dB/K and Boltzmann’s constant of -228.6 dBW K-Hz are entered into the Eq. (7), obtaining an uplink carrier-to-noise density ratio of 64.6 dB-Hz. This density ratio is sufficient to support the data rate that the system requires for successful operation.

For the downlink, the same calculations are performed to obtain the downlink carrier-to-noise density ratio. A FSPL of 198 dB is calculated for the carrier frequency of 5150 MHz. A satellite EIRP of 36 dBW and a control station $G/T$ of 20 dB·K are used to calculate the downlink carrier-to-noise density ratio. These values are standard satellite EIRP and station $G/T$ values.

The total carrier-to-noise density ratio is calculated using Eq. (8) found in [Ref. 3].

$$\left( \frac{C}{N} \right) = \left[ \left( \frac{C}{N} \right)_u^{-1} + \left( \frac{C}{N} \right)_d^{-1} \right]$$

8
To solve for the total carrier-to-noise density ratio, the values of the carrier-to-noise density ratios must be converted to real numbers, substituted into Eq. (8), the equation solved and the answer converted to the logarithm form of dB-Hz.

A margin of 2 dB is further engineered into the system design to allow for interferences that would degrade the operation of the system. The data rate is calculated by subtracting the 2 dB margin from the total carrier-to-noise ratio, converting that dB value to a factor and then dividing that factor by the equivalence of the required signal-to-noise ratio which is set to two representative values of 15 and 20 dB for this system design. These required signal-to-noise ratios are typical satellite navigation signal-to-noise ratios; each will be used for design calculations.

To calculate the data rate for a lower required signal-to-noise ratio of 15 dB, the 2 dB margin is subtracted from the total carrier-to-noise ratio of 64.57 dB-Hz, giving 62.57 dB-Hz. This ratio is converted to a value of 1.807 x 10³ and is divided by 31.26 (value of 15 dB) which in turn yields the data rate of approximately 57 kbps. When the required signal-to-noise ratio is increased to 20 dB (factor of 100), the data rate is lowered to 18 kbps. The link analysis is tabulated in Table 1.

2. User-to-Control Station Data

After receiving 3 or 4 satellites' ranging signals and measuring time differentials, the user must transmit the information to the control station for calculation. The transmission is Code Division Multiple Access (CDMA) as required by the Federal Communications Commission (FCC) and is designed to provide the system with a bit error probability of 10⁻⁶ to 10⁻⁻. (A bit error probability of 10⁻⁶ and 10⁻⁻ provides the reliability needed for a successful system.) The data stream must include a synchronous code word in the user transmission for the control station to synchronize to the transmission. Once the the control station has synchronized itself to the data transmission, the control station can identify the information that the user is sending to the control station from within the data frame. A Barker code word of 13 bits would provide the synchronization that the control station equipment requires and enable the control station to lock on to the transmission and recover the user transmitted data [Ref. 4].

Other information that is sent includes the user ID, the ID of the satellite that was received first, the second and third satellites IDs and the time differentials ΔT in symbols between receipt of the signals of the satellites. These time differentials are determined by the correlator. The minimum distance that a satellite may be from the user is 35,786 km which corresponds to a minimum propagation delay of 119.287 ms. The maximum distance is 41,678.82 km which corresponds to the maximum propagation
Table 1. SYSTEM I USER-TO-CONTROL STATION LINK ANALYSIS

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Uplink Frequency</td>
<td>1618 MHz</td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>4 dB</td>
</tr>
<tr>
<td>Transmitted Power</td>
<td>80 W</td>
</tr>
<tr>
<td>Carrier EIRP</td>
<td>23 dBW</td>
</tr>
<tr>
<td>Free Space Path Loss</td>
<td>190 dB</td>
</tr>
<tr>
<td>Satellite G T</td>
<td>+3 dB K</td>
</tr>
<tr>
<td>Boltzmann’s Constant</td>
<td>-228.6 dBW K-Hz</td>
</tr>
<tr>
<td>Uplink Carrier-to-Noise Density Ratio</td>
<td>64.6 dB-Hz</td>
</tr>
<tr>
<td>Downlink Frequency</td>
<td>5150 MHz</td>
</tr>
<tr>
<td>Satellite EIRP</td>
<td>36 dBW</td>
</tr>
<tr>
<td>Free Space Path Loss</td>
<td>198 dB</td>
</tr>
<tr>
<td>Control Station G T</td>
<td>20 dB K</td>
</tr>
<tr>
<td>Downlink Carrier-to-Noise Density Ratio</td>
<td>86.6 dB-Hz</td>
</tr>
<tr>
<td>Total Carrier-to-Noise Density Ratio</td>
<td>64.57 dB-Hz</td>
</tr>
<tr>
<td>Margin</td>
<td>2 dB</td>
</tr>
<tr>
<td>If required $\frac{E_b}{N_0}$</td>
<td>15 dB 20 dB</td>
</tr>
<tr>
<td>Then Data Rate can be</td>
<td>57 kbps 18 kbps</td>
</tr>
</tbody>
</table>

delay of 138.929 ms. A maximum time differential between the minimum and maximum time delays is 19.642 ms. We require an accuracy of 0.1 μsec. This allows a quantization interval of 0.2 μsec. The number of 0.2 μsec intervals in 19,642 μsec is 98,210. To represent this value, $2^{18} = 262,144$ will easily suffice to represent the maximum time differential. Thus there are 18 bits required to represent the maximum differential time measurement between the first satellite and each of the other satellites and 2 additional bits to identify the satellite for a total of 20 bits used. In the three-satellite design, the altitude of the user must also be transmitted to the control station for position determination. An altitude of 100,000 meters with 0.1 meter accuracy can be represented with
20 bits. For the four-satellite case, the field for altitude is used for satellite 4 ID and differential time measurement. In all cases, 20 bits for additional information are designed into the frame for future uses such as emergency reporting of position location when the user may be in need of it. See Figure 2 for an example of a data frame.

![Data Frame Diagram](image)

**Figure 3. Data Frame (User to Control Station)**

To increase the number of channels that could exist on the system, Forward Error Correction (FEC) is used. The use of FEC lowers the required signal-to-noise ratio and thus allows more CDMA channels to be used. A convolution code with Viterbi decoding and soft decision will increase the total number of channels [Ref. 5]. There are various combinations of convolution code rates and constraint lengths that can be examined to provide the optimum data rate for the system.

To calculate the number $K+1$ of CDMA channels that can successfully exist within the bandwidth at a bit error probability of $10^{-6}$ and $10^{-7}$, the following results from Refs. 5 and 6 were used. To calculate the number $K+1$ of CDMA channels that can be successfully utilized simultaneously, Eq. (9) is used.

$$K = 3N \left( \frac{\eta_0}{2E_b^*} - \frac{N_0}{2E_b^*} \right)$$  \hspace{1cm} (9)

The chips per bit ($N$), the design bit energy-to-noise ratio ($\frac{E_b^*}{\eta_0}$) and the bit energy-to-noise plus user interference density ratio ($\frac{E_b^*}{\eta^*_u}$) must be known to solve for $K$. For
reliability, bit energy-to-noise ratios of 15 and 20 dB were selected for the system. The chips per bit (N) is 1023. The value for the bit energy-to-noise plus user interference density ratio is a function of other parameters as follows.

\[ p = Q\left(\sqrt{2R \frac{E_b}{\eta_o}}\right) \] (10)

where \( Q(x) \) is defined as

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{u^2}{2}} du \] (11)

The code rate (R) and the channel transition probability (p) must be known to calculate the energy-to-noise plus user interference density ratio. A code rate (R) of 1,2 to 7,8 was used in the calculations to determine the bit rate that will support the maximum number of CDMA channels. The probability (p) is a function of parameters that are determined by the code rate and system design. To calculate p, Eq. 12 is used [Ref. 6].

\[ P_{b,\text{coded}} < \frac{1}{k} B_{d_{\text{free}}} 2^{d_{\text{free}}} p^{\frac{d_{\text{free}}}{2}} \] (12)

For System I design, coded bit error probabilities \((P_{b,\text{coded}})\) of \(10^{-6}\) and \(10^{-7}\) are used to insure a reliable system for the user. The number k of encoder input bits and the number n of encoder output bits establish the code rate (R), where \( R = \frac{k}{n} \). The variable \( B_{d_{\text{free}}} \) used to calculate the channel transition probability (p) is a calculated value that is determined by the code rate, constraint length, and the free distance of the code. The variable \( d_{\text{free}} \) is the free distance of the convolutional code and is determined by the code rate and the constraint length. The values \( B_{d_{\text{free}}} \) and \( d_{\text{free}} \) have been previously calculated in [Refs. 5,6] at a constraint length of 7 for the code rates of 1/2 to 7/8.

To calculate the number of simultaneous CDMA channels that the system can support, the channel transition probability (p) must be found with Eq. (12). The number k of encoder input bits, \( B_{d_{\text{free}}} \), \( d_{\text{free}} \) and the coded bit error probability \((P_{b,\text{coded}})\) are entered into Eq. (12) to solve for the channel transition probability (p). Then the bit energy-to-noise plus user interference density ratio \( \left( \frac{E_b}{\eta_u} \right) \) is substituted into Eq. (10) along with the given chips per bit (N) of 1023, and the design bit energy-to-noise density.
ratio \( \left( \frac{E_b}{N_0} \right) \) of 15 and 20 dB. In Tables 2 and 4, the number \( K + 1 \) of channels that can successfully be used with hard decision Viterbi coding at a bit error probability of \( 10^{-6} \) and \( 10^{-7} \) respectively are shown. For example, given the code rate \( (R) \) of 1/2, the encoded bit \( k \) is 1, \( B_{d_free} \) is 36, and \( d_{free} \) is 10. For a \( P_{b_coded} \) of \( 10^{-6} \) a channel transition probability \( (p) \) of 0.00703 is calculated. With a \( p \) of 0.00703 and a code rate \( (R) \) of 1/2, a bit energy-to-noise plus user interference density ratio \( \left( \frac{E_b}{\eta_s} \right) \) of 5.87 is calculated. This value, the chips per bit \( (N) \) of 1023, and the design energy-to-noise density ratio of 20 dB (100) yields a value \( K \) of 246. The number of channels \( K + 1 \) is then 247. An inspection of the tables reveals that a code rate of 1/2 yields the largest number of simultaneous channels that can be used successfully. When soft decision Viterbi coding is used, a decrease in the bit energy-to-noise plus user interference density ratio of 2 dB is realized. Thus soft decision Viterbi coding further increases the number of CDMA channels that can be used which is shown in Tables 3 and 5 for \( 10^{-6} \) and \( 10^{-7} \) respectively. The different bit error probabilities of \( 10^{-6} \) and \( 10^{-7} \) and soft and hard Viterbi decisions are used in the calculations to provide the reader an indication of four possible system capabilities.

The major limitation in using lower code rates of 1/2 is that the bandwidth of the system that is available to the user is severely reduced. Although the number of CDMA simultaneous channels is greatly increased, the bandwidth of the system is decreased. Further research is required in the area of the coding techniques to find the maximum number of CDMA channels that can be used with the highest coding rate to allow for the greatest user bandwidth.

3. **Satellite-to-Control Station Downlink**

The frequency bandwidth allocated for the satellite-to-control station downlink is 16.5 MHz from 5150 to 5165.5 MHz. The satellite serves as a relay for the user-to-control station data link.

**D. POSITION DETERMINATION**

After receipt of the time differentials, the central control station computes the coordinates of the user from the measured time differentials, and the known location of each of the satellites.

The x, y, z position determination of the user is defined in a geocentric coordinate system as shown in Figure 1. The x axis is the intersection of the equatorial plane and the Greenwich Meridian plane and is oriented from the center of the earth. The y axis
Table 2. NUMBER OF SIMULTANEOUS CDMA CHANNELS WITH FEC

(Hard Decision) at Coded Bit Error probability of $10^{-6}$

<table>
<thead>
<tr>
<th>Code Rate $R$</th>
<th>1/2</th>
<th>2/3</th>
<th>3/4</th>
<th>4/5</th>
<th>5/6</th>
<th>6/7</th>
<th>7/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$B_{d_{free}}$</td>
<td>36</td>
<td>3</td>
<td>42</td>
<td>12</td>
<td>92</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>$d_{free}$</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>CDMA Channels $K$ for $\frac{E_b}{N_0} = 100$ or $20$ dB</td>
<td>247</td>
<td>237</td>
<td>185</td>
<td>172</td>
<td>157</td>
<td>147</td>
<td>145</td>
</tr>
<tr>
<td>CDMA Channels $K$ for $\frac{E_b}{N_0} = 32$ or $15$ dB</td>
<td>213</td>
<td>204</td>
<td>152</td>
<td>139</td>
<td>124</td>
<td>114</td>
<td>112</td>
</tr>
</tbody>
</table>

completes the cartesian coordinate system with the x axis. The z axis is the north to south axis connecting the two poles.

Although actual distance is not measured by the user equipment, the distance can be calculated from the time it took for the other satellites signals to reach the user versus the first satellite signal that was received. The time differentials are converted to differential distances by multiplying the time differentials by the speed of light ($c$); approximately $3 \times 10^8$ m/s. The accuracy of the synchronization of the satellites transmissions plays an important part in the users position determination. If the satellites do not transmit at the exact same time, the time differentials will not be correct and thus neither will the distance differentials.

The distance between the unknown user position $(x, y, z)$ and each of the satellites $(x_i, y_i, z_i)$ where $i = 1, 2, \ldots, n$ can be calculated from

$$d_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \quad i = 1, 2, \ldots, n.$$  \hspace{1cm} (13)

We further define the distance measurements

$$a_i = d_1 - d_i \quad i = 1, 2, \ldots, n$$  \hspace{1cm} (14)
Table 3. NUMBER OF SIMULTANEOUS CDMA CHANNELS WITH FEC  
(Soft Decision) at Coded Bit Error probability of $10^{-6}$

<table>
<thead>
<tr>
<th>Code Rate R</th>
<th>$1/2$</th>
<th>$2/3$</th>
<th>$3/4$</th>
<th>$4/5$</th>
<th>$5/6$</th>
<th>$6/7$</th>
<th>$7/8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$B_{d_{free}}$</td>
<td>36</td>
<td>3</td>
<td>42</td>
<td>12</td>
<td>92</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>$d_{free}$</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>CDMA Channels K for $\frac{E_b}{N_0} = 100$ or 20 dB</td>
<td>400</td>
<td>384</td>
<td>302</td>
<td>281</td>
<td>258</td>
<td>242</td>
<td>238</td>
</tr>
<tr>
<td>CDMA Channels K for $\frac{E_b}{N_0} = 32$ or 15 dB</td>
<td>366</td>
<td>351</td>
<td>269</td>
<td>247</td>
<td>225</td>
<td>209</td>
<td>206</td>
</tr>
</tbody>
</table>

as the difference in distance from one satellite designated as satellite one to the user and the other satellites distance to the user.

For a four satellite case, $n = 4$, equations (13) and (14) are combined to form four independent equations and four unknowns, $x$, $y$, $z$, and $d_i$. From the known satellite positions and the time differences which have been measured by the user, the four unknowns can be solved. Sign ambiguities can be corrected as long as the user knows what quadrant he is in.

The coordinates of the satellites $(x_i, y_i, z_i)$ can be calculated from its longitude eastward of zero degrees by

$$x_i = r \cos \theta_i$$  \hspace{1cm} (15)

$$y_i = r \sin \theta_i$$  \hspace{1cm} (16)

$$z_i = 0$$  \hspace{1cm} (17)

The term $z$, will remain zero as the satellites are in an equatorial orbit with a latitude of zero degrees.
Table 4. **NUMBER OF SIMULTANEOUS CDMA CHANNELS WITH FEC**

(Hard Decision) at Coded Bit Error probability of $10^{-7}$

<table>
<thead>
<tr>
<th>Code Rate R</th>
<th>1/2</th>
<th>2/3</th>
<th>3/4</th>
<th>4/5</th>
<th>5/6</th>
<th>6/7</th>
<th>7/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$B_{d_{free}}$</td>
<td>36</td>
<td>3</td>
<td>42</td>
<td>12</td>
<td>92</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>$d_{free}$</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

CDMA Channels $K$ for $\frac{E_s}{N_o} = 100$ or 20 dB

| 215 | 200 | 159 | 147 | 135 | 123 | 122 |

CDMA Channels $K$ for $\frac{E_s}{N_o} = 32$ or 15 dB

| 181 | 167 | 126 | 112 | 102 | 89  | 88  |

The four non-linear independent equations are

$$(d_i - a_i)^2 = (x - x_i)^2 + (y - y_i)^2 + z^2 \quad i = 1, 2, 3, 4$$

Equation (18) can be reduced to 3 linear independent equations by substituting equation (14) into equation (13) and subtracting the $(i = 1)$ equation from the other 3 equations. The remaining three equations are

$$(x_i - x_1)x + (y_i - y_1)y - a_i d_i = -\frac{a_i^2}{2} \quad i = 2, 3, 4.$$  \hspace{1cm} (19)

The 3 equations can then be entered into matrix form to solve x, y, and $d_i$ as shown:

$$
\begin{bmatrix}
x \\
y \\
d_i
\end{bmatrix} = -\frac{1}{2}
\begin{bmatrix}
x_2 - x_1 & y_2 - y_1 & -a_2 \\
x_3 - x_1 & y_3 - y_1 & -a_3 \\
x_4 - x_1 & y_4 - y_1 & -a_4
\end{bmatrix}^{-1}
\begin{bmatrix}
a_2^2 \\
a_3^2 \\
a_4^2
\end{bmatrix}
\hspace{1cm} (20)
$$

$z$ can be calculated by substituting equation (20) into equation (18)
Table 5. NUMBER OF SIMULTANEOUS CDMA CHANNELS WITH FEC
(Soft Decision) at Coded Bit Error Probability of $10^{-7}$

<table>
<thead>
<tr>
<th>Code Rate R</th>
<th>1/2</th>
<th>2/3</th>
<th>3/4</th>
<th>4/5</th>
<th>5/6</th>
<th>6/7</th>
<th>7/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$B_{d_{free}}$</td>
<td>36</td>
<td>3</td>
<td>42</td>
<td>12</td>
<td>92</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>$d_{free}$</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>CDMA Channels K for $\frac{E_s}{N_0} = 100$ or 20 dB</td>
<td>349</td>
<td>326</td>
<td>261</td>
<td>239</td>
<td>223</td>
<td>203</td>
<td>201</td>
</tr>
<tr>
<td>CDMA Channels K for $\frac{E_s}{N_0} = 32$ or 15 dB</td>
<td>316</td>
<td>292</td>
<td>228</td>
<td>206</td>
<td>190</td>
<td>170</td>
<td>168</td>
</tr>
</tbody>
</table>

$$z = \left[ (d_i - a_f)^2 - (x - x_i)^2 - (y - y_i)^2 \right]^{\frac{1}{2}}. \tag{21}$$

From equation (20) and (21), the user position $(x, y, z)$ can be solved to obtain an initial approximation. The actual position must be solved iteratively or by utilizing a Kalman filter. From $(x, y, z)$, the users altitude $h$, longitude and latitude can be derived.

$$x = (R + h) \cos \theta_L \cos \theta_l \tag{22}$$

$$y = (R + h) \sin \theta_L \cos \theta_l \tag{23}$$

$$z = (R + h) \sin \theta_l \tag{24}$$

where $\theta_L, \theta_l, \text{ and } h$ are solved by

$$\theta_L = \tan^{-1} \left( -\frac{y}{x} \right) \tag{25}$$

$$\theta_l = \tan^{-1} \left( z \frac{\cos \theta_L}{x} \right) \tag{26}$$
For a three satellite system, the position solution algorithm is much the same. The following equation substitutes for the fourth satellite equation

\[ x^2 + y^2 + z^2 = (R + h)^2 \]  

(28)

As in the four satellite case, these four equations must be solved iteratively to obtain the users position. In the three satellite case, the user must report his altitude because only the longitude and latitude of his position can be determined from three satellites. The four non-linear equations are

\[(d_i - \alpha_i)^2 = (x - x_i)^2 + (y - y_i)^2 + z^2 \quad i = 1, 2, 3\]  

(29)

\[ x^2 + y^2 + z^2 = (R + h)^2. \]  

(30)

As in the four satellite case, the equation with index = 1 is subtracted from the other two equations.

\[ d_1 = \alpha_1 x + B_1 y + \gamma_1 \]  

(31)

\[ d_1 = \alpha_2 x + B_2 y + \gamma_2 \]  

(32)

where

\[ \alpha_1 = \frac{x_2 - x_1}{a_2} \quad \alpha_2 = \frac{x_3 - x_1}{a_3} \quad B_1 = \frac{y_2 - y_1}{a_2} \]  

(33)

\[ B_2 = \frac{y_3 - y_1}{a_3} \quad \gamma_1 = \frac{x_1^2 + y_1^2 - x_2^2 - y_2^2 + a_2^2}{2a_2} \]  

(34)

\[ \gamma_2 = \frac{x_1^2 + y_1^2 - x_3^2 - y_3^2 + a_3^2}{2a_3}. \]  

(35)

When we solve for \( y \) in terms of \( x \), we get

\[ y = \mu_1 x + \mu_2 \]  

(36)

where
\[
\mu_1 = \frac{y_1 - y_2}{B_2 - B_1}, \quad \mu_2 = \frac{y_1 - y_2}{B_2 - B_1}.
\]

(37)

The substitution of (36) into (31) produces

\[
d_1 = \mu_3 x + \mu_4
\]

(38)

where

\[
\mu_3 = \frac{\alpha_1 B_2 - \alpha_2 B_1}{B_2 - B_1}, \quad \mu_4 = \frac{\alpha_1 B_2 - \alpha_2 B_1}{B_2 - B_1}.
\]

(39)

The term \(z^2\) can be expressed as

\[
z^2 = d_1^2 - x^2 - y^2 + 2x_1 x + 2y_1 y - x_1^2 - y_1^2.
\]

(40)

Substitution of (36), (38), and (40) into (28) produces

\[
a x^2 + b x + c = 0
\]

(41)

where

\[
a = \mu_3^2
\]

(42)

\[
b = 2 \mu_3 \mu_4 + 2y_1 \mu_1 + 2x_1
\]

(43)

\[
c = \mu_4^2 + 2y_1 \mu_2 - x_1^2 - y_1^2 - (R + h)^2.
\]

(44)

The value of \(x, y,\) and \(z\) can be calculated from (36), (40), and (41) subject to the limitation \(x < 6378\) km provided the user knows what quadrant of the earth he is located in.

**E. POSITION ERROR**

Even though the satellite is geostationary, the satellite does move about its nominal position. The drift of the satellite can be calculated to less than 20 meters from its nominal position. The error that results in the user position determination can be calculated. The range error can be expressed as user position error \((\Delta x, \Delta y, \Delta z).\)
\[
\Delta d_i = \frac{\partial d_i}{\partial x} \Delta x + \frac{\partial d_i}{\partial y} \Delta y + \frac{\partial d_i}{\partial z} \Delta z = \frac{x-x_i}{d_i} \Delta x + \frac{y-y_i}{d_i} \Delta y + \frac{z-z_i}{d_i} \Delta z \quad i = 1, 2, 3, 4.
\] 

Equation (45) can be transformed into matrix form as \( Av = u \) and with equation (14),

\[
A = \begin{bmatrix}
\frac{x-x_1}{d_1} & \frac{y-y_1}{d_1} & \frac{z-z_1}{d_1} \\
\frac{x-x_2}{d_1-a_2} & \frac{y-y_2}{d_1-a_2} & \frac{z-z_2}{d_1-a_2} \\
\frac{x-x_3}{d_1-a_3} & \frac{y-y_3}{d_1-a_3} & \frac{z-z_3}{d_1-a_3} \\
\frac{x-x_4}{d_1-a_4} & \frac{y-y_4}{d_1-a_4} & \frac{z-z_4}{d_1-a_4}
\end{bmatrix}
\] 

\[
v = \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}, \quad u = \begin{bmatrix}
\Delta d_1 \\
\Delta d_2 \\
\Delta d_3 \\
\Delta d_4
\end{bmatrix}
\] 

The user position error can then be simulated by letting the range error be

\[
\Delta d_i = \left[ (x-x'_i)^2 + (y-y'_i)^2 + (z-z'_i)^2 \right]^{\frac{1}{2}} - \left[ (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \right]^{\frac{1}{2}} \quad i = 1, 2, 3, 4
\]

where

\[
(x'_i, y'_i, z'_i) = (x_i + \Delta x_i, y_i + \Delta y_i, z_i + \Delta z_i).
\]

The variable \( \Delta d_i \) is the difference between the distance from the user to the nominal satellite position and the distance from the user to the satellite position that the satellite may have drifted to. The variables \((x'_i, y'_i, z'_i)\) are the coordinates of the nominal satellite position with the drift distances \((\Delta x_i, \Delta y_i, \Delta z_i)\) added to the nominal satellite position coordinates. The error vector \( v \) is the linear least-squares solution of \( Av = u \) where \( v = (A^T A)^{-1} A^T u \) and the mean squared error is \( e = (Av - u)^T (Av - u) \). The values \((\Delta x, \Delta y, \Delta z)\) are calculated using Eq. (46) and (47). The error contribution of the z co-
ordinate is much less than the x or y coordinate as the user position approaches the equator. This ill-conditioned problem results in a poor estimation of \( \Delta z \) which can be minimized by modifying the least-squares method of solution. By first calculating the least-squares estimate of \( \Delta x \) and \( \Delta y \) and ignoring the \( \Delta z \) component of the total error and calculating \( \Delta z \) after \( \Delta x \) and \( \Delta y \) are found, a better estimate of the total error can be found.

\[
\begin{bmatrix}
\Delta x \\
\Delta y
\end{bmatrix} = (A_1^T A_1)^{-1} A_1^T u
\tag{50}
\]

\[
\Delta z = (A_2^T A_2)^{-1} (A_2^T u - A_2^T A_1 v_1)
\tag{51}
\]

where

\[
A_1 = \begin{bmatrix}
\frac{x - x_1}{d_1} & \frac{y - y_1}{d_1} \\
\frac{x - x_2}{d_1 - a_2} & \frac{y - y_2}{d_1 - a_2} \\
\frac{x - x_3}{d_1 - a_3} & \frac{y - y_3}{d_1 - a_3} \\
\frac{x - x_4}{d_1 - a_4} & \frac{y - y_4}{d_1 - a_4}
\end{bmatrix}
\tag{52}
\]

\[
A_2 = \begin{bmatrix}
\frac{z - z_1}{d_1} \\
\frac{z - z_2}{d_1 - a_2} \\
\frac{z - z_3}{d_1 - a_3} \\
\frac{z - z_4}{d_1 - a_4}
\end{bmatrix}
\]

\[
u_1 = \begin{bmatrix}
\Delta d_1 \\
\Delta d_2 \\
\Delta d_3 \\
\Delta d_4
\end{bmatrix}
\tag{53}
\]

For a three satellite system, the range errors \( \Delta d, i = 1, 2, 3 \) are calculated from equation (45) and the user altitude error \( \Delta h \) is calculated from equation (30) as
\[
\Delta h = \frac{x(\Delta x) + y(\Delta y) + z(\Delta z)}{\left[ x^2 + y^2 + z^2 \right]^{\frac{1}{2}}}
= \frac{x(\Delta x) + y(\Delta y) + z(\Delta z)}{R + h}
\] (54)

The user position error vector \( v = [\Delta x, \Delta y, \Delta z]^T \) is the least-squares solution of \( Bv = w \) which is \( v = (B^T B)^{-1} B^T w \), where

\[
B = \begin{bmatrix}
\frac{x - x_1}{d_1} & \frac{y - y_1}{d_1} & \frac{z - z_1}{d_1} \\
\frac{x - x_2}{d_1 - a_2} & \frac{y - y_2}{d_1 - a_2} & \frac{z - z_2}{d_1 - a_2} \\
\frac{x - x_3}{d_1 - a_3} & \frac{y - y_3}{d_1 - a_3} & \frac{z - z_3}{d_1 - a_3} \\
\frac{x}{R + h} & \frac{y}{R + h} & \frac{z}{R + h}
\end{bmatrix}
\]
\[
w = \begin{bmatrix}
\Delta d_1 \\
\Delta d_2 \\
\Delta d_3 \\
\Delta h
\end{bmatrix}
\] (55)

For the three satellite case, the least-squares method is also modified to provide a better estimate of \( v \). The variable \( v \) is calculated from \( v = [v_1, \Delta z]^T \) where

\[
v_1 = \begin{bmatrix}
\Delta x \\
\Delta y
\end{bmatrix} = (B_1^T B_1)^{-1} B_1^T w
\] (56)

\[
\Delta z = (B_2^T B_2)^{-1} (B_2^T w - B_2^T B_1 v_1)
\] (57)

and

\[
B_1 = \begin{bmatrix}
\frac{x - x_1}{d_1} & \frac{y - y_1}{d_1} \\
\frac{x - x_2}{d_1 - a_2} & \frac{y - y_2}{d_1 - a_2} \\
\frac{x - x_3}{d_1 - a_3} & \frac{y - y_3}{d_1 - a_3} \\
\frac{x}{R + h} & \frac{y}{R + h}
\end{bmatrix}
\]
\[
B_2 = \begin{bmatrix}
\frac{z - z_1}{d_1} \\
\frac{z - z_2}{d_1 - a_2} \\
\frac{z - z_3}{d_1 - a_3} \\
\frac{z}{R + h}
\end{bmatrix}
\] (58)
Equations (50), (51), (56), and (57) calculate the upper limit of the error in the user position provided the satellites position errors are known in either the three of four satellite case. The average error for various users positions has been compiled in [Ref. 1]. The average error in the users position increases as the user approaches the equator. The average error ranges from 14.2 to 61.5 meters depending on how close to the equator the user is. The closer the user is to the equator, the worse the error which indicates a weakness in the system.

F. CONTROL STATION-TO-USER DATA LINK

1. Uplink Bandwidth

The frequency bandwidth allocated for the control station-to-satellite uplink is 6524.5 to 6530.77 MHz. The 6.27 MHz bandwidth coincides with the available bandwidth of the satellite-to-user downlink part of the relay.

After the control station has completed the calculations and the user position is determined, the position coordinates have to be sent to the user. A TDM channel is used to transport the user information to the user. To calculate the total carrier-to-noise density ratio of the control station-to-user link. Eqs. (5) - (8) are used as was discussed in the user-to-control link.

The uplink frequency is 6533 MHz with an antenna gain ($G_u$) of 44.5 dB, which is a standard antenna gain for a ground station antenna. The antenna gain is much higher than for a mobile vehicle as one is able to use a large dish. The power out ($P_u$) is set at only 1 watt per channel since there would be more than one channel simultaneously transmitted by the control station. For example, if 200 CDMA channels were used, then 200 watts of power would be required for transmission of the channels. The lower power out is standard for a ground station. For this link design, the satellite G T is -1.5 dB K. The power out and the satellite G/T values are design parameters determined by judgement and current systems. For the control station-to-user link, the total carrier-to-noise density ratio is 66.25 dB-Hz is calculated by Eq. (8). A carrier-to-noise density ratio of 66.25 dB-Hz is sufficient for a successful link. A 5 dB margin is engineered into the system to allow for the cross-interference degradation of the system by the multiple carriers that are transmitted by the control station. After subtracting the 5 dB margin from the total carrier-to-noise density ratio, converting to a factor and dividing the result by the required noise ratio factor, the data rate for the bit error probabilities of $10^{-6}$ and $10^{-7}$ are 42.2 kbps and 13.3 kbps, respectively. The calculations for the control station-to-user link utilize Eqs. (5) - (8), which have been discussed earlier.
These are the same equations as Table 1 with only the input values differing. The control station-to-user link analysis is tabulated in Table 6.

Table 6. SYSTEM 1 CONTROL STATION-TO-USER LINK ANALYSIS

<table>
<thead>
<tr>
<th></th>
<th>Upplink Frequency</th>
<th>6533 MHz</th>
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<tbody>
<tr>
<td>Antenna Gain</td>
<td></td>
<td>44.5 dB</td>
</tr>
<tr>
<td>Transmitted Power</td>
<td></td>
<td>1 W</td>
</tr>
<tr>
<td>Carrier EIRP</td>
<td></td>
<td>44.5 dBW</td>
</tr>
<tr>
<td>Free Space Path Loss</td>
<td></td>
<td>199 dB</td>
</tr>
<tr>
<td>Satellite G/T</td>
<td></td>
<td>-1.5 dB K</td>
</tr>
<tr>
<td>Boltzmann’s Constant</td>
<td></td>
<td>-228.6 dBW-K-Hz</td>
</tr>
<tr>
<td></td>
<td>Upplink</td>
<td>72.6 dB-Hz</td>
</tr>
<tr>
<td></td>
<td>Carrier-to-Noise</td>
<td>72.6 dB-Hz</td>
</tr>
<tr>
<td></td>
<td>Density Ratio</td>
<td>72.6 dB-Hz</td>
</tr>
<tr>
<td>Downlink Frequency</td>
<td></td>
<td>2491 MHz</td>
</tr>
<tr>
<td>Satellite EIRP</td>
<td></td>
<td>53.6 dBW</td>
</tr>
<tr>
<td>Free Space Path Loss</td>
<td></td>
<td>191 dB</td>
</tr>
<tr>
<td>User G/T</td>
<td></td>
<td>-23.8 dB K</td>
</tr>
<tr>
<td></td>
<td>Downlink</td>
<td>67.4 dB-Hz</td>
</tr>
<tr>
<td></td>
<td>Carrier-to-Noise</td>
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</tr>
<tr>
<td></td>
<td>Density Ratio</td>
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</tr>
<tr>
<td>Total</td>
<td></td>
<td>66.25 dB-Hz</td>
</tr>
<tr>
<td></td>
<td>Carrier-to-Noise</td>
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</tr>
<tr>
<td></td>
<td>Density Ratio</td>
<td>66.25 dB-Hz</td>
</tr>
<tr>
<td>Margin</td>
<td></td>
<td>5 dB</td>
</tr>
<tr>
<td>If Required $\frac{E_b}{N_o} =$</td>
<td>15 dB</td>
<td>20 dB</td>
</tr>
<tr>
<td>Then Data Rate can be</td>
<td>42.2 kbps</td>
<td>13.3 kbps</td>
</tr>
</tbody>
</table>

2. Control Station-to-User Data

Using the available data bit rate, the user coordinates, identification tag and synchronization bits must be transmitted to each user that has requested a position fix. There are 58 bits required to represent the latitude, longitude, and altitude. Seven bits are allowed for degrees (where $2^7 = 128$ which will suffice to represent the user’s position if the user supplies to his machine the quadrant that he is in). There are 6 bits for minutes, and 6 bits for seconds, where $2^6 = 64$ will suffice to represent 60 values for both
latitude and longitude. Another 20 bits are required to represent the user altitude. In Figure 3, a 40 bit code word is used for synchronization of the TDM frame. A 40 bit code word is a design parameter which is used in other communication systems. Each user information frame consists of two 8 bit flags, position information of 58 bits, general information of 20 bits and a user ID of 10 bits. The 8 bit flags provide the start and end of the user data.

![TDM Frame Diagram](image)

**Figure 4.** TDM Frame (Control Station to Users)

3. **Satellite-to-User Downlink**

The data link from the control station to the user is completed with the satellite-to-user downlink at a frequency bandwidth of 6.27 MHz between 2493.73 MHz and 2500 MHz. The satellite provides relay service only and does not process the data that is passed through.
III. SYSTEM II

A. SYSTEM II DESCRIPTION

System II will employ 3 or 4 satellites in the same arrangement and positions as System I. System II differs from System I in the technique by which the differential distances are obtained. System II users will transmit a user unique pseudorandom ranging code to the control station via a satellite relay. The time delays between the arrivals of the codes from each of the satellites will determine the differential distances of the paths. Each time the user transmits a code, the code will travel to the satellites at the speed of light. The satellite that is closest will receive the code first and relay the code to the central control station. The closest satellite will provide the shortest time delay. The satellite that is furthest away from the user and the central control station will have the longest delay. It is these differences in the times of arrivals that determine the distance differentials as we did in System I. In System II, a total user-to-satellite and satellite-to-control station differential distance is calculated from the total time differential. The central control station assumes the task of measuring the differences in the round trip distance between itself and the user via one satellite, arbitrarily designated as satellite one, and each of the other satellites. We define these differential distances as

\[ b_i = (l_i + d_i) - (l_i + d_i), \quad i = 1, 2, ..., n \]  

(59)

where \( l_i \) is the distance from the central control station to the satellite

\[ l_i^2 = (x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2, \quad i = 1, 2, ..., n \]  

(60)

and \( d_i \) is the distance from the user to the satellite

\[ d_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2, \quad i = 1, 2, ..., n \]  

(61)

For \( n = 4 \), Eqs. (9) - (11) combine to form a set of four independent equations and four unknowns, namely, \( x, y, z, \) and \( d_i \) These four equations are

\[ (d_i - b_i + l_i - l_i)^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2, \quad i = 1, 2, 3, 4 \]  

(62)

For \( n = 3 \), Eq. (4) is used in addition to Eq. (12) for \( i = 1, 2, 3 \) assuming the user altitude \( h \) is known.
After the position solution is obtained, the control station relays the user position to the user via a satellite link as in System I. The frequency assignment and bandwidth allocated for each of the satellite links listed below are specified by the Federal Communications Commission in [Ref. 2].

- **User-to-Control Station**
  
  **User-to-Satellite Uplink**
  
  Frequency: 1610 - 1626.5 MHz  
  Bandwidth: 16.5 MHz  

  **Satellite-to-Control Station Downlink**
  
  Frequency: 5150 - 5166.5 MHz  
  Bandwidth: 16.5 MHz

- **Control Station-to-User**
  
  **Control Station-to-Satellite Uplink**
  
  Frequency: 6524.5 - 6541 MHz  
  Bandwidth: 16.5 MHz

  **Satellite-to-User Downlink**
  
  Frequency: 2483.5 - 2500 MHz  
  Bandwidth: 16.5 MHz

- **Control Station-to-Satellite Command Link**
  
  Frequency: 6526.5 - 6541.5 MHz  
  Bandwidth: 15 MHz

### B. USER-TO-CONTROL STATION LINK

1. **Uplink Bandwidth**

   The frequency bandwidth allocated for the user-to-control station data link is 16.5 MHz from 1610 MHz to 1626.5 MHz. Within this bandwidth, the user must transmit a PN ranging code, an identification code, general information, and user altitude in the three satellite configuration. The user will transmit a standard navigation 1,023 chip Gold code sequence as the ranging code with the other data. The calculations for the user-to-control station are completed as was discussed in System I, utilizing Eqs. (5) - (8). System II design parameters are the same as System I. The total carrier-to-noise ratio less the 2 dB margin will support data rates of 57 kbps and 18 kbps for required signal-to-noise ratios of 15 dB and 20 dB, respectively. The link analysis is tabulated in Table 7.

2. **User-to-Control Station Data**

   As in System I, the user must transmit a 10 to 17 bit user identification number, 20 bits for additional information, and 20 bits for altitude if only three satellites are used. Using a standard modulation scheme, this information would be modulated by QPSK and
Table 7. **SYSTEM II USER-TO-CONTROL STATION LINK ANALYSIS**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Uplink Frequency</td>
<td>1618 MHz</td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>4 dB</td>
</tr>
<tr>
<td>Transmitted Power</td>
<td>80 W</td>
</tr>
<tr>
<td>Carrier EIRP</td>
<td>23 dBW</td>
</tr>
<tr>
<td>Free Space Path Loss</td>
<td>190 dB</td>
</tr>
<tr>
<td>Satellite G T</td>
<td>+3 dB K</td>
</tr>
<tr>
<td>Boltzmann's Constant</td>
<td>-228.6 dBW K-Hz</td>
</tr>
</tbody>
</table>

Uplink

| Carrier-to-Noise Density Ratio | 64.6 dB-Hz |

Downlink Frequency | 5150 MHz |
| Satellite EIRP    | 36 dBW   |
| Free Space Path Loss | 198 dB  |
| Control Station G T | 20 dB K  |

Downlink

| Carrier-to-Noise Density Ratio | 86.6 dB-Hz |

Total

| Carrier-to-Noise Density Ratio | 64.57 dB-Hz |

| Margin                  | 2 dB        |
| If Required $\frac{E_b}{N_0} = 15$ dB | 20 dB       |
| Then Data Rate can be   | 57 kbps     | 18 kbps     |

spread with the 1023 chip Gold code. The number of CDMA channels that the system will support is the same as in System I.

3. **Satellite-to-Control Station Downlink**

The frequency bandwidth allocated for the satellite-to-control station downlink of the user-to-control station data link is 16.5 MHz from 5150 to 5165.5 MHz. The satellite functions as a relay only as in System I.

**C. POSITION DETERMINATION**

The position of the user is determined in much the same manner as was done in the System I design. The main difference is that there are two distances that must be ac-
counted for in the calculation. In System I, the satellites transmit the ranging code and the differential distances between the satellite and the user had to be accounted for in position error. In System II, the user transmits the code so the user-to-satellite and satellite-to-control station differential distances must be taken into consideration when calculating the users position. The differential distance measurements are made using Eq. (59).

The central control station-to-satellite differential distance error does not contribute to the user position error as greatly as do the satellite-to-user distance differentials. The central control station would be suitably located away from the equator to minimize the control station-to-satellite differential distance measurement errors that are inherent to the system as was discussed in System I. In the three satellite configuration, where altitude must be entered for position determination, the central control station knows its exact altitude, so the only altitude error entering into the position determination is from the user. In the four satellite configuration, satellite position errors between the user-to-satellite and central control station-to-satellite do increase the user position error but not as greatly as first thought. After conducting a computer simulation model run of 1000 iterations with randomly varying satellite position errors between + 20 and - 20 meters as was conducted in [Ref. 1]; approximately a 1 to 2 meter additional error was added to the overall position error of System II in comparison with System I user position error. It was originally thought that a doubling of the position error would occur since an error in the satellite position affects both the user-to-satellite and the central control station-to-satellite paths, but was not the case. For this reason, the System II design is a viable alternative to System I design.

D. CONTROL STATION-TO-USER DATA LINK

1. Uplink Bandwidth

The System II control station-to-user data link is designed the same as the data link in System I. The big advantage in System II is that the bandwidth allocated for the satellite-to-user downlink is used solely for the control station to user data link. The total bandwidth of 16.5 MHz can be utilized for data transmission because there is no ranging code to be transmitted by the satellites. In System I, the ranging code occupied 10.23 MHz of the 16.5 MHz bandwidth allocated for the satellite to user link, leaving only 6.27 MHz for the control station-to-user data link.
2. Control Station-to-User Data

As in System I, the user ID, latitude, longitude, altitude and general information must be transmitted to the user. The System I TDM frame structure will also be utilized in System II. In System II, the number of users that can be serviced each second could be increased even further due to the 10.23 MHz increase in the available bandwidth. The increase in the number of users will depend on the protocols used and is beyond the scope of this thesis. The link calculations are completed using Eqs. (5) - (8) and System I link parameters. The values obtained in Table 8 are identical to those in System I for the same link parameters. Table 8 is furnished to present the fact that the link calculations do not change.

3. Satellite to User Downlink

The satellite to user downlink occupies the full bandwidth of 16.5 MHz from 1610 to 1626.5 MHz. The satellite operates as a passive relay only.
Table 8. SYSTEM II CONTROL STATION-TO-USER LINK ANALYSIS

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td><strong>Uplink Frequency</strong></td>
<td>6533 MHz</td>
</tr>
<tr>
<td><strong>Antenna Gain</strong></td>
<td>44.5 dB</td>
</tr>
<tr>
<td><strong>Transmitted Power</strong></td>
<td>1 W</td>
</tr>
<tr>
<td><strong>Carrier EIRP</strong></td>
<td>44.5 dBW</td>
</tr>
<tr>
<td><strong>Free Space Path Loss</strong></td>
<td>199 dB</td>
</tr>
<tr>
<td><strong>Satellite G T</strong></td>
<td>-1.5 dB K</td>
</tr>
<tr>
<td><strong>Boltzmann’s Constant</strong></td>
<td>-228.6 dBW K-Hz</td>
</tr>
<tr>
<td><strong>Uplink</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Carrier-to-Noise</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Density Ratio</strong></td>
<td>72.6 dB-Hz</td>
</tr>
<tr>
<td><strong>Downlink Frequency</strong></td>
<td>2491 MHz</td>
</tr>
<tr>
<td><strong>Satellite EIRP</strong></td>
<td>53.6 dBW</td>
</tr>
<tr>
<td><strong>Free Space Path Loss</strong></td>
<td>191 dB</td>
</tr>
<tr>
<td><strong>User G T</strong></td>
<td>-23.8 dB K</td>
</tr>
<tr>
<td><strong>Downlink</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Carrier-to-Noise</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Density Ratio</strong></td>
<td>66.25 dB-Hz</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Carrier-to-Noise</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Density Ratio</strong></td>
<td>61.73 dB-Hz</td>
</tr>
<tr>
<td><strong>Margin</strong></td>
<td>5 dB</td>
</tr>
<tr>
<td><strong>If Required</strong></td>
<td></td>
</tr>
<tr>
<td>$\frac{E_s}{N_0}$</td>
<td>15 dB</td>
</tr>
<tr>
<td><strong>Then Data Rate can be</strong></td>
<td>42.2 kbps</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Then Data Rate can be</strong></td>
<td>13.3 kbps</td>
</tr>
</tbody>
</table>
IV. SYSTEM COMPARISONS AND CONCLUSIONS

System II is the simpler system in many ways. A System II user sends the ranging code to the control station so the time of transmission does not require any synchronization. The satellite does not require an on-board clock or a code generator and transponder to send a ranging code to the user. There exists the possibility of using existing satellites or sharing future satellites. The user equipment is also simpler in that the user does not have to make time measurements of the satellite-to-user time differentials. By reducing the complexity of the user equipment, the cost to each user would also be reduced.

The number of users that could simultaneously be serviced by System II is also greater because the available bandwidth of the control station to user is dedicated totally to the link. In System I, the satellite-to-user bandwidth must also support the transmission of the ranging code from the satellite to the user reducing the bandwidth of the control station-to-user link. Although there is a slight increase in position error that is inherent to System II, the position accuracy is well within expected tolerances required of many civilian users.

This thesis has proposed the concepts of two Global Satellite Location and Tracking Systems designed for the civilian user. The advantages of the systems are the low cost, user equipment simplicity, and accurate position determination. The user systems are cheaper because the computational hardware and software are maintained at the central control station. This eliminates the need for installing an expensive computer in each user transceiver.

System II provides many possibilities for future design due to the fact that satellites already in space could be used as communications relays for the system. This eliminates the need to design and launch special satellites into space for location and tracking purposes only.

Geostationary Satellite Location and Tracking Systems are viable systems for civilian position determination.
V. LIST OF REFERENCES


## VI. INITIAL DISTRIBUTION LIST

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<thead>
<tr>
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<th>Copies</th>
<th>Distribution List</th>
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| 4.  | 5      | Professor Tri T. Ha, Code 62Ha  
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